Area Conditions and Positive Incentives: Engaging Local Communities to Protect Forests

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Millions of smallholder farmers produce global commodities, and some burn forest to expand farms and escape poverty. This paper guides organizations seeking to prevent deforestation and benefit local farmers in a target area. The usual approach is to offer each farmer incentives (e.g., price premiums or farm inputs) if he foregoes deforestation. An alternative is to offer incentives to local farmers collectively, conditional on no deforestation in the area. We propose area *no-use*: offer incentives collectively if no one generates income from (recently) deforested land in the area. This novel condition enables forest regeneration after accidental fire and sourcing from the area in compliance with new EU regulations. We model local farmers' decisions on deforestation and whether to block others from using deforested land for income, allowing cooperation within limited groups. With innovations in cooperative game theory, we identify area characteristics for which conditional incentives prevent deforestation and, ideally, compensate every farmer for forgoing deforestation. With full cooperation, area no-deforestation is best. With limited cooperation, area no-use is best for preventing deforestation when blocking use is easy and compensating locals is not required; otherwise, perfect individual incentives may be needed. Lowering blocking costs and improving cooperation help, but not universally. Ensuring that blocking attempts succeed is important and cannot always be replaced by lower blocking costs. With limited cooperation, it is more effective to combine area conditions with efforts to help farmer groups coordinate on mutually beneficial outcomes or to condition incentives on smaller sub-areas wherein locals fully cooperate.

1. Introduction

Commodity buyers including Unilever, Cargill, and Nestlé have pledged to have zero deforestation and uplift the smallholder farmers in their supply chains (Newton and Benzeev 2018). They are struggling to meet these dual commitments in the tropics, where millions of smallholders who produce agricultural commodities live in poverty, and some burn forest (often illegally) to expand their farms as a pathway out of poverty (Tyukavina et al. 2018, Lambin and Furumo 2023). Tropical deforestation causes enormous emissions of CO_2 and smoke, and harms public health, biodiversity and watersheds (Balboni et al. 2023). Motivated by practitioners' calls for guidance (Denier et al. 2015) and by field research in Indonesia and Thailand, this paper proposes a novel conditional incentive and a framework for choosing the best conditional incentive to prevent deforestation and raise incomes for all farmers in a sourcing area of interest.

Commodity buyers have attempted to meet their dual commitments with an *individual conditional incentive*, whereby a farmer whose land plots remain deforestation-free is rewarded with a positive incentive such as a price premium, subsidized inputs or equipment, a loan, or a cash payment. Unilever's palm oil program in Indonesia exemplifies the approach: in collaboration with NGO partners, Unilever has mapped the plots of 47,000 smallholders and monitors these daily for deforestation via a live dashboard that integrates satellite and radar feeds; farmers whose plots remain deforestation-free are eligible for certification to receive a price premium for fruit produced on those plots (Unilever 2025a). Appendix §EC.1 provides other examples.

In practice, such individual schemes face multiple obstacles. To participate, a smallholder must demonstrate clear property rights over his land plot, which is difficult in the many deforestation-prone regions with ambiguous or overlapping property rights. Plot mapping, verification and enrollment typically leverage GPS technology but require extensive local work and manual processes. This is costly, and the cost is multiplied by the need to enroll large numbers of smallholders and many small plots. Unless all forested land in his area is mapped and enrolled, an individual participant might clear and produce on an *unmapped* forest plot (or source from a neighbor who does so) and sell the additional output under the guise of improved productivity on his mapped plot. That would violate the commodity buyer's commitment to zero deforestation. Individuals often clear and farm plots within national parks and other *publicly-owned* forest; an individual scheme cannot stop that illegal deforestation. Lastly, a major obstacle is that the incentive that a commodity buyer can offer is typically *imperfect*: it has too little value for some farmers, for whom forest clearing is more profitable. For example, Unilever and other prominent palm oil buyers rely on the Roundtable on Sustainable Palm Oil (RSPO) to determine the price premium for certified palm farmers in $ per tonne of fresh fruit bunches; the RSPO price premium incentive has too little value especially for the farmers with the smallest landholdings, as we also find in our empirical analysis in §EC.7.

An emerging alternative is an *area* no-deforestation scheme: smallholders within a specified area receive a positive incentive if and only if *no deforestation* occurs in the area. The area may be a jurisdiction. For example, in the Sabah province of Malaysia, the Sabah government and palm oil buyers are leading an initiative to certify *all* palm oil producers located in Sabah province so they gain access to the RSPO price premium provided that no deforestation occurs in that entire province. As an alternative, the area may be the landholdings of the farmers in an agricultural cooperative. The RSPO website lists cooperatives with Independent Smallholder [Group] Certification, which qualifies all cooperative members to receive the RSPO price premium provided that no deforestation occurs on any cooperative member's landholdings. §EC.2 describes these and other examples in detail.

The success of an area no-deforestation scheme may hinge on the ability of the smallholders in the area to cooperate. In some rural communities, smallholders can coordinate on key decisions for the management of agriculture and natural resources, and share in the costs and benefits. Such cooperation can arise within an extended family or ethnic group, in a cohesive village, or in a farmer organization like an agricultural cooperative or savings and loan association (Durlauf and Fafchamps 2005), and is fostered by a traditional culture of mutual assistance in many rural settings. But where social ties are weak, a single farmer for whom the incentive carries insufficient benefits might clear forest and thereby void the benefits for everyone.

An area no-deforestation condition could be violated by one individual setting a fire. In Indonesia and Thailand, among many other deforestation-prone regions, locals ordinarily use fire to clear forest and then establish agriculture on the cleared land (van Wees et al. 2021). Setting fire in forest is illegal yet common because this is the quick and easy way to clear forest, and a culprit is difficult to catch in the act of ignition, as would be necessary to hold him responsible and prevent any forest loss. Especially in Indonesia's peat soil forest, extinguishing a fire is difficult even with a rapid, resource-intensive response from a crew trained and equipped to fight fire (Kopansky 2018). In a randomized controlled trial in Indonesia, Falcon et al. (2022) offered villages a cash incentive conditional on no fire-induced deforestation within a village's administrative area. Overall, Falcon et al. did not find significant prevention of fire-induced deforestation from the conditional incentive, but the effects were heterogeneous. Some villages earned the conditional incentive through deliberate socialization and coordination among the people of a village, but in larger villages, a small number of individuals set fires and voided the incentive for all farmers in the village.

We propose a novel *area no-use* condition. The idea is to reward all smallholders in an area if *no one derives economic benefit* from clearing forest within the area. This allows more flexibility than the area no-deforestation condition: compliance is achieved either by preventing any forest clearing or – if clearing occurs – by ensuring that no timber is extracted and no production takes place on cleared land. Especially for areas with clearing by fire or illegal use of public land, this distinction matters because preventing clearing may be impossible, whereas preventing use of deforested plots can be easier. For instance, depending on local circumstances, communities can prevent use by reporting culprits to authorities, applying peer pressure (e.g., public censure or ostracism), or through material sanctions like occupying the cleared land, cutting down seedlings in a new plantation, or blocking the harvest and transport of produce (Villadiego 2017, Berenschot et al. 2021). An area no-use condition also allows smallholders to retain the reward in the event of accidental fire, if they prevent use of cleared land. It is similar in spirit to the recovery-plan policies adopted by some commodity buyers: Unilever, for example, allows a suspended supplier who cleared forest to regain market access by ceasing production on the cleared land and undertaking verified actions to restore the forest (Unilever 2022). However, unlike a recovery plan for an *individual* supplier, an incentive conditional on *area* no-use could motivate collective action by smallholders in the area to prevent anyone who cleared forest from using cleared land for income. For brevity, we use *"block"* to refer to the act of preventing an individual who cleared forest from thereby generating income.

We develop a stylized model to evaluate individual and area conditional incentives. We consider a given area wherein *locals* could clear forest to produce an agricultural commodity. The locals are offered a positive incentive if they comply with a specified condition. Locals are heterogeneous in the value they derive from the incentive vs. from clearing forest in the area, and some locals may prefer the latter. Locals are organized into an exogenous partition of *families*; locals in the same family form coalitions wherein they *cooperate* by coordinating decisions and sharing benefits and costs, whereas no cooperation is possible across coalitions (and therefore, across families). The resulting coalitions engage in a non-cooperative game: each decides whether to clear forest and then decides whether to incur costs to block locals outside the coalition who cleared forest. Under an area no-use condition, all locals retain the incentive if all locals who cleared forest are blocked. The model captures important primitives, including limitations on cooperation (family structure), each local's net income from the incentive vs. from deforestation, and the blocking cost. We study how these primitives and the conditional incentive influence the *outcome* in terms of coalitions formed, allocation of net income among the locals in each coalition, and whether or not deforestation occurs in the area.

To predict the possible outcomes, we contribute a new solution concept: the pessimistic recursive core for a game with multiple partition functions and limited cooperation. This generalizes a concept due to Kóczy (2007) to allow for limitations on cooperation and multiple equilibria in the non-cooperative game. The premise of the recursive core is that locals who form a coalition anticipate that locals outside that coalition will act to maximize their own net income by engaging in a smaller, "residual cooperative game." The recursive core is obtained by solving recursively over residual games, and contains those outcomes at which no locals think that they could obtain higher net income by forming different coalitions. In case the core of a residual game has multiple elements or the non-cooperative game among coalitions has multiple equilibria, we assume that locals forming a coalition have *pessimistic* beliefs about which outcome will occur, and thus plan on the worst-case possible net income for their coalition. Pessimism gives rise to the largest possible recursive core, and we make conservative predictions about the performance of a conditional incentive by requiring such performance in *all* outcomes in the pessimistic recursive core.

We consider two performance requirements for a conditional incentive: to *prevent deforestation* in the area and to *compensate* locals for the opportunity cost of forgone deforestation. The first requirement is motivated by commodity buyers' commitments to zero deforestation and the European Union's Deforestation-Free Regulation (EUDR), which requires firms selling products derived from palm oil, coffee, cocoa, cattle, rubber, soy, or wood to ensure that their inputs are not produced on land deforested after 2020 (EU 2023). Because the incentive is a positive reward, adopting the incentive and avoiding deforestation in the area will improve each local's income, but the improvement may be smaller than he could have obtained through deforestation. Related literature makes a strong ethical case for the additional requirement – compensation – and criticizes the EUDR for preventing deforestation without compensation (Wunder et al. 2018, Zhunusova et al. 2022). Not every commodity buyer will adopt the compensation requirement; this is most appropriate for firms whose mission is centered on fair treatment for farmers, like chocolate manufacturer Tony's Chocolonely.

Considering the requirement to prevent deforestation and the (added) idealistic requirement to compensate, we compare several conditional incentive schemes and develop guidelines for the best choice. The three schemes are (1) area no-deforestation, (2) area no-use and (3) an individual no-deforestation condition for each local in the area. For each scheme, we characterize the (pessimistic recursive) core and – if the core is nonempty – conclude that the scheme prevents deforestation if all outcomes in the core have no deforestation, and it compensates if all outcomes in the core make each local better off than with deforestation. We characterize the cases (set of model parameters) in which each scheme prevents deforestation and the subset of those cases wherein it also compensates. We then identify the scheme that is *best*, i.e., feasible for the largest set of model parameters, and discuss implications related to cost-effectiveness.

The degree of local cooperation plays a critical role in our analysis; we distinguish two cases, depending on the number of families in our model. With a single family, locals are capable of forming any coalition, including the grand coalition, and we say that the area has "*full cooperation*." This case is exemplified in the small, cohesive villages wherein locals cooperated to win the cash incentive by preventing fire-induced deforestation (Falcon et al. 2022). The full cooperation case presumably also arises when the area is the landholdings of members of a smaller agricultural cooperative. In the alternative case, with two or more families, we say that the area has "*limited cooperation*." This case presumably arises when the area is large and has many locals like Sabah province, which has tens of thousands of smallholder palm farmers. This could also arise in large agricultural cooperatives that have many distinct farmer groups.[1] The limited cooperation case may also arise because the area has an entrant, i.e., new immigrant or investor who lacks social ties in the area.

For an area with full cooperation, the *area no-deforestation condition* is best to prevent deforestation and to compensate. This works with any incentive that is more valuable for the locals in the area overall than to engage in deforestation. Locals redistribute the incentive if needed for each local to willingly forgo deforestation. Therefore, in this case with full cooperation, the requirement to compensate does not increase the cost to prevent deforestation.

For an area with limited cooperation, if the blocking cost is not prohibitively high and compensation is not required, the *area no-use condition* is best. This prevents deforestation with any incentive that – for some family – is preferable to engaging in deforestation, and is sufficient to motivate some members of that family to enforce the area no-use condition by blocking use of deforested land by all other locals. The caveat to preventing deforestation with an incentive conditional on area no-use is in distributional equity. Locals with the highest opportunity cost to forgo deforestation – typically the poorest locals, with the least land – may be under-compensated. In fact, the most cost-effective

[1] §EC.2 discusses examples of Indonesian cooperatives, including a 29-member cooperative (arguably a case with full cooperation) and a 508-member cooperative, organized into 34 groups (arguably a case with limited cooperation).

scheme favors locals or families with low values from deforestation, or large families that would be costly to block, while leaving other locals unpaid, thereby widening local inequalities.

In all other cases with limited cooperation – specifically, if the blocking cost is prohibitively high or compensation is required – *individual conditions* outperform the area conditions. Furthermore, the requirement to compensate would not increase the cost to prevent deforestation. To prevent deforestation with individual conditions requires a perfect incentive: each local's gain with the incentive must exceed his opportunity cost from forgone deforestation. That also makes such incentives impractical, due to the expense or difficulty of directly providing enough value to each individual.

Therefore we also study hybrid schemes that partition the area and tailor conditions to sub-areas. We prove that a hybrid scheme yields no benefit in an area with full cooperation. But in an area with limited cooperation, we prove that implementing area no-deforestation conditions on smaller sub-areas chosen to enable full cooperation can prevent deforestation and compensate locals.

We show that our main results also hold in other settings: when forest is cleared through logging (rather than fire) and attempts to clear forest can be blocked, when blocking use can fail probabilistically, with heterogeneous blocking costs, or if locals are optimistic rather than pessimistic.

We calibrate our model with data that we collected in 58 villages in East Kalimantan, Indonesia, a region prone to deforestation for palm oil expansion and characterized by strong social ties within villages. We develop a detailed model of a household's decision-making and use survey data to estimate our model parameters. To make conservative predictions regarding the feasibility for a conditional incentive to prevent deforestation, we use an estimation procedure based on robust data envelopment analysis to deliberately overestimate the profits from converting forest to palm oil farms. As incentive, we consider a price premium paid per tonne of palm fresh-fruit bunches. East Kalimantan is divided into villages as smallest administrative units. Each village is a clearly-defined area that encompasses forest and palm farms. Therefore we consider applying area no-deforestation and area no-use conditions at the level of each village. We find that a single price premium sufficient to prevent deforestation in all 58 villages must vastly exceed RSPO price premiums. In contrast, much lower village-specific price premiums conditioned on area no-use can prevent deforestation in most villages and are also effective at deterring entrants from clearing forest.

This paper provides guidance for commodity buyers, governments, NGOs, carbon offset providers, water funds and other organizations that aim to prevent tropical deforestation and improve incomes for smallholder farmers (Denier et al. 2015). Following a literature review, §2 formulates our model. §3 defines the pessimistic recursive core and characterizes the set of model parameters (circumstances in the target area) for which each conditional incentive prevents deforestation, with vs. without compensation. §3.4 identifies the best conditional incentive for robust performance. §4 explains how our results generalize with alternative interpretations and model extensions. All proofs and the model calibration for villages in East Kalimantan are in the Electronic Companion. §5 draws conclusions.

1.1. Literature Review

Our study is broadly related to a growing literature in OM on agricultural value chains, surveyed in Sodhi and Tang (2014), Swaminathan and Deshpande (2021), Sunar and Swaminathan (2022), Dong (2021). Closest to our study are the sub-streams focused on preventing deforestation, helping farmers, and managing agricultural cooperatives.

On preventing deforestation, Orsdemir et al. (2019) show how a firm avoids buying illegal wood by instead buying a mill and its log inputs, and selling wood to competitors. Agrawal et al. (2022) analyze contract terms for offset credits, including for avoided deforestation. For an individual forest owner with private information about his potential income from clearing his forest, Li et al. (2022) recommend an effective linear payment scheme for forest conserved.

A growing body of OM research studies interventions to improve farmers' welfare. Some studies have focused on government tools, such as taxes or subsidies (Akkaya et al. 2021, Tang et al. 2023), financial access (Pay et al. 2022), and the dissemination of market-price information (Chen and Hall 2007). Other studies have considered interventions by private parties: NGOs that enable farmers to share information (Xiao et al. 2020) or get certified (Agrawal and Zhang 2024); commodity buyers that redesign sourcing contracts (de Zegher et al. 2019, Hu et al. 2019), or pay premiums and subsidize inputs (Chintapalli and Tang 2021). All of these studies model farmers as *non-cooperative* agents and offer incentives *unconditionally*; the research challenge is to find settings where such incentives do not backfire–e.g., by inducing oversupply, worsening equity, or spurring environmental harm. The positive incentive in our model could, in principle, correspond to any of these interventions. Different from these studies, we model *conditions* tied to the incentive that prevent negative spillover effects on the environment (deforestation) or on distributive justice, and we study how farmers' ability to cooperate influences the effectiveness of interventions.

Our work is also related to OM literature that discusses how farmers can improve their welfare by cooperating, typically through agricultural cooperatives. An et al. (2015) show how farmers can increase their profits when jointly choosing their production quantities, and Li et al. (2024) show how cooperation can improve farmers' joint decisions of how much land to allocate to distinct crops. Qian and Olsen (2020, 2022) model how farmers coordinate their quality, quantity, and financial decisions through a cooperative and provide an excellent survey of the literature on farmers' cooperatives. Boyabatli et al. (2021), chapters 3 and 12, consider cooperation in sharing water and knowledge. Different from these papers, we model an explicit lever (the families) that controls the farmers' ability to cooperate and we allow for arbitrary limited coalitions to form, leveraging cooperative game theory concepts to examine how this influences outcomes.

Cooperative game theory, surveyed by Nagarajan and Sošić (2008), has longstanding importance in OM literature, but has not been applied in agricultural OM. Recent advances involve the use of *farsighted* solution concepts in which players are strategic in forming a coalition and anticipate other players' response; see Tian et al. (2019). Most cooperative games in OM literature have characteristic function form: a coalition's net income depends only on that coalition. The only OM paper that we are aware of analyzing a cooperative game in partition function form is Fang and Cho (2020), wherein manufacturers partition into coalitions to audit suppliers, and coalitions' net incomes are determined by the *unique* non-cooperative equilibrium in the coalitions' auditing efforts. This paper innovates in cooperative game theory by formulating a class of games with externalities, transferable utility, *limited cooperation*, and *multiple partition functions*, demonstrating the practical importance of this class of games, and generalizing the recursive core solution concept of Kóczy (2007). Because many OM models – including those related to agriculture, such as de Zegher et al. (2019), Pay et al. (2022), Alizamir et al. (2022) – exhibit such multiplicity, we hope that our novel solution concept proves useful and encourages broader use of cooperative game theory.

Our work also contributes to the literature on Payments for Ecosystem Services (PES), which studies schemes that pay landholders for verifiable conservation actions. Appendix §EC.3 reviews this extensive literature and situates our contribution within it.

1.2. Notation

For a set N, $\mathbb{R}^N$ denotes the set of vectors with real components indexed by the elements of N, and $\{0,1\}^N$ denotes the set of vectors with binary components indexed by the elements of N. For a vector x with components indexed by some set N and a subset $S \subseteq N$, we define $x(S) := \sum_{i \in S} x_i$ and x_S as the subvector of x with components corresponding to indices $i \in S$. Also, x_{-S} denotes the vector obtained by removing components $i \in S$ from x, and $[x_S, x_{-S}]$ denotes the vector where components in S are replaced with values x_S. (This notation also applies to matrices, interpreted on rows: for $M \in \mathbb{R}^{m \times n}$, $[m_i, M_{-i}]$ represents the matrix obtained from M by replacing its i-th row with the row vector m_i.) A *partition* π of a set N is a set of mutually exclusive subsets whose union includes all the elements of N, i.e., $\bigcup_{S \in \pi} S = N$ and $S \cap H = \emptyset$ for all $S, H \in \pi$ with $S \neq H$. The set of all partitions of set N is denoted by Π_N. For $R \subseteq N$ and partitions $\pi \in \Pi_R$ and $\bar{\pi} \in \Pi_N$, we say π is "finer than" $\bar{\pi}$ if for every $S \in \pi$, there exists $S' \in \bar{\pi}$ such that $S \subseteq S'$. We use $\Pi_R^{\preceq \bar{\pi}} \subseteq \Pi_R$ to denote the set of all partitions $\pi_R \in \Pi_R$ finer than $\bar{\pi}$.

2. Model Formulation

Consider the challenge of preventing deforestation in an area by offering a positive incentive to "locals" who meet a specified condition. (By "locals," we refer to all parties who could profit by deforesting land in the area.) In response to the conditional incentive, locals may form coalitions. Next, locals decide whether to deforest land in the area. Lastly, locals observe those who deforest land, and decide whether to block them from using this land to generate income. We describe this formally below.

Coalitions. Locals can form limited coalitions that allow them to cooperate. With $\mathcal{L}$ denoting the set of all locals, a *coalition* $S \subseteq \mathcal{L}$ is a set of locals who *cooperate*, i.e., make decisions to maximize their aggregate net income and make transfers among themselves to implement any agreed-upon allocation of that aggregate net income. Coalition formation is constrained: $\mathcal{L}$ has an exogenous partition $\bar{\pi} \in \Pi_{\mathcal{L}}$ and every coalition S must satisfy $S \subseteq F$ for some $F \in \bar{\pi}$. We refer to any $F \in \bar{\pi}$ as a "family." Coalition formation thus results in a partition π of locals that is finer than $\bar{\pi}$, so that $\pi \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$. As an extreme case, the partition could contain coalitions consisting of a single local, $\pi = \{\{\ell\} : \ell \in \mathcal{L}\}$. As the opposite extreme case, the partition could be $\pi = \bar{\pi}$. To simplify notation, we omit repeating the domain $\Pi_{\mathcal{L}}^{\prec\bar{\pi}}$ from which π is chosen when no confusion can arise.

Deforestation Decisions. In each coalition $S \in \pi$, the locals jointly decide whether each local $\ell \in S$ will deforest land in the area to *potentially* generate income – an action denoted by $d_\ell = 1$, with cost $c_\ell > 0$ – or not, denoted by $d_\ell = 0$. We use $d_H := (d_\ell)_{\ell \in H} \in \{0,1\}^H$ to denote the deforestation decisions of all locals in set $H \subseteq \mathcal{L}$, and $d := d_{\mathcal{L}}$ to denote the deforestation decisions of all locals. All locals observe the deforestation decisions d.

Blocking Decisions. To "*block*" a local is to prevent him from using land that he deforested to generate income. In each coalition $S \in \pi$, the locals jointly decide whether to block other locals $\ell \in \mathcal{L} \setminus S$. We represent the blocking decisions by the locals i in a coalition S as the matrix $B_S(d) \in \{0,1\}^{S \times \mathcal{L}}$ and the blocking decisions for all locals $\ell \in \mathcal{L}$ as the matrix $B(d) \in \{0,1\}^{\mathcal{L} \times \mathcal{L}}$. In either matrix, $B_{i\ell} = 1$ if and only if i blocks ℓ, $B_{i\ell} = 0$ if i and ℓ belong to the same coalition $S \in \pi$, and $\max_{i \in \mathcal{L}} B_{i\ell} = 1$ indicates that ℓ is blocked from using deforested land. B and B_S depend on d because blocking is done after observing deforestation decisions; when no confusion can arise, we omit the dependency on d. The cost to block *one* local is $\eta > 0$, so local $i \in \mathcal{L}$ incurs total blocking cost $\eta \cdot \sum_{\ell \in \mathcal{L}} B_{i\ell}$.

Conditions. Locals receive the incentive if and only if their deforestation and blocking decisions comply with the specified condition $\mathbb{C}(d, B) \in \{\text{yes}, \text{no}\}^{\mathcal{L}}$, where $\mathbb{C}_\ell(d, B)$ indicates whether local ℓ receives the incentive. We consider three conditions:

1. *Individual condition,* $\mathbb{I}$. Local ℓ receives the incentive if and only if he does not individually deforest land: $\mathbb{I}_\ell(d, B) = \text{yes} \;\Leftrightarrow\; d_\ell = 0$.
2. *Area no-deforestation condition,* $\bar{\mathbb{D}}$. Each local $\ell \in \mathcal{L}$ receives the incentive if and only if no local deforests land: $\bar{\mathbb{D}}_\ell(d, B) = \text{yes} \;\Leftrightarrow\; d_i = 0, \forall i \in \mathcal{L}$.
3. *Area no-use condition,* $\bar{\mathbb{U}}$. Each local $\ell \in \mathcal{L}$ receives the incentive if and only if no local uses land that he deforested to generate income: $\bar{\mathbb{U}}_\ell(d, B) = \text{yes} \;\Leftrightarrow\; d_i \cdot \left(1 - \max_{g \in \mathcal{L}} B_{gi}\right) = 0, \forall i \in \mathcal{L}$.

With the individual Condition, whether a local receives the incentive depends only on his individual deforestation decision, and any interactions between locals are therefore irrelevant.

With either area condition, whether a local is compliant depends on the decisions of other locals. The area no-deforestation Condition $\bar{\mathbb{D}}$ requires that no locals deforest land, so blocking use of deforested land is irrelevant. The area no-use Condition $\bar{\mathbb{U}}$ is weaker and is satisfied if $\bar{\mathbb{D}}$ is satisfied or if every local that deforested land is blocked.

Locals' Incomes and Allocations. Let v_ℓ^{sq} denote the *status quo* income that local ℓ would obtain without deforestation and without the incentive.

If local ℓ receives the incentive, his income increases by $I_\ell \geq 0$, and his *net income* – i.e., his income net of any deforestation and blocking costs – is:

$$v_\ell^{\text{sq}} + I_\ell - c_\ell \cdot d_\ell - \eta \sum_{i \in \mathcal{L}} B_{\ell i}.$$

Notice that in this case, local ℓ does not use deforested land to generate income. This arises because under any condition $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$, $\mathbb{C}_\ell(d,B) = \text{yes}$ implies that either $d_\ell = 0$ or $\max_{g \in \mathcal{L}} B_{g\ell} = 1$.

Conversely, if local ℓ deforests land and uses this to generate income, then he does not receive the incentive, and his net income is:

$$v_\ell^{\text{sq}} + D_\ell - \eta \sum_{i \in \mathcal{L}} B_{\ell i},$$

where D_ℓ is the net income gain with deforestation, which we assume to be strictly positive $D_\ell > 0$.

If locals do not collectively receive the incentive under an area condition, we assume that they have recourse and act rationally, so that they subsequently deforest land.[2]

In summary, accounting for incentive and deforestation gains, the net income of each local $\ell \in \mathcal{L}$ is:

$$V_\ell(d,B) := \begin{cases} v_\ell^{\text{sq}} + I_\ell - c_\ell \cdot d_\ell - \eta \sum_{i \in \mathcal{L}} B_{\ell i} & \text{if } \mathbb{C}_\ell(d,B) = \text{yes} \\ v_\ell^{\text{sq}} + D_\ell - \eta \sum_{i \in \mathcal{L}} B_{\ell i} & \text{if } \mathbb{C}_\ell(d,B) = \text{no}. \end{cases} \tag{1}$$

In each coalition $S \in \pi$, the locals $\ell \in S$ make joint deforestation decisions d_S and blocking decisions B_S to maximize their coalition's aggregate net income $V_S(d,B) = \sum_{\ell \in S} V_\ell(d,B)$. They also jointly decide how to split that aggregate net income, i.e., they decide the net income allocation a_ℓ that each local $\ell \in S$ should receive, so that $\sum_{\ell \in S} a_\ell = V_S(d,B)$.

Decision Timeline and Overall Game Formulation. Given a conditional incentive $(I, \mathbb{C})$, the locals first organize themselves into a partition $\pi \in \Pi_{\mathcal{L}}^{\prec \bar{\pi}}$ of coalitions. The initial formation of coalitions and allocation of net income a_ℓ to each ℓ within each coalition is a cooperative game with transferable utility and externalities, which we refer to as *the cooperative game*. Subsequently, the coalitions $S \in \pi$ engage in a two-stage, non-cooperative game – which we refer to as *the non-cooperative game* – wherein the locals in each coalition $S \in \pi$ choose their deforestation decisions d_S followed by their blocking decisions $B_S(d)$ to maximize their coalition's net income $V_S(d,B)$.

[2] When locals do not receive the incentive, any local ℓ who initially set $d_\ell = 0$ would earn net income $v_\ell^{\text{sq}} - \eta \sum_{i \in \mathcal{L}} B_{\ell i}$ if he did *not* subsequently deforest land, whereas he would earn a larger net income with deforestation because $D_\ell > 0$.

An *outcome* of the overall game is a partition π, allocation of net income $a \in \mathbb{R}^{\mathcal{L}}$, and a subgame-perfect equilibrium for the non-cooperative game corresponding to partition π.

Performance Requirements. We define two performance requirements for an outcome:

- The outcome has no deforestation: $d_\ell = 0$ for all $\ell \in \mathcal{L}$.
- The outcome *compensates*: $a_\ell \geq v_\ell^{\mathrm{sq}} + D_\ell$ for all $\ell \in \mathcal{L}$.

To meet a commodity buyer's zero-deforestation commitment, a conditional incentive must induce an outcome with no deforestation. We will prove that in an outcome with no deforestation, a commodity buyer meets a dual commitment to increase locals' incomes in the weak sense that $a_\ell \geq v_\ell^{\mathrm{sq}}$ for every $\ell \in \mathcal{L}$, meaning that each local's net income is at least his status quo net income without the incentive and without deforestation. This weak income improvement arises because the incentive is positive ($I \geq 0$), yet proof is required to establish this for each area condition.

The more just, stronger version of a commodity buyer's dual commitments is to induce an outcome with no deforestation that *compensates*, meaning that each local is reimbursed for their opportunity cost of forgoing deforestation. The net income a_ℓ allocated to each local exceeds the net income that he would have obtained with deforestation.

We formalize the non-cooperative game in §3.2 and formalize the cooperative game and recursive core solution concept in §3.3 to predict the possible outcomes induced by a conditional incentive.

2.1. Discussion of Modeling Assumptions

Deforestation and Blocking Decisions. We focus on the decision of *whether* to deforest land because that is consistent with buyers' *zero* deforestation commitments. However, the key parameters c_ℓ and D_ℓ account for the *optimal amount* of land to deforest and *optimal decisions* regarding land use that maximize local ℓ's net income. Our model and results are general because we impose no assumptions on the underlying mechanisms that lead to these parameters. In §EC.7, we demonstrate how to estimate such parameters through field study and a detailed structural model of a local's decision-making; the reader may find it helpful to briefly refer to §EC.7 to better understand these parameters. We do not explicitly model any punitive measures on a local ℓ who generates income through deforestation, but those could be represented by decreasing D_ℓ.

Our model is highly relevant for an area where forest is cleared by fire to establish agriculture, illegally, on public land or on land without a clear property right – a common setting for tropical deforestation. For such an area, c_ℓ primarily reflects the cost of establishing agriculture on cleared land (e.g., planting palm seedlings) because igniting a fire requires little cost or effort. Blocking deforestation by fire is extremely difficult – infeasible in our main model – whereas blocking illegal *use* of deforested land for agricultural production is more sensible. To block a local from producing commodities might be accomplished, for example, by reporting him to authorities, applying peer pressure, or through material sanctions as discussed in the introduction. One may interpret η as the cost to block one local by whatever manner is easiest and least costly. This is an important area characteristic. (In an area with strong property rights, blocking a local from generating income by clearing forest on his own land may be impossible, which corresponds to $\eta = +\infty$.) §EC.6.4 extends our results when the blocking cost depends on the family that engages in the blocking, i.e., the cost is η_F for all $\ell \in F$ and all $F \in \bar{\pi}$. Our results extend to cases where the cost depends on the local who is blocking and who is being blocked ($\eta_{\ell i}$ for $\ell, i \in \mathcal{L}$) at the expense of more complex analysis.

Incentive. Summarizing the incentive's effect through I_ℓ without specific functional forms makes our model broadly applicable. For instance, the incentive could be a price premium (as in our empirical case study), access to subsidized inputs or credit, or a carbon offset payment commensurate with the CO_2 emissions avoided by preventing deforestation. Alternatively, the incentive could be collective in nature, such as an investment in assets shared by the local community (schools, roads, equipment) or granting the community land tenure and land-use rights.

In §4, we discuss how our results extend as we relax several of our assumptions.

3. Main Results

For any given incentive with positive values I and condition $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$, §3.1-3.3 characterize the possible outcomes in terms of the performance requirements. §3.1 focuses on the individual condition. §3.2-3.3 focus on the area conditions. In §3.1-3.3, because the given incentive is fixed, we abbreviate statements like "the incentive with condition $\mathbb{C}$ [...]" by just referring to the condition.

§3.4 distills recommendations on how to choose an incentive and a condition $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$.

3.1. Analysis of Individual Condition $\mathbb{I}$

Under the individual condition $\mathbb{I}$, whether local ℓ complies is determined solely by his deforestation decision ($\mathbb{I}_\ell(d, B) = \text{yes} \Leftrightarrow d_\ell = 0$), regardless of the partition π into coalitions and of other locals' deforestation and blocking decisions. Consequently, every local's deforestation decision is set independently of all other deforestation decisions and no blocking would occur, $B = 0$. By (1), local ℓ prefers to set $d_\ell = 0$ and earn the incentive rather than set $d_\ell = 1$ and deforest land if and only if $I_\ell > D_\ell$. Therefore, the individual condition $\mathbb{I}$ prevents deforestation – and also compensates locals – if and only if every local earns higher net income with the incentive than with deforestation, $I_\ell > D_\ell, \forall \ell \in \mathcal{L}$.

In reality, such "perfect" incentives are rare. Practical incentives like the ones discussed in our introduction can rarely be tailored to ensure that *every* local prefers the incentive to deforestation because data challenges make it hard to precisely estimate every local's value from the incentive or his opportunity costs, and budget constraints limit payments and outreach efforts.

Building on this intuition, we define $\mathcal{G} := \{\ell \in \mathcal{L} : I_\ell > D_\ell\}$ as the set of locals who prefer the incentive individually and we say that coalition $S \subseteq \mathcal{L}$ *prefers the incentive* if $I(S) := \sum_{\ell \in S} I_\ell > D(S) := \sum_{\ell \in S} D_\ell$, it *prefers deforestation* if $D(S) \geq I(S)$, and it *strictly prefers deforestation* if $D(S) > I(S)$. For expositional simplicity, we assume that every local has a strict preference: $I_\ell \neq D_\ell, \forall \ell \in \mathcal{L}$.

3.2. The Non-Cooperative Game (of Deforestation and Blocking) under $\bar{\mathbb{D}}, \bar{\mathbb{U}}$

Under either area condition $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$, consider a fixed partition $\pi \in \Pi_{\mathcal{L}}^{\preceq\bar{\pi}}$ and the non-cooperative game among the coalitions $S \in \pi$. A subgame-perfect equilibrium of that game is a set of deforestation decisions $d^\star$ and blocking policies $B^\star(d)$ that satisfy, for each $S \in \pi$:

$$d_S^\star \in \underset{d_S \in \{0,1\}^S}{\arg\max}\, V_S\big([d_S, d_{-S}^\star], B^\star\big([d_S, d_{-S}^\star]\big)\big) \tag{2a}$$

$$B_S^\star(d) \in \underset{B_S \in \{0,1\}^{S\times\mathcal{L}}}{\arg\max}\, V_S\big(d, [B_S, B_{-S}^\star(d)]\big). \tag{2b}$$

In the first stage, each coalition S chooses its deforestation decisions d_S to maximize its net income given other coalitions' deforestation decisions $d_{-S}^\star$, and anticipating the second-stage optimal blocking response $B^\star(\cdot)$. In the second stage, having observed the deforestation decisions of all locals d, each coalition S chooses its blocking policy $B_S(d)$ to maximize its net income $V_S(d, B)$, given all deforestation decisions d and other coalitions' blocking decisions $B_{-S}^\star(d)$. If coalition S is indifferent between setting $d_\ell = 0$ and $d_\ell = 1$ for some $\ell \in S$ in (2a), we assume it sets $d_\ell = 1$.

We use $\mathcal{Q}(\pi, \mathbb{C})$ to denote the set of all subgame-perfect equilibria $(d^\star, B^\star(d))$ that satisfy (2b)-(2a) for partition π and condition $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$.

Lemma 1 establishes that in equilibrium, no blocking occurs and either no local deforests land or all locals deforest land.

LEMMA 1. *For any partition π and condition $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$ and in any subgame-perfect equilibrium of the non-cooperative game, the deforestation decisions of all locals are the same and there is no blocking: $d^\star \in \{0,1\}$, $B^\star(d^\star) = 0$, $\forall \big(d^\star, B^\star(d^\star)\big) \in \mathcal{Q}(\pi, \mathbb{C})$.*

Blocking does not occur in any subgame-perfect equilibrium. Under $\bar{\mathbb{D}}$, blocking would not influence compliance, so no blocking occurs because blocking is costly. Under $\bar{\mathbb{U}}$, in any equilibrium in which a coalition that decided to deforest land would be blocked, the coalition would anticipate that blocking, so it would decide *not* to deforest land in the first place, avoiding the cost to deforest land. Therefore blocking would not actually occur. A credible threat of blocking (from one or more coalitions that would profitably block a coalition that decided to deforest land) can deter deforestation.

Only the two extreme types of deforestation decisions occur in equilibrium because each coalition is more inclined to deforest land when other coalitions do so. Under $\bar{\mathbb{D}}$, deforestation by any coalition prevents all other locals from earning the incentive, so their best response is to deforest land, too. Under $\bar{\mathbb{U}}$, deforestation by more locals increases the total cost a coalition would incur to block them and earn the incentive, which incentivizes that coalition to deforest land rather than block the other locals who deforested land.

Lemma 1's characterization of the two types of equilibrium enables simple exposition. The equilibrium deforestation decisions $d^\star \in \{0,1\}$ identify the equilibrium between the two types. Therefore we will simply write *"equilibrium with $d^\star=0$"* to refer to the type of equilibrium with no deforestation, and *"equilibrium with $d^\star=1$"* to refer to the type of equilibrium with deforestation. We use $T(\pi,\mathbb{C}) := \{d^\star \in \{0,1\} : (d^\star, B^\star \equiv 0) \in \mathcal{Q}(\pi,\mathbb{C})\}$ to represent all of the equilibria that arise with partition π and condition $\mathbb{C}$. When the condition $\mathbb{C}$ is clear from context, we drop it from the notation.

Lemma 2 characterizes the set of problem parameters for which each type of equilibrium exists.

Lemma 2. *Consider any partition $\pi \in \Pi_{\mathcal{L}}$. Under the area no-deforestation condition $\bar{\mathbb{D}}$,*

$$T(\pi,\bar{\mathbb{D}}) = \begin{cases} \{0\} & \text{if } \pi = \{\mathcal{L}\} \text{ and } I(\mathcal{L}) > D(\mathcal{L}) \\ \{1\} & \text{if } I(S) \le D(S) \text{ for some coalition } S \in \pi \\ \{0,1\} & \text{otherwise.} \end{cases}$$

Under the area no-use condition $\bar{\mathbb{U}}$,

$$T(\pi,\bar{\mathbb{U}}) = \begin{cases} \{0\} & \text{if } \eta < \eta_1(\pi) := \sup\{\eta : \exists S \in \pi \text{ with } I(S) - D(S) > \eta\,|\mathcal{L}\setminus S|\} \\ \{1\} & \text{if } \eta > \eta_2(\pi) := \sup\left\{\eta : \sum_{S\in\pi: I(S)>D(S)} \left\lfloor \frac{I(S)-D(S)}{\eta} \right\rfloor \ge \max_{H\in\pi: I(H)\le D(H)} |H|\right\} \\ \{0,1\} & \text{otherwise.} \end{cases}$$

The thresholds $\eta_1(\pi)$ and $\eta_2(\pi)$ satisfy $\eta_1(\pi) \le \eta_2(\pi)$.

Under $\bar{\mathbb{D}}$, equilibria with $d^\star=1$ exist unless locals are in the grand coalition ($\pi = \{\mathcal{L}\}$) and collectively prefer the incentive. If a coalition $S \in \pi$ prefers deforestation, then *only* equilibria with $d^\star=1$ exist because under $\bar{\mathbb{D}}$ there is no credible threat of blocking that could deter coalition S, so deforestation is the best response for all other coalitions.

Under $\bar{\mathbb{U}}$, a credible threat of blocking can sustain equilibria with $d^\star=0$. When $\eta < \eta_1(\pi)$, some coalition of locals $S \in \pi$ could profitably block all other locals $\mathcal{L}\setminus S$ because $I(S) - \eta|\mathcal{L}\setminus S| > D(S)$, so coalition S's net benefit with the incentive and blocking exceeds its benefit from deforestation. Given this credible threat of blocking, no local would deforest land. When $\eta > \eta_2(\pi)$, some coalition $H \in \pi$ that prefers deforestation will deforest land, knowing that the remaining coalitions will not block it subsequently. This is because the coalitions preferring the incentive could block at most $\sum_{S\in\pi: I(S)>D(S)} \lfloor (I(S)-D(S))/\eta \rfloor$ locals, where the floor operator $\lfloor\cdot\rfloor$ is used because utility transfer occurs only within coalitions, not across coalitions. When $\eta \in [\eta_1(\pi), \eta_2(\pi)]$, both equilibria with $d^\star=0$ or $d^\star=1$ exist because no single coalition could profitably block all other locals, whereas two or more coalitions jointly could.

The thresholds $\eta_1(\pi)$ and $\eta_2(\pi)$ depend on the partition π. If locals form the grand coalition ($\pi = \{\mathcal{L}\}$) and prefer the incentive, then only equilibria with $d^\star=0$ exist irrespective of the blocking cost, so $\eta_1(\pi) = \eta_2(\pi) = +\infty$. With a partition π so that *all* coalitions prefer the incentive, equilibria with no deforestation exist irrespective of the blocking cost, so $\eta_2(\pi) = +\infty$. With a partition π so that *all* coalitions prefer deforestation, only equilibria with deforestation exist irrespective of the blocking cost, so $\eta_1(\pi) = \eta_2(\pi) = -\infty$. In all other cases, the types of equilibria depend on the blocking cost, so $\eta_1(\pi)$ and $\eta_2(\pi)$ are strictly positive and finite. The thresholds $\eta_1(\pi)$ and $\eta_2(\pi)$ are non-monotonic in π. As the partition π becomes finer, a coalition can either increase the threat of blocking by removing locals from the coalition who have a high preference for deforestation, or decrease the threat of blocking by removing a local who prefers the incentive.

3.3. The Cooperative Game (of Coalition Formation) under $\bar{\mathbb{D}}, \bar{\mathbb{U}}$

To formalize the cooperative game through which locals organize themselves into coalitions under a given condition $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$, we must first assign a value to each coalition $S \in \pi$ in every feasible partition π. Consider any subgame-perfect equilibrium $(d^\star, B^\star(d)) \in \mathcal{Q}(\pi, \mathbb{C})$ in the non-cooperative game corresponding to $\mathbb{C}$ and π. By Lemma 1, this is either a no-deforestation or a deforestation equilibrium, as indicated by $d^\star \in T(\pi, \mathbb{C}) \subseteq \{0,1\}$, and no blocking occurs, $B^\star(d^\star) = 0$. Therefore, in this equilibrium, the net income for each coalition $S \in \pi$ is $V_S(d^\star, 0)$, by (1). We define the following function that returns the net income of each coalition $S \in \pi$ in the equilibrium $d^\star \in T(\pi, \mathbb{C})$:

$$w(\,\cdot\,; d^\star) : \pi \to \mathbb{R}, \quad w(S; d^\star) = V_S(d^\star, 0), \, \forall S \in \pi. \tag{3}$$

The function w (known as the *partition function* in the cooperative games literature) is not uniquely determined by the partition π because the non-cooperative game between coalitions may admit multiple subgame-perfect equilibria, which happens when $T(\pi, \mathbb{C}) = \{0,1\}$. However, the equilibrium indicator $d^\star \in T(\pi, \mathbb{C})$ uniquely identifies a specific equilibrium and the net income for each coalition $S \in \pi$. That is our rationale for simplifying the notation by omitting π as an argument for w and writing $w(\,\cdot\,; d^\star)$, with the implicit understanding that $d^\star \in T(\pi, \mathbb{C})$.

We can now formalize our *TU Cooperative Game with Multiple Partition Functions and Limited Cooperation*. This game is characterized by a triplet $(\mathcal{L}, \bar{\pi}, W)$, where W summarizes all the partition functions, i.e., W is the correspondence defined as $W(\pi, \mathbb{C}) = \{w(\,\cdot\,; d^\star) \, : \, d^\star \in T(\pi, \mathbb{C})\}$, $\forall \pi \in \Pi_{\mathcal{L}}^{\preceq \bar{\pi}}$. An *outcome* of this game is a triple $(\pi, d^\star, a)$: a partition π, an equilibrium indicator $d^\star \in T(\pi)$, and an allocation of net income to each local $a \in \mathbb{R}^{\mathcal{L}}$ satisfying $\sum_{\ell \in S} a_\ell = w(S; d^\star)$ for every $S \in \pi$. To predict which outcomes can occur, we extend the pessimistic recursive core solution concept due to Kóczy (2007) to our setting with non-unique partition functions and limited cooperation. The premise of the recursive core is that when some locals form a specific configuration of coalitions, they anticipate that the other "residual" locals will act to maximize their own net income by engaging in a smaller "residual" game that treats this configuration as fixed. Through recursion over residual games, one solves for the recursive core: the set of outcomes at which no locals would expect to achieve higher net income by forming different coalitions. If the core of a residual game contains many outcomes, we assume that locals who form coalitions have *pessimistic* beliefs about how residual locals will act. The next definitions formalize this; we simplify notation by not including the condition $\mathbb{C}$, but it should be understood that the definitions apply separately to $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$.

Definition 1 (Residual Game). Consider a set of "residual" locals $R \subseteq \mathcal{L}$ faced with a fixed partition $\pi_{\mathcal{L}\setminus R} \in \Pi^{\prec\bar{\pi}}_{\mathcal{L}\setminus R}$ of the other locals. In response to $\pi_{\mathcal{L}\setminus R}$, the locals in R play a *residual game*: the partition $\pi_R \in \Pi^{\prec\bar{\pi}}_R$ formed by the locals in R together with the partition $\pi_{\mathcal{L}\setminus R}$ of the other locals and the equilibrium indicator $d^\star \in T(\pi_R \cup \pi_{\mathcal{L}\setminus R})$ determine the net income $w\big(S; d^\star\big)$ for each coalition $S \in \pi_R$. An outcome for the residual game is a partition $\pi_R \in \Pi^{\prec\bar{\pi}}_R$, an equilibrium indicator $d^\star \in T(\pi_R \cup \pi_{\mathcal{L}\setminus R})$, and an allocation $a \in \mathbb{R}^R$ satisfying $a(S) = w\big(S; d^\star\big)$ for every $S \in \pi_R$.

Definition 2 (Pessimistic Recursive Core). We define this recursively, based on the size of the residual set R. For a residual game where one local $R = \{\ell\}$ responds to a partition $\pi_{\mathcal{L}\setminus\{\ell\}}$ of the other locals, define the pessimistic recursive core as $C(\{\ell\}; \pi_{\mathcal{L}\setminus\{\ell\}}) = \big\{\big(\{\{\ell\}\}, d^\star, a_\ell\big) : a_\ell = w\big(\{\ell\}; d^\star\big),\ d^\star \in T(\{\{\ell\}\} \cup \pi_{\mathcal{L}\setminus\{\ell\}})\big\}$. For some integer $k < |\mathcal{L}|$, suppose that pessimistic recursive cores are defined for every possible residual game faced by at most k residual locals, i.e., $C(R; \pi_{\mathcal{L}\setminus R})$ is defined for any $R \subseteq \mathcal{L}$ with $|R| \leq k$ and any partition $\pi_{\mathcal{L}\setminus R} \in \Pi^{\prec\bar{\pi}}_{\mathcal{L}\setminus R}$. For a residual game with $|R| = k+1$, the pessimistic recursive core $C(R; \pi_{\mathcal{L}\setminus R})$ is the set of outcomes that are *not dominated*; an outcome $(\pi_R, d^\star, a)$ is *dominated* if there exists a group of locals $H \subseteq R$ from the same family ($H \subseteq F$ for some $F \in \bar{\pi}$) who can form a partition $\pi_H \in \Pi^{\prec\bar{\pi}}_H$ so that

$$w\big(S; d\big) > a(S), \forall S \in \pi_H,\ \forall d^\star \in A\big(R \setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R}\big), \tag{4}$$

where the set of $\underline{\text{A}}$ssumptions $A(\cdot;\cdot)$ contains all equilibrium indicators of plausible outcomes, and is defined for any residual set of locals $Z \subseteq \mathcal{L}$ and any partition $\pi_{\mathcal{L}\setminus Z} \in \Pi^{\prec\bar{\pi}}_{\mathcal{L}\setminus Z}$ of the other locals as:

$$A\big(Z; \pi_{\mathcal{L}\setminus Z}\big) = \begin{cases} \big\{d^\star \in \{0,1\} : \exists \tilde{\pi}_Z, \exists \tilde{a} \text{ with } \big(\tilde{\pi}_Z, d^\star, \tilde{a}\big) \in C(Z; \pi_{\mathcal{L}\setminus Z})\big\} & \text{if } Z \neq \emptyset \text{ and } C(Z; \pi_{\mathcal{L}\setminus Z}) \neq \emptyset \\ \big\{d^\star \in \{0,1\} : \exists \tilde{\pi}_Z \text{ with } d^\star \in T(\tilde{\pi}_Z \cup \pi_{\mathcal{L}\setminus Z})\big\} & \text{otherwise.} \end{cases} \tag{5}$$

The pessimistic recursive core of the TU cooperative game among all locals is then $C(\mathcal{L}; \emptyset)$.

Notice how the definition of recursive cores and dominated outcomes represents *pessimism*: locals who deviate anticipate their *worst-case* net income when residual locals act to maximize their own net income. By (4), for coalition H to deviate, it must form a new partition π_H in which every coalition $S \in \pi_H$ has strictly greater net income under *every* plausible outcome that could emerge in the residual game played by locals in $R \setminus H$. Plausible outcomes are summarized by the deforestation indicators $d^\star$ in the Assumption set $A\big(R \setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R}\big)$, which correspond to all core outcomes of the residual game if that core is non-empty, or to any equilibrium outcomes otherwise, by (5).

§EC.6.7 instead assumes *optimism*: locals anticipate their *best-case* net income when residual locals act to maximize their own net income. Proposition EC.10 in §EC.6.7 shows that the resulting optimistic recursive core is a subset of the pessimistic recursive core. That motivates our subsequent focus on the pessimistic recursive core. We will characterize the model parameters for which *all* outcomes in the pessimistic recursive core meet the performance requirements (which by Proposition EC.10 implies that all outcomes in the optimistic recursive core meet the performance requirements).

We subsequently use "core" to refer to the pessimistic recursive core.

Our first result shows that a core outcome with $d^\star\!=\!0$ weakly benefits locals.

Proposition 1. *In any core outcome with $d^\star\!=\!0$, every local's allocation weakly exceeds his status-quo income without the incentive and without deforestation, i.e., $a_\ell \geq v_\ell^{\mathrm{sq}}, \forall \ell \in \mathcal{L}$ holds for every $(\pi, d^\star, a) \in C(\mathcal{L}; \emptyset)$ with $d^\star = 0$.*

Proposition 1 implies that in a core outcome with $d^\star\!=\!0$, a buyer meets a dual commitment to zero deforestation and to improve smallholders' incomes, but the improvement may be weak. In contrast, our second, stronger performance requirement is that an outcome *compensates*: $a_\ell \geq v_\ell^{\mathrm{sq}} + D_\ell \;\; \forall \ell \in \mathcal{L}$.

Our next results characterize the performance of the core outcomes under each area condition. We determine whether all, some, or none of the core outcomes have $d^\star\!=\!0$ (no deforestation), and whether these compensate. This performance depends on particular circumstances in the area. For example, a key insight is that under either $\bar{\mathbb{D}}$ or $\bar{\mathbb{U}}$, the performance of the core outcomes depends on the binary distinction in family structure $\bar{\pi} = \{\mathcal{L}\}$ vs. $\bar{\pi} \neq \{\mathcal{L}\}$. In the "*full cooperation*" case $\bar{\pi} = \{\mathcal{L}\}$, $\bar{\pi}$ contains a single family with all locals so any coalition of locals is possible and π can be any partition. In the "*limited cooperation*" case $\bar{\pi} \neq \{\mathcal{L}\}$, $\bar{\pi}$ contains at least two families.

Theorem 1 **(Area No-Deforestation)**. *Under a conditional incentive $(I, \bar{\mathbb{D}})$:*

A. The core is non-empty, and all core outcomes with $d^\star\!=\!0$ compensate.

B. Suppose $\bar{\pi} = \{\mathcal{L}\}$. All core outcomes have $d^\star\!=\!0$ if $I(\mathcal{L}) > D(\mathcal{L})$, and none have $d^\star\!=\!0$ otherwise.

C. Suppose $\bar{\pi} \neq \{\mathcal{L}\}$. At least one core outcome has $d^\star\!=\!0$ if and only if $I(F) > D(F)$ for all $F \in \bar{\pi}$, but not all core outcomes have $d^\star\!=\!0$.

Under $\bar{\mathbb{D}}$, a core outcome with $d^\star\!=\!0$ must compensate – otherwise a local could unilaterally deviate, deforest land, and obtain strictly higher income. A core outcome with $d^\star\!=\!0$ is maintained through utility transfers within each coalition: locals who prefer the incentive transfer utility to coalition members who prefer deforestation so that those members are compensated for not deforesting land.

With full cooperation, the locals' collective preference determines core outcomes. If locals collectively prefer the incentive, $I(\mathcal{L}) > D(\mathcal{L})$, a core outcome exists with the grand coalition setting $d = 0$ and compensating every local. That outcome is not dominated because any partition with two or more coalitions has a non-cooperative equilibrium with $d^\star\!=\!1$ that would yield lower income for every coalition. In that case, all core outcomes have $d^\star\!=\!0$ because any outcome with $d^\star\!=\!1$ is dominated by the grand coalition forming and setting $d^\star\!=\!0$. Conversely, if locals collectively prefer deforestation, a core outcome with $d^\star\!=\!1$ exists, and no core outcome has $d^\star\!=\!0$.

With limited cooperation, the core under $\bar{\mathbb{D}}$ always has outcomes with $d^\star\!=\!1$, and also has outcomes with $d^\star\!=\!0$ if and only if every family prefers the incentive, $I(F) > D(F)$ for every $F \in \bar{\pi}$. Core outcomes with $d^\star\!=\!1$ arise due to a coordination problem: with multiple families, the non-cooperative game always has an equilibrium with $d^\star\!=\!1$. In the case that every family prefers the incentive, a core outcome exists with $d^\star\!=\!0$, partition $\bar{\pi}$, and utility transfer within each family so that all members are compensated for not deforesting land. That outcome is not dominated because with any deviation the non-cooperative equilibrium with $d^\star\!=\!1$ would arise and yield lower income for all coalitions.

In contrast, the performance of core outcomes under the area no-use condition $\bar{\mathbb{U}}$ may depend on the detailed family structure and blocking cost η. In preparation for our complex Theorem 2 on the area no-use condition $\bar{\mathbb{U}}$, we define and discuss the four key thresholds for the blocking cost:

$$\overline{\eta} := \max_{F,S,\eta}\Big\{\eta \;:\; I(S) - D(S) \geq \eta,\, F \in \bar{\pi}, S \subseteq F\Big\}, \tag{6}$$

$$\eta_2 := \min_{F',H}\left\{\max_{\eta}\left\{\eta : \sum_{\substack{F\in\bar{\pi}:\\ I(F\setminus H)>D(F\setminus H)}} \left\lfloor \frac{I(F\setminus H) - D(F\setminus H)}{\eta}\right\rfloor \geq |H|\right\} : F' \in \bar{\pi},\, H \subseteq F',\, I(H) \leq D(H)\right\}, \tag{7}$$

$$\overline{\eta}_1 := \max_{F,S,\eta}\Big\{\eta : I(S) - D(S) \geq \eta|\mathcal{L}\setminus S|,\, F \in \bar{\pi},\, S \subseteq F,\, S \neq \mathcal{L}\Big\}, \tag{8}$$

$$\underline{\eta}_1 := \max_{F,S,\eta}\Big\{\eta : I(S) - D(S) \geq \eta|\mathcal{L}\setminus S|,\, F \in \bar{\pi},\, S \subseteq F,\, S \neq \mathcal{L},\, I(F) > D(F)\Big\}. \tag{9}$$

The four thresholds describe how blocking affects the core.

If $\eta > \overline{\eta}$, no coalition could profitably block even one local, so the area no-use condition $\bar{\mathbb{U}}$ acts equivalently to the area no-deforestation condition $\bar{\mathbb{D}}$. This $\overline{\eta}$ is the largest of the four thresholds.

If $\eta \leq \eta_2$, the core has at least one outcome with $d^\star\!=\!0$. η_2 is the minimum value of $\eta_2(\pi)$ (defined in Lemma 2) among partitions π in which some coalition H prefers deforestation and all other locals remain in their families. Thus, when at least one family prefers the incentive, $\eta \leq \eta_2$ guarantees a core outcome with $d^\star\!=\!0$, maintained through credible threats that coalition H would be blocked.

If $\eta < \underline{\eta}_1$, all core outcomes have $d^\star\!=\!0$, whereas if $\eta \geq \overline{\eta}_1$, at least one core outcome has $d^\star\!=\!1$. The thresholds $\underline{\eta}_1$ and $\overline{\eta}_1$ split the role of $\eta_1(\pi)$ in the cooperative game. The threshold $\overline{\eta}_1$ is the maximum $\eta_1(\pi)$ among partitions with a coalition $S \neq \mathcal{L}$ that can profitably block all other locals; $\underline{\eta}_1$ is the same maximum restricted to coalitions S drawn from families that prefer the incentive. Hence, $\overline{\eta}_1 \geq \underline{\eta}_1$, with equality if every family prefers the incentive. When $\eta < \underline{\eta}_1$, any core outcome must have $d^\star\!=\!0$, because an outcome with $d^\star\!=\!1$ is dominated by one with $d^\star\!=\!0$ maintained through such a blocking threat. When $\eta \geq \overline{\eta}_1$, no coalition can profitably replace the transfers required for compensation with a threat to block all other locals. This condition therefore matters both for core non-emptiness and for whether core outcomes with $d^\star\!=\!0$ compensate.

Theorem 2 characterizes the core under $\bar{\mathbb{U}}$. The theorem also distinguishes between full cooperation and limited cooperation, but unlike with $\bar{\mathbb{D}}$, an important distinction arises between the cases $\mathcal{G} = \emptyset$, $\mathcal{G} = \mathcal{L}$, and $\mathcal{G} \notin \{\emptyset, \mathcal{L}\}$. The last case is of practical importance because an incentive may provide more value than deforestation to some, but not all, locals. In that case, the relevant comparisons are between η and the four thresholds defined above, which satisfy $\overline{\eta} \geq \eta_2$ and $\overline{\eta} \geq \overline{\eta}_1 \geq \underline{\eta}_1$.

Theorem 2 **(Area No-Use)**. *Under the conditional incentive $(I, \bar{\mathrm{U}})$:*

A. Suppose $\mathcal{G} \notin \{\emptyset, \mathcal{L}\}$. The core is nonempty if $\eta \notin (\eta_2, \overline{\eta}_1)$; otherwise it can be *empty, even if $\bar{\pi} = \{\mathcal{L}\}$. The core outcomes, if any, satisfy the following:*

i. Suppose $\bar{\pi} = \{\mathcal{L}\}$. All outcomes have $d^\star = 0$ if $I(\mathcal{L}) > D(\mathcal{L})$; it is possible that no outcome has $d^\star = 0$ if $I(\mathcal{L}) = D(\mathcal{L})$; no outcome has $d^\star = 0$ if $I(\mathcal{L}) < D(\mathcal{L})$. If $I(\mathcal{L}) > D(\mathcal{L})$, at least one outcome with $d^\star = 0$ compensates if and only if $\eta \geq \overline{\eta}_1$, and all compensate if $\eta > \overline{\eta}$; if $I(\mathcal{L}) = D(\mathcal{L})$, no outcome with $d^\star = 0$ compensates.

ii. Suppose $\bar{\pi} \neq \{\mathcal{L}\}$, and $I(F) > D(F)$ for some $F \in \bar{\pi}$. At least one outcome has $d^\star = 0$ if $\eta \leq \eta_2$; otherwise, it is possible that no outcome has $d^\star = 0$. All outcomes have $d^\star = 0$ if $\eta < \underline{\eta}_1$; at least one outcome has $d^\star = 1$ if $\eta \geq \underline{\eta}_1$. At least one outcome with $d^\star = 0$ compensates if $I(F) > D(F)$ for all $F \in \bar{\pi}$ and $\eta \geq \overline{\eta}_1$; no outcomes with $d^\star = 0$ compensate if $I(F) < D(F)$ for some $F \in \bar{\pi}$; otherwise, it is possible that no outcomes with $d^\star = 0$ compensate. Not all outcomes have $d^\star = 0$ and compensate.

iii. Suppose $\bar{\pi} \neq \{\mathcal{L}\}$, and $I(F) \leq D(F)$ for all $F \in \bar{\pi}$. It is possible that no outcome has $d^\star = 0$. At least one outcome has $d^\star = 1$.

B. If $\mathcal{G} = \mathcal{L}$, the core is non-empty and at least one core outcome has $d^\star = 0$. All *core outcomes have $d^\star = 0$ if and only if $\bar{\pi} = \{\mathcal{L}\}$ or $\eta < \underline{\eta}_1$. All core outcomes with $d^\star = 0$ compensate.*

C. If $\mathcal{G} = \emptyset$, the core is non-empty and no core outcome has $d^\star = 0$.

Consider first the practically relevant case (A).

The core is non-empty if $\eta \leq \eta_2$ (when the core has an outcome with $d^\star = 0$) or if $\eta \geq \overline{\eta}_1$ (when the core has an outcome with $d^\star = 1$). A simple sufficient condition for non-emptiness for every η is that, within every family, either all members prefer the incentive or all prefer deforestation: $F \subseteq \mathcal{G}$ or $F \subseteq \mathcal{L} \setminus \mathcal{G}$ for every $F \in \bar{\pi}$. Then the family F with the largest value of $I(F) - D(F)$ also has the greatest ability to block all other locals. Because every coalition H that prefers deforestation lies outside F, whenever F can block all other locals it can also block H; hence, $\eta_2 \geq \overline{\eta}_1$.

If instead $\eta_2 < \overline{\eta}_1$, the core can be empty for $\eta \in (\eta_2, \overline{\eta}_1)$. To see why, consider full cooperation and $I(\mathcal{L}) > D(\mathcal{L})$. An outcome with $d^\star = 1$ is dominated by the grand coalition receiving the incentive and using transfers to sustain $d^\star = 0$. Some coalition $S \subset \mathcal{L}$ containing locals in $\mathcal{G}$ who make those transfers may then gain by withholding them and threatening to block all other locals, which is possible because $\eta < \overline{\eta}_1 = \underline{\eta}_1$. The resulting outcome with $d^\star = 0$ may in turn be dominated by a coalition H that prefers deforestation, is not compensated, and cannot be blocked because $\eta > \eta_2$. Similar cycles can arise when locals collectively prefer deforestation or when cooperation is limited.

When interpreting the statements in parts (A)-(i-iii) of Theorem 2, consider glancing at Figure 3.1, which illustrates the core outcomes in the more complex case $\eta_2 < \overline{\eta}_1$, when the core can be empty.

With full cooperation, corresponding to case (A)-(i) and the top row of Figure 3.1, the community's aggregate preference determines whether core outcomes have $d^{\star}=0$, whereas η determines whether such outcomes compensate. If $I(\mathcal{L})>D(\mathcal{L})$, all core outcomes have $d^{\star}=0$, because any outcome with $d^{\star}=1$ is dominated by the grand coalition receiving the incentive and compensating every local. At least one such outcome compensates if and only if $\eta\geq\overline{\eta}_1$, and all such outcomes compensate if $\eta>\overline{\eta}$. Otherwise, it is possible that no core outcome has $d^{\star}=0$, and even if some do, none compensates.

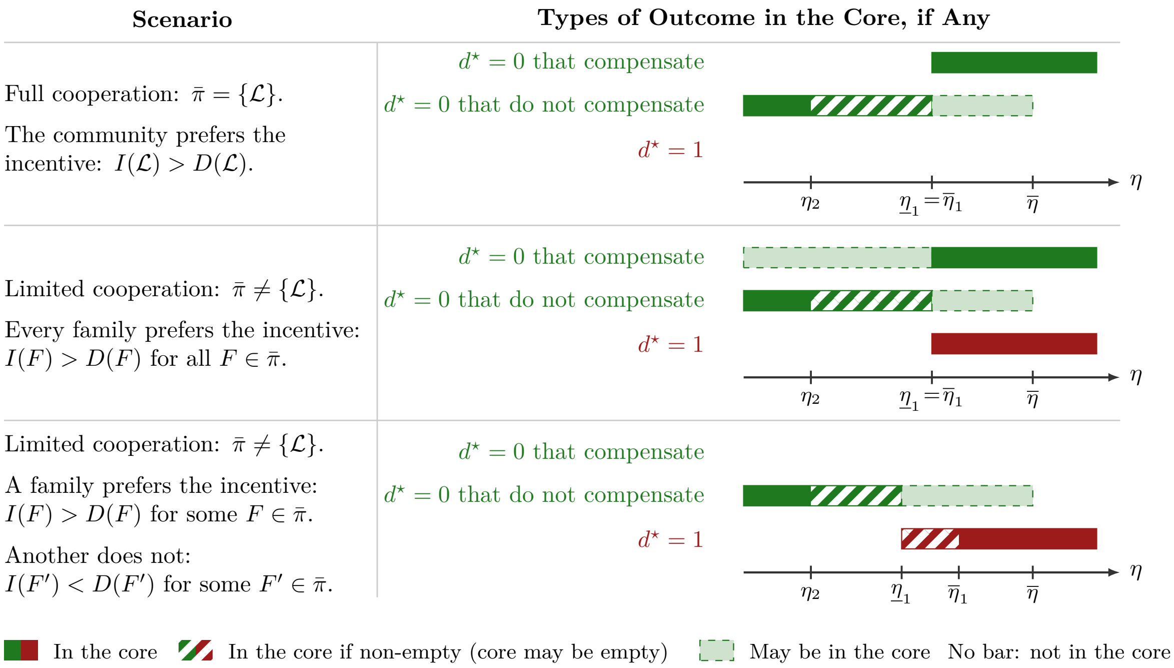

Figure 3.1 Existence and performance of core outcomes under the area no-use condition $\bar{\mathrm{U}}$ for the case $\eta_2<\underline{\eta}_1$.

With limited cooperation, suppose first that at least one family prefers the incentive, as in cases (A)-(ii) and the last two rows of Figure 3.1. If $\eta\leq\eta_2$, the core is non-empty and contains an outcome with $d^{\star}=0$, maintained by the credible blocking threats created jointly by the coalitions. If $\eta<\underline{\eta}_1$, *every* core outcome has $d^{\star}=0$, because some coalition S can profitably block all other locals. Thus, if both inequalities hold, the core is non-empty and all its outcomes have $d^{\star}=0$. If $\eta\in(\min(\eta_2,\underline{\eta}_1),\underline{\eta}_1)$, the core can be empty, although any non-empty core contains only outcomes with $d^{\star}=0$. If $\eta\geq\underline{\eta}_1$, a non-empty core contains an outcome with $d^{\star}=1$; even though for these blocking cost values, some coalition S could block all other locals, that coalition would be drawn from a family that prefers deforestation, so other members can transfer utility to S so that it accepts the outcome with $d^{\star}=1$. Once $\eta>\eta_2$, the core may contain no outcome with $d^{\star}=0$. And the core is also not guaranteed to contain any outcomes with $d^{\star}=0$ in case (A)-(iii), when no family prefers the incentive.

With limited cooperation, the core is guaranteed to contain an outcome with $d^{\star}=0$ that compensates – but never only such outcomes – if every family prefers the incentive and $\eta\geq\overline{\eta}_1$. The first condition gives every family enough net income with the incentive to compensate its members; the second condition prevents a coalition from increasing its net income by withholding transfers and relying on blocking threats instead. However, the core also contains other types of outcomes. In all other cases, the core is either guaranteed not to (or may not) contain an outcome with $d^\star\!=\!0$ that compensates.

In case (B) with a perfect incentive, the core is non-empty and has outcomes with $d^\star\!=\!0$ that compensate. In particular, the outcome with partition $\pi=\bar{\pi}$, in which every family chooses not to deforest land and every local ℓ receives $v_\ell^{\text{sq}}+I_\ell$, is in the core. All core outcomes with $d^\star\!=\!0$ compensate, because an uncompensated local ℓ could form a singleton coalition and guarantee at least $\min(v_\ell^{\text{sq}}+D_\ell, v_\ell^{\text{sq}}+I_\ell)=v_\ell^{\text{sq}}+D_\ell$. With full cooperation, all core outcomes have $d^\star\!=\!0$; with limited cooperation, this holds if and only if $\eta<\underline{\eta}_1$. Thus, when $\eta\geq\underline{\eta}_1$, the core also contains outcomes with $d^\star\!=\!1$, even though every local individually prefers the incentive.

In case (C) when no locals prefer the incentive, any outcome with $d^\star\!=\!0$ is dominated by an outcome in which every family deforests land, so the core only has outcomes with $d^\star\!=\!1$.

Under $\bar{\mathbb{U}}$, reducing the locals' ability to cooperate has ambiguous effects. If no family prefers the incentive – including the case with full cooperation and locals collectively preferring deforestation – the core need not contain an outcome with $d^\star=0$. Refining the partition of families $\bar{\pi}$ may nevertheless isolate a sufficiently small family that prefers the incentive and thereby create more problem instances with such outcomes. Once some family F prefers the incentive, however, further refinement that separates locals who prefer the incentive from F is generally detrimental and can eliminate problem instances in which the core contains an outcome with $d^\star=0$.

Reducing the blocking cost η also has ambiguous effects: it worsens performance with full cooperation or if compensation is required, but it can lead to core outcomes with $d^\star\!=\!0$ in more problem instances with limited cooperation. With full cooperation and $I(\mathcal{L})>D(\mathcal{L})$, $\eta>\overline{\eta}$ makes $\bar{\mathbb{U}}$ equivalent to $\bar{\mathbb{D}}$: the core is non-empty and all its outcomes have $d^\star=0$ and compensate. Reducing η may eliminate all core outcomes that compensate when $\eta<\overline{\eta}_1$ and can make the core empty when $\eta\in(\eta_2,\overline{\eta}_1)$. If locals collectively prefer deforestation, reducing η does not qualitatively improve performance. With limited cooperation, reducing η likewise weakens compensation: when every family prefers the incentive, $\eta\geq\overline{\eta}_1$ guarantees at least one outcome with $d^\star=0$ that compensates, and $\eta>\overline{\eta}$ makes all such outcomes compensate, whereas lower values may yield none. However, for preventing deforestation, a lower η makes the conditions $\eta\leq\eta_2$ and $\eta<\underline{\eta}_1$ easier to satisfy, thereby yielding more problem instances wherein a non-empty core contains at least one (or only) outcomes with $d^\star\!=\!0$. The caveat is that lowering η from above $\overline{\eta}_1$ into $(\eta_2,\overline{\eta}_1)$ can also make the core empty.

3.4. Recommendations for Choice of Condition and Incentive

We take a robust approach to identify the best condition $\mathbb{C}\in\{\mathbb{I},\bar{\mathbb{D}},\bar{\mathbb{U}}\}$ and corresponding incentive values I, depending on the performance requirement. The basic performance requirement is to prevent deforestation (induce $d^\star = 0$). The stronger, more ethical performance requirement is to prevent deforestation and compensate. A *best* condition meets the performance requirement *for the largest set of incentive values* I. More than one condition can be best when those sets are not nested. In assessing that an area condition $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$ is best, we interpret Theorems 1 and 2 conservatively: we deem as feasible only those incentive values I for which the theorems conclusively state that the core is non-empty and *all* core outcomes meet the performance requirement. Figure 3.2 reports the best condition(s) and corresponding feasible incentive values I.

With **full cooperation**, the area no-deforestation condition $\bar{\mathbb{D}}$ is the unique best condition. With any incentive that is more valuable to locals collectively than deforestation, $I(\mathcal{L}) > D(\mathcal{L})$, $\bar{\mathbb{D}}$ prevents deforestation and meets the stronger ethical performance requirement. The alternatives $\bar{\mathbb{U}}$ and $\mathbb{I}$ work with strictly smaller sets of incentive values. With the area no-use condition $\bar{\mathbb{U}}$ and an incentive satisfying $I(\mathcal{L}) > D(\mathcal{L})$, all core outcomes have $d^\star = 0$ (like with $\bar{\mathbb{D}}$) but the core can be empty for intermediate blocking costs, $\eta_2(I) < \eta < \overline{\eta}_1(I)$. (We append (I) to highlight the dependence of the thresholds η_2 and $\overline{\eta}_1$ defined in (7) and (8) on the incentive values.) Moreover, with an imperfect incentive, $\bar{\mathbb{U}}$ would not compensate locals if blocking costs are low, $\eta < \overline{\eta}_1(I)$. Both $\bar{\mathbb{D}}$ and $\bar{\mathbb{U}}$ are more robust than the individual condition $\mathbb{I}$, which requires a perfect incentive.

To **prevent deforestation** with **limited cooperation**, either the area no-use condition $\bar{\mathbb{U}}$ or the individual condition $\mathbb{I}$ is best. The individual condition works only with a perfect incentive, but is the best condition when blocking costs η are high. (Note that when $\eta_2(I) < \eta < \overline{\eta}_1(I)$, the core under $\bar{\mathbb{U}}$ could be empty, whereas for $\eta > \overline{\eta}_1(I)$, it would contain an outcome with $d^\star = 1$.) In contrast, $\bar{\mathbb{U}}$ is best with an imperfect incentive. Under $\bar{\mathbb{D}}$, the core always has outcomes with deforestation, even if the incentive is perfect, due to failed coordination.

To **prevent deforestation and compensate** with **limited cooperation**, the individual condition $\mathbb{I}$ is best, with a perfect incentive. The area no-use condition $\bar{\mathbb{U}}$ only meets the performance requirement with a perfect incentive and only if the blocking cost is small, $\eta < \underline{\eta}_1(I)$.

Scenario	Prevent Deforestation	... and Compensate
Full cooperation: $\bar{\pi} = \{\mathcal{L}\}$.	Area No-Deforestation $\bar{\mathbb{D}}$ with: $I(\mathcal{L}) > D(\mathcal{L})$	Area No-Deforestation $\bar{\mathbb{D}}$ with: $I(\mathcal{L}) > D(\mathcal{L})$
Limited cooperation: $\bar{\pi} \neq \{\mathcal{L}\}$.	Area No-Use $\bar{\mathbb{U}}$ with: $I(F) > D(F)$ for some $F \in \bar{\pi}$, and $\underline{\eta}_1(I) > \eta$ and $\eta_2(I) \geq \eta$ or Individual $\mathbb{I}$ with: $I_\ell > D_\ell$ for all $\ell \in \mathcal{L}$	Individual $\mathbb{I}$ with: $I_\ell > D_\ell$ for all $\ell \in \mathcal{L}$

Figure 3.2 Best condition $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$ and corresponding feasible set of incentive values I. Each constraint has a distinct color to highlight similarities in feasible sets across different scenarios and performance requirements.

4. Alternative Interpretations and Model Extensions

This section offers new interpretations of our results and model, and insights from model extensions.

4.1. Inducing the $d^\star=0$ Type Among Core Outcomes

Now suppose that under $(I,\mathbb{C})$, if Theorems 1 and 2 guarantee the core is non-empty and contains *at least one* outcome with $d^\star=0$, then a core outcome with $d^\star=0$ will occur. For an area with limited cooperation and each area condition $\mathbb{C}\in\{\bar{\mathbb{D}},\bar{\mathbb{U}}\}$, this greatly expands the feasible set of incentives to prevent deforestation (and that, for $\bar{\mathbb{D}}$, also compensate). In contrast, for an area with full cooperation, this is practically irrelevant.[3] Therefore the remainder of this section focuses on the **limited cooperation** scenario. Figure 4.3 reports the best condition(s) and corresponding feasible incentive values, for comparison with those for the limited cooperation scenario in Figure 3.2.

The important change in our recommendations is that the area no-deforestation condition $\bar{\mathbb{D}}$ replaces the individual condition $\mathbb{I}$ as best to prevent deforestation, and also to compensate. Under $\bar{\mathbb{D}}$, with any incentive that is more valuable than deforestation for every family, $I(F)>D(F)$ for all $F\in\bar{\pi}$, the core is non-empty and contains an outcome with $d^\star=0$ that compensates. This is a strictly larger feasible set of incentives than the perfect incentive needed with $\mathbb{I}$. Accordingly, $\bar{\mathbb{D}}$ joins $\bar{\mathbb{U}}$ as a best condition to prevent deforestation, and $\bar{\mathbb{D}}$ is the unique best condition to also compensate.

A commodity buyer who offers a feasible incentive conditional on $\bar{\mathbb{D}}$ could induce $d^\star=0$ by enabling coordination across families. All families will be strictly better off with $d^\star=0$ than $d^\star=1$. The buyer conceivably could convene representatives of all families to enable their coordination on $d^\star=0$.

Scenario	Prevent Deforestation	... and Compensate
Limited cooperation: $\bar{\pi}\neq\{\mathcal{L}\}$. If core has an outcome with $d^\star=0$, that type of outcome occurs.	Area No-Use $\bar{\mathbb{U}}$ with: $I(F)>D(F)$ for some $F\in\bar{\pi}$, and $\eta_2(I)\geq\eta$ or Area No-Deforestation $\bar{\mathbb{D}}$ with: $I(F)>D(F)$ for all $F\in\bar{\pi}$	Area No-Deforestation $\bar{\mathbb{D}}$ with: $I(F)>D(F)$ for all $F\in\bar{\pi}$

Figure 4.3 Best condition $\mathbb{C}\in\{\mathbb{I},\bar{\mathbb{D}},\bar{\mathbb{U}}\}$ and corresponding feasible set of incentive values I.

4.2. Minimum Targeted Cash Payments

Suppose the buyer targets cash payments to locals so as to minimize his cost $I(\mathcal{L})$. §EC.6.1 derives the best conditions and cash payments to meet each performance requirement at minimum cost.

To prevent deforestation at minimum cost, if the blocking cost η is not too large, the buyer employs $\bar{\mathbb{U}}$ and payments that are *not equitable* for locals. Under $\bar{\mathbb{D}}$, preventing deforestation requires compensating every local. Under $\bar{\mathbb{U}}$, the buyer substitutes a credible threat of blocking for compensation, at a cost of η per local blocked, leading to locals who receive no incentive. The buyer

[3] With $\bar{\pi}=\{\mathcal{L}\}$, the core has outcomes with $d^\star=0$ and with $d^\star=1$ only under $\bar{\mathbb{U}}$, in the edge case $I(\mathcal{L})=D(\mathcal{L})$ and $\eta<\eta_2$.

pays locals and families who are less costly to pay than to block. Within the family that supplies the blocking threat, the locals who receive payment are those with $D_\ell < \eta$. Any other family receives the incentive only if it is larger than every family left unpaid and its average value from deforestation is below the blocking cost, $D(F)/|F| < \eta$. The remaining families receive no payment. Small families and families whose members value deforestation highly are therefore the ones left out.

4.3. Deforestation by Logging and Blocking Deforestation Attempts

When deforestation is done through *logging* rather than fire, it may be possible to block deforestation attempts before any forest is cleared. §EC.6.2 formally extends our model to capture this possibility by allowing locals to block attempts to deforest land. The decision $d_\ell = 1$ represents an *attempt to deforest* land, c_ℓ is the cost of that attempt, and D_ℓ is the net income gain if deforestation succeeds and the resulting timber or land is used. After observing the deforestation attempts, coalitions can first *block deforestation attempts* at cost $\eta_{\bar{\mathbb{D}}}$ (per local blocked) and they can subsequently *block use* at cost $\eta_{\bar{\mathbb{U}}}$ (per local blocked). Because measures that prevent deforestation also prevent any associated economic use, whereas use may be blocked through less costly actions (like those discussed in our base model), we assume $\eta_{\bar{\mathbb{U}}} < \eta_{\bar{\mathbb{D}}}$. Under $\bar{\mathbb{D}}$, compliance requires that no deforestation attempt should result in forest clearing; under $\bar{\mathbb{U}}$, compliance requires that every deforestation attempt be blocked before clearing or before the resulting timber or land is used.

Under each area condition, the extended game reduces to that in our base model under $\bar{\mathbb{U}}$, with distinct blocking cost. Under $\bar{\mathbb{U}}$, coalitions block by the least costly means, so they block use rather than block deforestation; the game thus reduces to that in our base model under $\bar{\mathbb{U}}$, with blocking cost $\eta_{\bar{\mathbb{U}}}$. Under $\bar{\mathbb{D}}$, only blocking a deforestation attempt is relevant to compliance, so the game reduces to that in our base model under $\bar{\mathbb{U}}$ with blocking cost $\eta_{\bar{\mathbb{D}}}$. Therefore, our results in §3 for $\bar{\mathbb{U}}$ apply for both area conditions, with those distinct blocking costs.

Each core of interest corresponds to the core under $\bar{\mathbb{U}}$ in the base model with distinct blocking cost η ($\eta = \eta_{\bar{\mathbb{D}}}$ and $\eta = \eta_{\bar{\mathbb{U}}}$ for $\bar{\mathbb{D}}$ and $\bar{\mathbb{U}}$ in the extension, and $\eta = +\infty$ for $\bar{\mathbb{D}}$ in the base model). Hence an understanding of how the potential to block deforestation affects the core under $\bar{\mathbb{D}}$ and a buyer's best choice of condition follows directly from the effects of lowering η on the core under $\bar{\mathbb{U}}$ in the base model, explained at the end of §3.3: Less costly blocking can help maintain no-deforestation outcomes under limited cooperation (if the core is non-empty), but can make the core empty or allow no-deforestation outcomes that do not compensate.

Paradoxically, the potential to block deforestation makes $\bar{\mathbb{D}}$ *less* appealing. This change makes $\bar{\mathbb{D}}$ more similar to $\bar{\mathbb{U}}$. This weakens $\bar{\mathbb{D}}$ under full cooperation and when compensation is required, which were exactly the circumstances in which $\bar{\mathbb{D}}$ was preferred in the base model; the change strengthens $\bar{\mathbb{D}}$ in the limited-cooperation cases where inexpensive blocking helps prevent deforestation, but $\bar{\mathbb{U}}$ remains more effective in precisely those cases because blocking use is less costly. Consequently, our recommendations for a context with potential to block deforestation add alternatives or further restrict the feasible set of incentive values I in scenarios where $\bar{\mathbb{D}}$ was previously recommended, without adding $\bar{\mathbb{D}}$ to the limited-cooperation recommendations for preventing deforestation.

4.4. When Blocking Use May Not Succeed

In practice, blocking use of deforested land – if attempted – may not always succeed. Section EC.6.3 extends the base model to a case where all attempts to block use succeed with probability $p \in (0,1]$ and all attempts fail with probability $1-p$. Coalitions incur the blocking cost regardless of whether their attempts succeed. Under $\bar{\mathbb{U}}$, if successful blocking attempts are made on all locals who deforest land, the area complies and locals receive the incentive; otherwise, locals obtain their net incomes from deforestation. Coalitions maximize their *expected* net income.

Allowing blocking to fail changes the non-cooperative game in an important way: equilibria may feature both deforestation and blocking attempts (unlike the base model, where no blocking occurs). In particular, a coalition S may deliberately take a gamble and let a subset of its members $R \subseteq S$ deforest even though it anticipates that another coalition will subsequently attempt to block them.

The threshold p_2 (defined in §EC.6.3) identifies when this mechanism disappears. If $p > p_2$, blocking is sufficiently likely to succeed that every coalition would rather comply than undertake even the cheapest possible gamble on blocking failure. Consequently, the complex equilibria described above are eliminated: every equilibrium has either $d^\star = 0$ or $d^\star = 1$, and no blocking attempts are made on the equilibrium path. More strongly, Proposition EC.1 shows that the entire cooperative game is then equivalent to the base-model game under $\bar{\mathbb{U}}$ with blocking cost η/p. In this parameter regime, a probabilistic failure therefore has the same effect as increasing the blocking cost.

With $p > p_2$, all the results in Theorem 2 and in §3.4 then apply by taking a blocking cost η/p.

The situation is substantially more complex when $p \leq p_2$. The complex nature of the equilibria in the non-cooperative game makes it challenging to even fully encode the cooperative game, let alone solve for its recursive core. We instead focus on proving a subset of the results in Theorem 2: we seek sufficient conditions under which the core only contains outcomes with $d^\star = 0$.

With full cooperation and when locals collectively prefer the incentive, Proposition EC.1 shows that every core outcome, if any, has $d^\star = 0$ for every $p \leq p_2$, which mirrors the findings in Theorem 2.

To describe what can happen with limited cooperation, Proposition EC.1 introduces a second threshold $p_1 \leq p_2$. If $p < p_1$, blocking is sufficiently unreliable that some subcoalition S prefers to deforest and risk being blocked even when it is offered the largest allocation compatible with sustaining the rest of its family in an outcome with $d^\star = 0$. Under limited cooperation, Proposition EC.1 shows that no core outcome has $d^\star = 0$ if $p < p_1$.

For intermediate probabilities $p_1 \leq p < p_2$, the conclusions are instance-dependent. We construct instances in which, for every blocking cost $\eta > 0$, the core is nonempty and contains an outcome with some deforestation, $d^\star \neq 0$. At low blocking cost, some coalitions are willing to attempt blocking, but a deforesting coalition continues to prefer a gamble on blocking failure; at high blocking cost, the blocking threat itself becomes weak.

Finally, if no family strictly prefers the incentive, Proposition EC.1 shows that for $p \leq p_2$, every nonempty core contains an outcome with universal deforestation, $d^\star = 1$.

4.5. Hybrid Conditions

Whereas our base model applies a single condition uniformly across the target area, §EC.6.6 considers hybrid schemes that partition the area into non-overlapping sub-areas and apply one condition to all locals in each sub-area. The question is whether (and when) this added flexibility can make hybrid schemes more robust than the base-model "uniform" conditional incentives, which are special cases.

With full cooperation, sub-area conditions offer *no benefit* relative to a single, area-wide no-deforestation condition $\bar{\mathbb{D}}$. To have only outcomes with $d^\star=0$ under a hybrid scheme requires that every sub-area satisfy $I(H) > D(H)$ for all locals H in that sub-area, because otherwise the grand coalition $\{\mathcal{L}\}$ could deforest only in that sub-area while retaining the incentive in other sub-areas. This would imply that $I(\mathcal{L}) > D(\mathcal{L})$ also holds, so the area-wide condition $\bar{\mathbb{D}}$ would yield a non-empty core that has only outcomes with $d^\star = 0$. Therefore, the area-wide $\bar{\mathbb{D}}$ would be more robust because $I(\mathcal{L}) > D(\mathcal{L})$ may hold even though $I(H) > D(H)$ does not hold for some $H \subseteq \mathcal{L}$.

With limited cooperation, a hybrid scheme could be beneficial by eliminating the need to induce a preferred core outcome. To see this, consider a special setting wherein a sub-area is defined for each family $F \in \bar{\pi}$. A hybrid scheme that applies $\bar{\mathbb{D}}$ separately to each sub-area would lead to a non-empty core containing *only* outcomes with $d^\star=0$ if and only if $I(F) > D(F)$ for every family. Under the same inequalities, applying the uniform area condition $\bar{\mathbb{D}}$ also leads to core outcomes with $d^\star=0$, but also to core outcomes with $d^\star=1$. The hybrid scheme would therefore increase the set of instances wherein *only* core outcomes with $d^\star=0$ exist, eliminating the need to induce a specific outcome type.

More generally, a hybrid scheme with separate no-deforestation conditions $\bar{\mathbb{D}}$ applied to sub-areas defined for selected families and with a single no-use condition $\bar{\mathbb{U}}$ applied in aggregate to the remaining sub-area could prevent deforestation in more problem instances. If $I(F) > D(F)$ for each selected family F, the hybrid scheme will lead to (only) outcomes with $d^\star = 0$ exactly when the core for the no-use condition $\bar{\mathbb{U}}$ over the smaller sub-area has the corresponding property. This can strictly outperform uniform conditions because removing selected families reduces the number of outsiders whom an enforcing coalition in the remaining sub-area must credibly threaten to block. However, this could also remove families that might contribute to blocking threats, so the partitioning should be done carefully because an arbitrary split would not always strictly improve robustness.

Hybrid conditions could also be beneficial when area-wide deforestation prevention fails. Then, sub-areas governed by successful separate conditions could remain protected. Hybrid schemes may therefore reduce total forest loss, although formalizing that objective would require accounting for the amount of forest at risk in each sub-area, which could significantly complicate the overall game.

5. Conclusion

This paper offers guidance for a commodity buyer (or other organization) on how to prevent deforestation in a target area and benefit smallholder farmers, by offering a positive incentive tied to a condition for the area. We refer to all individuals who could profit from deforesting land in the target area as *locals*. A *family* is a set of locals who can coordinate their decisions and transfer utility, as in an extended family, agricultural cooperative, or neighborhood community. With *full cooperation* any set of locals can coordinate and transfer utility among its members, whereas with *limited cooperation* this is confined within distinct families; §EC.2 provides illustrative examples.

In an area with full cooperation the buyer should use an area no-deforestation condition and incentive that is more valuable to the locals, collectively, than to deforest land in the area. This prevents deforestation even with a real-world imperfect incentive that for some locals is less valuable than deforestation. In this case, locals redistribute the incentive so that those who prefer deforestation are compensated for forgoing it. By compensation, we mean that each local is at least as well off as he could have been by engaging in deforestation, rather than simply being better off than before the program. With full cooperation, the area no-deforestation condition outperforms area no-use and individual no-deforestation conditions for both robustness and cost-effectiveness. It is best even if the buyer's sole objective is to prevent deforestation, as the compensation requires no additional payment from the buyer. Compensation may be particularly important for a buyer with a whose mission is centered on fair treatment for farmers, such as Tony's Chocolonely.

To use the area no-deforestation condition in an area with limited cooperation, the buyer must offer an incentive that is more valuable to each family than deforesting land in the area, and promote coordination across families. The buyer could convene representatives of the distinct families to enable them to discuss and jointly decide to forgo deforestation for mutual benefit. If the buyer induces coordination, area no-deforestation again provides the most robust and least-cost way to prevent deforestation, while compensating those who forgo it. If the buyer cannot induce such coordination and aims to compensate locals, the fall back is the conventional approach: individual condition with a perfect incentive that makes each local individually better off than with deforestation.

If the buyer lacks the means to compensate all locals, we recommend an area no-use condition. This is particularly relevant where forest is burned illegally for agriculture, and locals can "block" the use of cleared land for agricultural production. This is also especially helpful to deter potential entrants or other outsiders from clearing public forest: inability to use the land makes clearing unprofitable, even though such locals would not receive the buyer's incentive.

The area no-use condition works with any incentive that adequately rewards some locals to create a credible threat of blocking use. It may induce redistribution of the incentive, to increase the community's ability to block or to reduce the number of locals that would need to be blocked.

A no-use condition favors forest recovery. Following a forest fire caused by accident or nature, locals could remain compliant with a no-use condition. Forest can regenerate naturally after fire when the land is not used for agriculture. NGOs and carbon offset providers help forests regenerate.[4] However, the buyer should complement an area no-use implementation by providing resources and training for locals to help them limit the spread of fires.

Adopting a no-use condition also enables recovery from deforestation that occurred in the recent past, for compliance with EUDR. By tracing its inputs to the entire area and ensuring that any land deforested after 2020 is kept out of production, a buyer would be compliant with EUDR. An *area* – rather than an *individual* – implementation of a no-use condition would be particularly effective, because verifying individual compliance would be fraught with difficulty, especially if a farmer's plots are small or if noncompliant production of the commodity occurs nearby. Instead, by incentivizing and verifying no-use across a *large* contiguous area, commodity buyers can reduce the cost and difficulty of verifiable traceability and remain compliant with EUDR.

An important caveat for area no-use concerns distributive justice. This approach may prevent deforestation without compensating the locals who would have gained most from deforestation, who are typically the poorest of smallholders, with the least land holdings. These locals may receive the least value from the practical incentives that commodity buyers can offer. For example, a price premium for RSPO-certified palm oil is worth least to a farmer with least production, and unattainable for one who cannot afford certification or join a cooperative.[5] A *least-cost* no-use program would *deepen* this disparity. To create a credible threat of blocking, this would target incentives to the locals with relatively little to gain from deforestation, whose participation could be secured at relatively little cost. It may also target incentives to large families whose use of cleared land would be more difficult to block. As a result, individuals with most to gain from deforestation may receive nothing at all. In practice, this least-cost targeting could favor large, established agricultural cooperatives with mature holdings – including land cleared well before 2020 – over smaller cooperatives and individual smallholders. Thus, a buyer may continue sourcing from the area under a no-use condition while placing a disproportionate share of the compliance burden on the poorest smallholders. This tension matters for buyers seeking to combine EUDR compliance with their due-diligence and living-income

[4] e.g. MORFO and Air Seed use drones to drop seedpods to accelerate forest regeneration, and collect carbon credits.

[5] Smallholder cooperatives seeking group certification often avoid admitting new members who have access to forested areas, for fear they may engage in deforestation and cause the entire group to lose its certification. §EC.2 discusses several such examples related to RSPO certification.

commitments (Lambin and Furumo 2023); it may also create challenges under future EU legislation such as the CSDDD, which calls for "fair prices" paid to suppliers.

Buyers should therefore distinguish clearly between a commitment to prevent deforestation and a commitment to benefit and compensate the smallholders who forgo it. Under individual and area no-deforestation conditions, those who forgo deforestation are compensated; under area no-use, some are instead deterred through blocking and receive no benefits.

Area no-use has a subtle dependence on the extent of cooperation in the target area. It requires that at least one family prefers the incentive. Reduction in cooperation – dividing a family – can isolate a smaller family that prefers the incentive, create a credible threat of blocking, and thus prevent deforestation. Further reduction in cooperation – splitting that smaller family – could destroy their interest and capability to enforce the area no-use condition. The buyer need not fully understand the structure of all families. The key is to ensure that at least one family prefers the incentive to deforestation and some members are sufficiently motivated, to create credible enforcement of no-use.

Area no-use also depends non-monotonically on the cost of blocking use. In an area with full cooperation, the compensation mechanism for compliance takes precedence if blocking is very costly, whereas blocking threats take precedence if blocking is cheap. At an intermediate level of blocking cost, however, locals may disagree over whether to share the incentive or retain more of it and rely on blocking. That might cause a failure to comply with no-use, even if the locals collectively prefer the incentive to deforestation and despite their full ability to cooperate. To avoid disagreements in such cases, buyers should invest substantially in reducing the blocking cost. This could involve, for instance, supporting local law enforcement, providing reporting hotlines or apps, and training locals to use Forest Watch or other free remote-sensing platforms to detect deforestation quickly (see §EC.4 for examples). Early intervention, before a farmer has established profitable agricultural operations, would make his relinquishing the land less costly and contentious, and facilitate enforcement.

A caveat with the no-use condition arises in case an attempt to block use is not sufficiently likely to succeed. In practice, blocking use may require intervention by local authorities such as courts or police, who might not always arrive even though locals notify them. In such cases, some locals may deforest land, anticipating that blocking attempts are likely to fail. Simply reducing the cost for locals to attempt to block deforestation would not prevent deforestation. The buyer must work with local authorities to ensure that they respond to notifications and support locals' enforcement efforts.

A caveat with the area no-deforestation condition arises in case locals can block others from deforesting land, as may be possible where deforestation occurs by logging. Paradoxically, this makes the area no-deforestation condition *less* appealing. Compliance may occur due to a credible threat of blocking of deforestation, rather than compensation, making no-deforestation operate much like no-use. That undermines $\bar{\mathbb{D}}$ as best condition to prevent deforestation and compensate locals in an area with full cooperation. With limited cooperation, a credible threat of blocking deforestation may help to prevent deforestation, yet an area no-use condition is better than the area no-deforestation condition to prevent deforestation, insofar as blocking use is easier than blocking deforestation.

For a large area with limited cooperation, the buyer should consider a hybrid design rather than uniform condition for the entire area. The buyer may be able to prevent deforestation and compensate by dividing the area into subareas, each a cohesive village, landholdings of an extended family, or other subarea with full cooperation among locals, and establishing a separate no-deforestation condition for each subarea. Alternatively, the buyer could apply the no-use condition separately to subareas, and potentially prevent deforestation with a practical incentive that would be insufficient with a no-use condition for the area as a whole. Research is needed to understand how to optimally subdivide an area to reflect actual spatial patterns of cooperation, account for the distribution of forest within the area, and target practical, imperfect incentives by subarea.

Area conditional incentives and the theory developed here may also be relevant to prevention of crop-residue burning, restoration of peatland, or other challenges that depend on decisions by smallholders. However, our pessimistic recursive core solution concept and theoretical results are based on strong rationality assumptions. More field research is needed to evaluate how smallholders perceive and respond to area conditional incentives, in order to test and improve theory and practice.

Lastly, our analysis points to a methodological direction for future research. The pessimistic recursive core has an intuitive rational micro-foundation and yields sensible, explainable predictions, but its recursive structure requires lengthy proofs even for stylized models. Future work could develop simpler solution concepts that approximate the recursive core while preserving qualitative insights.

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E-companion to Area Conditions and Positive Incentives: Engaging Locals to Protect Forests

EC Contents

Table EC.1: Summary of notation and definitions

Symbol / term	Meaning
Model primitives	
$\mathcal{L}$	Set of all locals.
ℓ	Index for an individual local, $\ell \in \mathcal{L}$.
$\bar{\pi}$	Exogenous partition of the locals into families, $\bar{\pi} \in \Pi_{\mathcal{L}}$.
$\Pi_R^{\prec\bar{\pi}}$	Set of all partitions $\pi_R \in \Pi_R$ that are finer than $\bar{\pi}$, for $R \subseteq \mathcal{L}$.
π	Feasible partition of the locals into coalitions, $\pi \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$.
F	A family, $F \in \bar{\pi}$.
S	A feasible coalition, with $S \subseteq F$ for some $F \in \bar{\pi}$; for a fixed partition π, each coalition satisfies $S \in \pi$.
$x(S)$	Sum of the components of x indexed by S, $x(S) = \sum_{i \in S} x_i$.
x_S	Subvector of x containing the components indexed by S.
Deforestation decisions	
d	Vector of deforestation decisions indexed by $\mathcal{L}$: $d_\ell = 1$ if local ℓ deforests and $d_\ell = 0$ otherwise.
c_ℓ	Cost incurred by local ℓ when engaging in deforestation.
D	Vector of net income gains from engaging in deforestation, with component $D_\ell > 0$ for local ℓ.
Blocking decisions	
$B(d)$	Blocking decisions of all locals after observing d, represented by a matrix in $\{0,1\}^{\mathcal{L}\times\mathcal{L}}$; $B_{i\ell}(d) = 1$ if and only if local i blocks local ℓ, and $B_{i\ell}(d) = 0$ whenever i and ℓ belong to the same coalition.
η	Cost to block one local.
Income, compliance, and incentives	
v_ℓ^{sq}	Status-quo income of local ℓ, without deforestation and without the incentive.
$V_\ell(d,B)$	Net income of local ℓ, given the deforestation and blocking decisions.
$\mathbb{C}_\ell(d,B)$	Compliance indicator for local ℓ; it equals yes when the deforestation and blocking profile satisfies the condition for local ℓ, and no otherwise.
$\mathbb{C}$	Condition (compliance rule).
$\mathbb{I}$	Individual condition: local ℓ receives the incentive if and only if $d_\ell = 0$.
$\bar{\mathbb{D}}$	Area no-deforestation condition: all locals receive the incentive if and only if no local deforests.
$\bar{\mathbb{U}}$	Area no-use condition: all locals receive the incentive if and only if no local uses deforested land to generate income.
I	Vector of net income gains from the incentive, with component $I_\ell \geq 0$ received by local ℓ when the applicable condition is satisfied.
$\mathcal{G}$	Set of locals who individually prefer the incentive to deforestation, $\mathcal{G} = \{\ell \in \mathcal{L} : I_\ell > D_\ell\}$.
Perfect incentive	Incentive satisfying $I_\ell > D_\ell$ for every local $\ell \in \mathcal{L}$, equivalently $\mathcal{G} = \mathcal{L}$.
Coalition net incomes and the non-cooperative game	

Symbol / term	**Meaning**
$V_S(d,B)$	Aggregate net income of coalition S, net of deforestation and blocking costs.
a_ℓ	Net income allocated to local ℓ; feasibility requires $a(S)=V_S(d,B)$ for every coalition $S\in\pi$.
$\mathcal{Q}(\pi,\mathbb{C})$	Set of subgame-perfect equilibria of the non-cooperative game for partition π under condition $\mathbb{C}$.
$d^\star$	Equilibrium deforestation decisions.
$B^\star(d)$	Equilibrium blocking policy as a function of the deforestation decisions.
Equilibrium with deforestation	Equilibrium with $d^\star=1$.
Equilibrium with no deforestation	Equilibrium with $d^\star=0$.
$T(\pi,\mathbb{C})$	Set of equilibrium indicators $d^\star\in\{0,1\}$ that can arise for partition π under condition $\mathbb{C}$.
Partition-function cooperative game	
$w(S;d^\star)$	Partition-function value giving coalition S's net income under equilibrium indicator $d^\star\in T(\pi,\mathbb{C})$, with the partition π understood implicitly.
$W(\pi,\mathbb{C})$	Correspondence collecting the partition functions associated with all equilibrium indicators in $T(\pi,\mathbb{C})$.
Outcome $(\pi,d^\star,a)$	A partition, equilibrium indicator, and feasible allocation of net income.
Residual-game and recursive-core notation	
$\pi_{\mathcal{L}\setminus R}$	Fixed partition of the locals outside the residual set $R\subseteq\mathcal{L}$.
$C(R;\pi_{\mathcal{L}\setminus R})$	Pessimistic recursive core of the residual game faced by the locals in R.
Core $C(\mathcal{L};\emptyset)$	Pessimistic recursive core of the full cooperative game.
Dominated outcome	Residual-game outcome from which some feasible group of locals can deviate so that every coalition formed by the deviators is strictly better off under every plausible residual response.
Residual game	Game faced by the locals in $R\subseteq\mathcal{L}$, given the fixed partition $\pi_{\mathcal{L}\setminus R}$ of the other locals.
Performance requirements	
Outcome with $d^\star=0$	Outcome in which no local deforests.
Compensating outcome	Outcome satisfying $a_\ell\geq v_\ell^{\mathrm{sq}}+D_\ell$ for every local $\ell\in\mathcal{L}$.
Prevents deforestation	The core is non-empty and every core outcome has $d^\star=0$.
Prevents deforestation and compensates	The core is non-empty and every core outcome has $d^\star=0$ and satisfies $a_\ell\geq v_\ell^{\mathrm{sq}}+D_\ell$ for every local $\ell\in\mathcal{L}$.
Blocking-cost thresholds under $\bar{\mathbb{U}}$	
$\eta_1(\pi)$	Supremum of blocking costs for which some coalition $S\in\pi$ can profitably block every local in $\mathcal{L}\setminus S$; if $\eta<\eta_1(\pi)$, only equilibria with $d^\star=0$ arise.

Symbol / term	Meaning
$\eta_2(\pi)$	Supremum of blocking costs for which the coalitions that prefer the incentive have enough aggregate blocking capacity to block a coalition that weakly prefers deforestation; if $\eta > \eta_2(\pi)$, only equilibria with $d^\star = 1$ arise.
$\overline{\eta}$	Largest blocking surplus of any feasible coalition; if $\eta > \overline{\eta}$, no coalition can profitably block even one local and $\bar{\mathbb{U}}$ is equivalent to $\bar{\mathbb{D}}$.
η_2	Core-level threshold such that $\eta \leq \eta_2$ guarantees at least one core outcome with $d^\star = 0$ when at least one family prefers the incentive.
$\overline{\eta}_1$	Maximum blocking surplus per outsider, $(I(S) - D(S))/\lvert\mathcal{L} \setminus S\rvert$, over all feasible non-grand coalitions S.
$\underline{\eta}_1$	Maximum blocking surplus per outsider over feasible non-grand coalitions S drawn from families that prefer the incentive.
Cooperation	
Full cooperation	The family partition is $\bar{\pi} = \{\mathcal{L}\}$, so any coalition of locals is feasible.
Limited cooperation	The family partition has at least two families, so coalition formation is constrained.

EC.1. Examples of Programs Implemented by Commodity Buyers

- **Unilever** *(Indonesia & Malaysia, palm oil).* Unilever engages tens of thousands of Indonesian smallholders through its three new smallholder hubs in Aceh, North Sumatra, and Riau. In collaboration with NGO partners, the company first creates GPS polygon maps for every farmer (47 000 mapped so far) and deforestation is checked with Unilever's dashboard, which combines satellite and radar alerts to flag any tree-cover loss in near-real time (Unilever 2025b). Mapped farmers receive agronomy training (26 000 trained) and help to gain RSPO Independent Smallholder (ISH) certification (17 500 certified). Unilever then purchases the farmers' ISH credits, which amounts to paying a premium for every palm fruit produced on certified plots (148 000 such credits were bought in 2024). If an alert confirms that forest on a mapped plot has been newly cleared, the farmer's credit income is suspended while the case is investigated (Unilever 2022b). Unilever suspends suppliers linked to deforestation but is open to allowing them to rejoin if they can demonstrate they have improved their practices in line with best industry standards, including providing a recovery plan for any recent deforestation or new development on peat in their supply chain (Unilever 2022b).
- **Charoen Pokphand (CP) Foods** *(Thailand, feed corn).* CP Foods has implemented a corn traceability system that combines blockchain technology, GPS mapping and monitoring, and a mobile app to ensure that maize used in its feed production is sourced from areas free of forest encroachment and stubble burning across Myanmar, Laos, and Vietnam (WBCSD 2020). Once verified, growers receive a comprehensive incentive bundle: free soil-fertility testing, hands-on

GAP (Good Agricultural Practices) training that helps them cut fertiliser use and plough crop residues back into the soil instead of burning them, on-farm drone-spraying demos and other precision-agriculture tools supplied via CP's True Digital arm, shared modern machinery, and pop-up buying stations that reduce transportation costs and pay transparent, pre-announced prices (WBCSD 2020).

- **Wilmar** *(Indonesia & Malaysia, palm oil).* Wilmar's smallholder scheme in Sabah links a package of cash and agronomic support to compliance with RSPO sustainability standards. Independent growers who join the Wild Asia Group Certification Scheme (WAGS) must keep their plots free of illegal forest clearance and follow RSPO good-practice criteria; in return they receive two rewards: a premium on every tonne of fresh-fruit bunches delivered to Wilmar mills and a second RSPO premium when downstream buyers purchase the certified palm oil. Certified members also gain continuous field advice, wholesale-price fertilizer with supervised application, and priority access to a replanting finance facility that Wilmar is designing with banks and NGOs to offer low-interest loans even to farmers lacking formal land titles (Wilmar International 2025).
- **Tony's Chocolonely** *(Ghana & Ivory Coast, cocoa).* Tony's pays cocoa farmers a Living Income Reference Price (LIRP) via farmer cooperatives and monitors individual farm plots in collaboration with Satelligence. The company also offers assistance through training in agroforestry methods and best practices (Tony's Chocolonely 2023).
- **Mondelez International** *(Ivory Coast & Ghana, cocoa).* Through its Cocoa Life initiative, the company pilots *payment-for-environmental-services* contracts: staged cash rewards to individuals for planting and nurturing shade trees and avoiding further forest clearance. Payments stop if the farmer expands into forest (Mondelez International 2021).
- **Cargill "Triple S"** *(Brazil, soy).* Cargill offers export premiums and priority contracts only to soy growers verified (via satellite and audit) as having zero post-2008 native-vegetation conversion; non-compliant farms lose the channel (Cargill, Inc. 2021).
- **Starbucks** *(Global origins, coffee).* C.A.F.E. Practices pays quality & sustainability premiums to farmers or cooperatives scoring high on its rubric; a zero-tolerance rule bars any farm with land deforested since 2004, and failure in audit removes the premium (Starbucks 2024).
- **Mars Inc** *(Ivory Coast & Ghana, cocoa).* Mars pays farm-gate premiums ($50–$120 per ton) solely for cocoa traced to farms verified deforestation-free via GPS and satellite; expansion into forest voids the premium (Ionova 2018, Askew 2022).
- **Hershey's** *(Ivory Coast, cocoa).* Hershey boosts cocoa-farmer livelihoods by helping smallholders expand Village Savings & Loan Associations for affordable credit, and funding community schools (Hershey 2023).

EC.2. Full vs. Limited Cooperation and Area-Based Schemes in Practice

This appendix places several of our model primitives in a more practical setting. §EC.2.1 discusses how cooperation can arise in smallholder communities and how commodity buyers or other organizations implementing conditional incentives might assess the degree of cooperation in a target area, and particularly whether full or limited cooperation is the more plausible approximation. §EC.2.2 then surveys several examples of area-based conditional incentives and discusses whether each is more naturally characterized by full or limited cooperation.

EC.2.1. Cooperation in Smallholder Farming Communities

Our model uses a "family" as a reduced-form representation of the limits on locals' ability to cooperate. Locals within a family can coordinate decisions and share benefits and costs, whereas such cooperation is not possible across families. Thus, a family need not correspond literally to a biological family. It may instead correspond to an extended kinship or ethnic group, a cohesive village community, members of a farmer organization who interact closely, or another group supported by sufficiently strong social ties and institutions. Likewise, "full cooperation" does not mean that all locals always agree or act altruistically. It means that, within the area under consideration, the institutional and social relationships are sufficiently strong that any subset of locals can potentially coordinate and make transfers among themselves. Limited cooperation instead recognizes boundaries across which such coordination and transfers are difficult or infeasible.

How cooperation can arise. A large literature on social capital provides mechanisms through which the cooperation represented in our model can emerge. Durlauf and Fafchamps (2005) survey this literature and emphasize the role of social networks, trust, shared norms, and repeated social interaction in supporting mutually beneficial behavior. In rural settings, these relationships may be embedded in kinship networks, villages, ethnic groups, farmer organizations, or informal associations. For example, existing evidence documents transfers and risk sharing within extended families (Rosenzweig 1988, Angelucci et al. 2018), rotating savings and credit groups (Geertz 1962, Ksoll et al. 2016), and collective action within villages (Falcon et al. 2022). Consistent with this interpretation, Fafchamps and Gubert (2007) find in rural communities that geographic proximity – and factors correlated with it, including kinship – is an important determinant of interpersonal risk-sharing relationships, and that such pre-existing relationships strongly predict subsequent gifts and informal loans. These forms of mutual assistance are particularly relevant to our transferable-utility formulation because they provide mechanisms for redistributing the benefits and costs of collective decisions within a group.

Cooperation can also be supported by formal or semi-formal institutions rather than social ties alone. Agricultural cooperatives, village organizations, savings-and-loan associations, and community natural-resource institutions can provide mechanisms for collective decision-making, transfers, monitoring, and enforcement. The extensive literature on collective management of common resources is particularly relevant. Ostrom (1990, 2000), for example, identifies institutional characteristics repeatedly associated with sustained collective action, including clearly defined group boundaries, participation by members in collective decisions, monitoring, sanctions for violations, and accessible mechanisms for resolving disputes. These institutions can help groups overcome collective-action problems even when cooperation does not arise solely from kinship or close personal relationships. Conversely, their absence can make full cooperation implausible even when all locals formally belong to the same village or cooperative. This is one reason that we do not equate formal membership in an organization with a particular cooperation structure.

How cooperation can be recognized. The theoretical family partition in our model is not directly observable, and we do not suggest that a buyer can readily identify this in practice. Instead, the buyer can assess whether full or limited cooperation is a plausible approximation by combining observable structural characteristics with evidence of actual collective behavior. Some useful structural indicators are the size and geographic concentration of the group; whether it consists of one cohesive community or several identifiable subgroups; kinship, ethnic, village, or organizational ties; stability of membership; the presence of recent entrants; and the existence of common governance institutions. These indicators speak to the opportunities for repeated interaction and the boundaries of the networks through which coordination and transfers can occur.

Behavioral evidence is even more directly informative for our model. Relevant questions include whether locals have previously undertaken joint projects; whether they contribute labor or money to common activities; whether they provide gifts, loans, or other transfers to one another; whether the group operates a common fund or redistributes collective benefits; whether members jointly respond to shocks or community problems; and whether collective decisions can be monitored and enforced. The randomized experiment of Falcon et al. (2022) illustrates the value of such behavioral evidence: some villages used deliberate socialization and coordination to obtain a collective incentive for avoiding fire, whereas collective-action failures prevented other villages from doing so. More generally, evidence that collective decisions and transfers occur predominantly within identifiable subgroups, rather than across the entire area, provides a natural empirical interpretation of the families in our limited-cooperation model.

Existing tools for measuring social capital provide one way to collect such information systematically. For example, the World Bank's Social Capital Integrated Questionnaire (Grootaert et al. 2004) separately elicits information on groups and networks, trust and solidarity, and, particularly relevant for our purposes, *collective action and cooperation.* Its questions examine participation in organizations and informal networks, contributions to and benefits from these groups, trust, and whether community members have worked together on joint projects or in response to common problems. Such survey information could be complemented by interviews with community leaders and members, participatory mapping of social and organizational relationships, and administrative information available to buyers or certification organizations. Importantly, trust or group membership alone is not sufficient for the cooperation assumed in our model. The most relevant evidence concerns whether members can actually coordinate decisions and share benefits and costs.

Industry proxies. Commodity buyers may already possess information that is useful for this assessment. The RSPO certification system provides a concrete example. As we discuss subsequently, RSPO classifies Independent Smallholder groups by risk for purposes of determining audit intensity. Its assessment considers group size and management capacity and distinguishes groups according to characteristics including geographic and socioeconomic homogeneity, geographic or jurisdictional separation, the presence of new members, the stability of group membership and management, and other forms of heterogeneity (Roundtable on Sustainable Palm Oil 2025). These risk ratings were not designed to measure the cooperation in our model, and we therefore do not interpret them as direct measures of the family partition. Nevertheless, several of their inputs are informative proxies: a small, stable, geographically concentrated and relatively homogeneous group with established governance is more plausibly approximated by full cooperation, whereas a large and dispersed group containing distinct subgroups or recent entrants more naturally motivates limited cooperation. We discuss concrete examples of cooperatives in §EC.2.2 to illustrate both possibilities even within the same formal institutional setting.

Assessment under uncertainty. Ultimately, classifying cooperation requires judgment, and we would not recommend attaching unwarranted precision to this difficult-to-measure characteristic. A buyer can triangulate information from group structure, geographic and social networks, governance institutions, observed transfers, and past collective behavior to identify plausible cooperation boundaries. When the evidence is ambiguous, the model can instead be applied under several plausible family partitions to examine whether the recommendation is robust to the cooperation structure. This is the approach we take in our empirical illustration in §EC.7, rather than claiming to estimate a unique family partition from limited survey information. Our analysis also suggests that the cooperation structure need not simply be treated as a constraint: because it depends partly on the boundary of the area, a buyer can sometimes define smaller subareas that better coincide with existing cooperative groups and apply different conditions across them, as in our analysis of hybrid schemes. Thus, assessing cooperation can inform both the choice of conditional incentive and the appropriate geographic and organizational scale at which it is implemented.

EC.2.2. Examples of Area-Based Schemes

1. RSPO Independent Smallholder (ISH) Certification Standard as an area condition with full or limited cooperation.

The 2024 RSPO ISH certification, described in Roundtable on Sustainable Palm Oil (2024), bears a natural interpretation as an area condition. The certification applies to independent smallholder *groups*: the unit of certification is the group manager together with all disclosed group members, and the certificate is awarded to the group as a whole (see section 3.1 of the standard). The certification process requires each group member to provide GPS coordinates and maps for each of their land plots and to outline any existing High Carbon Stock (HCS) or High Conservation Value (HCV) forested areas on the plots. The corresponding no-deforestation condition is that *no group member* may clear HCV/HCS forest anywhere on the entire certified land base (section 4 of the standard[6]). In the context of our model, the relevant "group" $\mathcal{L}$ is the cooperative membership and the relevant "area" is the union of all protected plots owned or operated by the group members.

The RSPO inspection framework, described in (Roundtable on Sustainable Palm Oil 2025), directly connects to our classification of a setting as having full or limited cooperation. In determining the sample size for how many group members to audit, the RSPO standard considers *a group's size* and a classification of the group as *low, medium, or high risk*. The latter risk classification depends on several factors related to the group's composition, management structure, and history, such as whether the group and its manager are well established, whether the group has new members, the degree to which group members are geographically or jurisdictionally separated from one another, the range of terrains on the group member's plots, the levels of experience with palm oil cultivation, the diverse sizes of the plantations, and a range of socioeconomic circumstances among the members (see section 6.15 of the standard). In the context of our model, a small group size and a "low-risk" classification would make the group a candidate for full cooperation, whereas a large group size and a "high-risk" rating would more likely resemble a limited-cooperation setting.

The RSPO website includes examples of certified cooperatives that could be plausible candidates for *full cooperation* or *limited cooperation*.[7] Koperasi Serba Usaha Suka Makmur in North Sumatra has only 29 smallholders covering 60.15 hectares, making it a good candidate for full cooperation Roundtable on Sustainable Palm Oil (n.d). KUD Karya Mulya in South Sumatra, with 129 smallholders on 496.93 hectares in one village, is larger but still small enough that full cooperation remains plausible; its own public account explains that the cooperative was set up in 1987 and is now seeking its second consecutive five-year RSPO certification, and stresses repeated training, good communication, transparency, trust, and many initiatives that benefit both members and non-members

[6] The standard also includes other requirements, e.g., related to operating palm plantations on peat or using fire for farm management. We restrict our discussion to aspects focused more narrowly on clearing forest.

[7] The information shared on the website does not include the groups' risk ratings.

of the group such as a fire-prevention unit, the construction of a small-sized mosque, or rural road projects (Roundtable on Sustainable Palm Oil 2021a). Other certified groups look more like the limited-cooperation case, even though they are formally a single cooperative. Koperasi Beringin Jaya in Siak has 209 smallholders organized into seven distinct groups, manages over 380 hectares, and achieved RSPO certification in 2024 (Roundtable on Sustainable Palm Oil n.d.). Similarly, KUD Tani Subur in Central Kalimantan had 508 certified smallholders organized into 34 distinct groups and 1,507 hectares under management in 2023, and its 2023 annual documentation states that 7 new members joined the group during the past year, adding 49 hectares to the group's land. The cooperative's journey report describes its efforts to diversify its activities and its focus on allowing members to have livelihoods beyond palm plantations Roundtable on Sustainable Palm Oil (n.d.). The size and separation into groups make this a good candidate for limited cooperation.

An alternative *hypothetical* implementation would be to define the area more widely, as a contiguous landscape containing the plots of cooperative members and nearby forest on public land or on land owned by farmers within the same village who are not members of the cooperative. Under such an implementation, the group would involve both members and non-members of the cooperative, and even a village with a small cooperative might have to be treated as a case with limited cooperation. The Suka Makmur example is illustrative for that point: although the cooperative itself is small and cohesive, its statement discusses how it had difficulty persuading other farmers in the village to join.

2. RSPO Jurisdictional Certification as an area condition with limited cooperation. The RSPO Jurisdictional Approach (JA) – described in Roundtable on Sustainable Palm Oil (2021b) – is a group certification framework operating at the scale of a government administrative area. The unit of certification is a *Jurisdictional Entity (JE)*, a legal entity comprising multiple participants in the palm-oil industry, such as smallholder farmers, collection centers, mills, crushers, and refiners. The JE is established within an administrative jurisdiction and governed by a multi-stakeholder supervisory board with government leadership. The relevant *area* is the jurisdiction itself, e.g., a country, state, province, or district. In the context of our model, the relevant "group" would then consist of all the actors participating in the JE, while the relevant "area" would be the full administrative territory covered by the JE. Pilots for such jurisdictional schemes are underway in Sabah, Malaysia; the Seruyan District of Central Kalimantan, Indonesia; and Ecuador.

The most natural interpretation of the JA in our model is as an area *no-deforestation* condition applied at the jurisdictional level. A key condition – which all participants in the JE must abide by – is to ensure that HCV/HCS areas within the jurisdiction are identified and protected or enhanced, and that no land clearing causes deforestation or damage to HCV/HCS forests (principle 7.12 of the standard). The JE is also required to establish jurisdiction-wide monitoring of deforestation and HCV/HCS areas, as well as jurisdictional "no-go" zones for oil palm expansion (section 3.4 of the standard). The JA framework clearly specifies that any non-compliance with the key principles and conditions – such as principle 7.12 on no-deforestation – would result in suspension or termination of the overall JE certificate (section 5.3 of the standard).

A JA implementation over a large area – such as the entire province of Sabah, Malaysia – would arguably give rise to a setting with *limited cooperation.* A jurisdiction is much larger than a farmer cooperative and includes many parties that may not have mutually strong ties. In fact, one of the core premises of the JA is that the relevant actors cannot normally coordinate among themselves, which is why the framework emphasizes the need for support from governmental bodies and key industry stakeholders to create and support the JE and its internal control systems and processes.

3. Area incentives based on the High Carbon Stock Approach (HCSA). The HCSA, which is described in (High Conservation Value Network 2023, HCSA Steering Group 2020b), is best understood as an integrated, participatory land-use planning approach for identifying forested areas that should be conserved and distinguishing them from land that may be developed. In practice, HCSA is implemented through HCV-HCS assessments combined with negotiated benefits agreements; in our terms, HCSA supplies the *area* and *forest-protection condition*, while the associated benefits agreement identifies the relevant group and supplies the positive incentive. Appendix 4 of the HCSA guide (HCSA Steering Group 2020a), which contains implementation details, maps closely to our framework. It stipulates that benefits and incentives should be provided *conditional on ongoing compliance* with conserving and enhancing HCV/HCS forests. Moreover, it stresses the importance of fairness in providing benefits: its design principles stipulate that overall benefits should outweigh the conservation costs and its illustrative notions of equity include directing greater benefits to the most needy, to those who bear the greatest costs, or where benefits have the greatest effect – notions that map closely to our requirement of *compensation.* (HCSA 2024), which is a specific guide for *smallholder palm oil farmer groups* in Indonesia, contains additional details that map onto premises of our model. The toolkit is built around identifying and mapping forests and land cover on *village lands*, which suggests a natural interpretation where the relevant "group" is the village smallholder group or local community, and the relevant "area" is the land controlled by the community. The toolkit's threat-assessment step, described in Section 6.2.2, explicitly asks the community to evaluate threats coming from inside the community or from *external outsiders*, to understand how forest protection could be undermined; and Section 6.2.3, focused on the development of a management plan, encourages the community to identify activities and establish teams and budgets that will implement its plan, including to address any identified threats to conservation, which can be thought of as providing the mechanism through which blocking arises in our model.

Specific examples of HCSA-based conditional incentives are:

- **New Britain Palm Oil (NBPOL)'s clan-based conservation rentals in Papua New Guinea as an area no-deforestation condition with limited cooperation.** In Papua New Guinea, 97% of the land is under customary ownership, so NBPOL implemented a scheme where payments were distributed according to *clan structures* (HCSA Steering Group 2020a, page 6). The scheme was implemented separately by clan; in each case, the relevant group involved all the families within the clan and NBPOL paid a rental per hectare for conservation areas, with payments made every three months and distributed to all families according to their documented rights. The payments were contingent on "no clearing" within HCV/HCS areas, which were mapped at contract inception. Because a clan involved multiple families, this could arguably constitute a setting with *limited cooperation*.
- **Golden Veroleum Liberia's (GVL) Production–Protection Agreements (PPA) in Liberia as an area no-deforestation scheme with compensation requirements.** In Liberia, the palm oil buyer GVL implemented a PPA where the relevant group was a community of smallholders (IDH 2017b,c, Fishman et al. 2017, IDH 2017a). The PPA required the community to protect five hectares of forest for every one hectare of community oil palm, and the incentive involved investment capital and loans, technical assistance, employment, guaranteed offtake, and an annual payment of 50 USD per hectare of oil palm developed. The workshop report IDH (2017c) is especially relevant for our purposes because it explicitly states that the incentive for the community as a whole should be greater than the opportunity cost of the land-use practices they would be asked to forgo to avoid deforestation, which closely resembles our requirement of *compensation* (albeit at a community level).
- **Mondelēz Cocoa Life / Nawa REDD+ PES pilot in Ivory Coast as area and individual schemes with compensation requirements.** In Ivory Coast, Mondelēz used HCSA to identify HCS forested areas, which were coupled with PES contracts that involved payments made in cash and/or in kind, conditional on compliance with a land-use plan and/or changes in agricultural practices (Mondelēz International and European Forest Institute 2018, European Forest Institute 2022). The approach combined individual and area approaches – the case study reports that the project planned agreements with 500 farmers and 5 communities – and involved three types of PES contracts, focused on agroforestry, reforestation, and conservation. Importantly for our model, a later evaluation suggested that the payments offered should exceed the potential revenue generated from new cocoa harvested from deforested areas, which is consistent with our definition of "compensation."

4. Landscape-level approaches as area schemes with limited cooperation. In collaboration with other commodity buyers, government agencies, civil society, and local communities, Unilever pursues several other "landscape-level" approaches (Unilever 2022a). Unlike the jurisdictional approaches discussed above, landscape-level approaches are focused on a particular forest ecosystem or peatland to be protected, and therefore could span several jurisdictions or only a part of one jurisdiction. Specific examples include RSPO certification schemes focused on protecting certain forest ecosystems such as the Leuser Ecosystem in Aceh Tamiang, Indonesia. Similarly, in Ivory Coast, Nestlé (in collaboration with the Earthworm Foundation) funds community livelihood projects and forest-restoration wages for villages around the Cavally Reserve, conditional on the prevention of new encroachment (Earthworm Foundation 2023). Similar to the jurisdictional approaches, these landscape-level approaches can be interpreted in the context of our model as area schemes in settings with limited cooperation.

EC.3. Relevant Literature on Payments for Ecosystem Services (PES)

PES schemes reward landholders for conservation, usually through monetary (or sometimes in-kind) transfers contingent on verifiable actions. Classic PES contracts target individual landowners, but practical experience has exposed several shortcomings of these individual schemes. First, such contracts, which require participants to have clear land titles, are rarely effective in areas where land tenure is ambiguous, which are often also those areas where deforestation is most acute (Wunder 2008, Börner et al. 2010). Second, payment targeting is often inefficient: too large for some recipients, who would have conserved anyway (which lowers the scheme's additionality), but insufficient for other participants, who still find clearing more profitable (Engel et al. 2008, Jack and Jayachandran 2019). Third, conservation in one location can simply induce forest clearing nearby, a phenomenon known as "leakage" (Alix-Garcia et al. 2012, Delacote et al. 2016). Fourth, enrolling and monitoring thousands of smallholders drives up transaction costs (Wunder et al. 2018). Finally, allocating payments in proportion to the forest area conserved often skews benefits toward wealthier landholders, exacerbating local inequities; this issue has been acknowledged amply in the PES literature and many have advocated for payments that compensate participants for their opportunity costs (Pascual et al. 2010, Wunder et al. 2018, Haas et al. 2019).

Collective or community-based PES (C-PES) schemes reward an entire community for conservation on communally- or publicly-held land, and evidence from diverse settings documents that these have important advantages (Brownson et al. 2019, Kaiser et al. 2023). C-PES is effective in settings where communities–but not individuals–have ownership or operating rights to land. Because the entire village (or jurisdiction) enrolls as a single unit, C-PES can reduce the potential for leakage, can reduce transaction costs, and can enable the conservation of large, contiguous blocks of forest (which is essential for watershed or biodiversity conservation efforts). C-PES is also well suited if the reward itself is collective, as with investments in schools, roads, shared equipment, or when granting communal land-use rights (Kerr et al. 2014, Suyanto et al. 2008, Pender et al. 2007, Knox et al. 2011).

However, despite their promise, C-PES schemes face three recurring pitfalls. First, collective action: communities must not only steward the resource but also decide how to use or distribute the payment for conservation–a task that succeeds when local institutions are strong but invites disagreement, conflict, and ultimately non-compliance when they are weak or contested (Hayes et al. 2019, Brownson et al. 2019). Second, elite capture: local power brokers can appropriate a disproportionate share of the funds, leaving poorer households undercompensated, which exacerbates local income inequalities and erodes the scheme's legitimacy (Hayes et al. 2019, Sommerville et al. 2010). Third, entrants: migrants or other outsiders who do not benefit from the PES payments may clear forest that the community is responsible for conserving, negating all the benefits for the community (Darmawan et al. 2016, Ruf et al. 2015, Carr 2009).

For a broad comparative survey of PES programs, see Wunder et al. (2008). The evidence reviewed above highlights cooperation and compensation as two central design issues for collective PES. The village-level payment for avoiding fire-induced deforestation studied in Falcon et al. (2022) illustrates that performance depends on which locals do and do not cooperate. Yet theoretical PES models typically either impose a single coalition, as in Zavalloni et al. (2019), Bareille et al. (2021), or preclude cooperation. We instead allow multiple coalitions to form endogenously, subject to limits on cooperation. This framework helps explain why collective PES schemes with area no-deforestation conditions succeed when social ties are strong and payments fully compensate participants' opportunity costs, while also showing that an area no-use condition can prevent deforestation without compensation.

Although compensation is not necessary for deforestation prevention under an area no-use condition, it remains an important ethical objective, as discussed above. Imposing that objective changes the set of optimal conditional incentives and, depending on local circumstances, may require individually tailored payments. The Vittel program in France illustrates this approach: negotiations with landowners produced personalized compensation Perrot-Maître (2006).

EC.4. Examples of Monitoring Platforms

We document publicly available and commercial platforms and datasets that monitor forest cover and detect deforestation, categorizing these based on **Geographic focus** (global or specific region) and **Spatial Resolution** (with the sensors used). The list is not exhaustive. Where a platform is documented in a peer-reviewed publication, we cite it alongside the platform's website.

Verifying an area no-use condition requires more than detecting canopy loss: cleared land that is subsequently planted with a tree crop such as oil palm, rubber, or cocoa must be distinguished from natural forest and from naturally regenerating forest, a distinction that tree-cover products alone do not make (Tropek et al. 2014, Berger et al. 2025). Advances in the last few years have made this distinction operational for several closed-canopy commodities and regions at resolutions relevant to smallholder plots, although limitations remain for shaded and agroforestry systems.

First, freely available imagery from the Sentinel-1 (radar), Sentinel-2 (10 m optical), and Landsat (30 m) missions is now processed jointly. Cloud-penetrating radar time series and dense optical observations therefore complement each other: radar-based alerts confidently detect disturbances of 0.2 ha every 6–12 days regardless of cloud cover (Reiche et al. 2021); a global 30 m alert now covers all vegetated land (Pickens et al. 2025); and Global Nature Watch (formerly Global Forest Watch) serves the optical and radar alerts as a single integrated 10 m layer (Global Forest Watch 2026).

Second, machine-learning classifiers trained on such multi-sensor stacks now map individual commodity crops at 10 m; the oil-palm and cocoa applications use convolutional neural networks. These classifiers map closed-canopy oil palm worldwide, distinguishing industrial from smallholder plantations at 10 m and estimating the establishment year of current plantations separately at 30 m (Descals et al. 2021, 2024); rubber across Southeast Asia (Wang et al. 2023); and cocoa in Côte d'Ivoire and Ghana (Kalischek et al. 2023). Related machine-learning approaches identify the land use or driver associated with forest loss. One such approach classifies the land use that follows deforestation into 15 classes at 5 m across Africa (Masolele et al. 2024). Another approach attributes newly detected radar alerts to smallholder agriculture, road development, selective logging, mining, or other causes, with timeliness and accuracy depending on the post-disturbance observation window (Slagter et al. 2023). A global neural-network model maps the dominant driver of forest loss at 1 km resolution (Sims et al. 2025). Separately, a global probability sample interpreted at 3–10 m distinguishes permanent conversion to pasture, cropland, and non-timber tree plantations from temporary canopy loss (Tyukavina et al. 2026).

Third, structural measurements provide information beyond spectral signatures. Sentinel-2 imagery trained against spaceborne lidar observations from GEDI yields a global canopy-height map at 10 m (Lang et al. 2023). Separately, models applied to very-high-resolution RGB imagery and trained against aerial lidar yield canopy-height maps at approximately 1 m (Tolan et al. 2024), while Sentinel-1 and Sentinel-2 imagery yield a 10 m map of tree extent in the tropics (Brandt et al. 2023). These products provide structural inputs that, when combined with land-use and plantation datasets, support distinctions among natural forest, regenerating forest, and tree crops. Such inputs have been combined into 10 m baseline maps of forest land use and forest type (primary, naturally regenerating, planted) for the EUDR cut-off year 2020 (Bourgoin et al. 2026a,b), into a 30 m map of natural versus non-natural land (Mazur et al. 2025), and into open tools that overlay multiple sources plot by plot (FAO Open Foris 2024).

Most recently, foundation-model embeddings that summarize optical, radar, and other measurements at 10 m are being used to produce annual maps of oil palm, cocoa, rubber, and coffee across the tropics (Brown et al. 2025, Forest Data Partnership 2026).

Limits remain: small-scale and agroforestry systems such as shaded cocoa and coffee are harder to separate from forest at 10 m and call for imagery finer than 5 m, and misclassifying agroforestry as forest is a recognized risk (Berger et al. 2025). For the closed-canopy commodities central to our setting, public data now permit plot-scale screening of whether cleared land has subsequently been put to agricultural use.

Platform (Provider)	**Region**	**Resolution (sensor)**	**Citation**
Global Nature Watch, formerly Global Forest Watch (WRI)	Global	30m (Landsat: tree cover loss, GLAD-L alerts; Landsat + Sentinel-2: DIST-ALERT, global), 10m (Sentinel-2: GLAD-S2; Sentinel-1 radar: RADD); integrated disturbance alerts served at 10m	World Resources Institute (2026), Global Forest Watch (2026)
UMD GLAD alerts (UMD)	Humid tropics, 30°N–30°S (GLAD-S2: Amazon basin)	30m (Landsat), 10m (Sentinel-2)	University of Maryland (2023), Hansen et al. (2016)
RADD alerts (Wageningen University and partners)	Humid tropics in Latin America, Africa, Southeast Asia and the Pacific	10m (Sentinel-1 C-band radar; 6–12 day revisit; cloud-penetrating)	Wageningen University & Research (2026), Reiche et al. (2021)
DIST-ALERT (NASA OPERA / UMD)	Global, all vegetated land	30m (Harmonized Landsat 8/9 and Sentinel-2)	University of Maryland GLAD (2026a), Pickens et al. (2025)
Hansen/UMD Global Forest Change (UMD/Google)	Global	30m (Landsat); annual, 2000–2025 (v1.13)	Hansen et al. (2013), University of Maryland GLAD (2026b)
EU Forest Observatory: Tropical Moist Forest and Global Forest Cover 2020 (European Commission JRC)	Global (GFC2020, forest types 2020); pantropical (TMF, 1990–)	10m (GFC2020, GFT2020), 30m (TMF, Landsat; annual)	European Commission Joint Research Centre (2026), Bourgoin et al. (2026b), Vancutsem et al. (2021)
FAO SEPAL (FAO Open Foris)	Global	10–30m (Sentinel-1/2, Landsat); alert module ingests GLAD, RADD and JJ-FAST alerts	FAO (2023)
Collect Earth Online (FAO, NASA/USAID SERVIR)	Global	Visual interpretation of high-resolution reference imagery (commercial basemaps, Sentinel-1/2, Landsat, user-supplied layers)	FAO and NASA/USAID SERVIR (2026)
JJ-FAST (JICA/JAXA)	Tropics: 78 countries until March 2024; Brazil only since April 2024	50m (ALOS-2 PALSAR-2 L-band ScanSAR); detects clearings $\geq$1.5 ha every $\sim$1.5 months	JICA and JAXA (2026), Watanabe et al. (2021)
PRODES (INPE, BiomasBR)	Brazil, all six biomes	20–30m (Landsat-class) through 2023; Sentinel-2 (10m) for all biomes from 2024; minimum mapped area 1 ha	INPE (2026b), Assis et al. (2019)
DETER (INPE)	Brazilian Legal Amazon (since 2004), Cerrado (2018) and Pantanal (2023)	55–64m (WFI on CBERS-4/4A and Amazônia-1); daily alerts $\geq$3 ha	INPE (2026a), Diniz et al. (2015)
Geobosques (MINAM Peru)	Peruvian Amazon	30m (Landsat) alerts; 10m (Sentinel-2) for verification	MINAM Peru (2023), Vargas et al. (2019)
WRI Forest Atlases (WRI with national governments)	Congo Basin (six countries), Liberia, Madagascar, Georgia	30m (GLAD alerts), 375m (VIIRS fire alerts)	WRI (2023)
Nusantara Atlas (TheTreeMap)	Indonesia, Malaysian Borneo, Brunei	10m alerts (RADD and GLAD harmonized); Planet NICFI 4.8m and Sentinel-2 10m for verification; annual oil palm and pulpwood plantation maps	TheTreeMap (2026), Gaveau et al. (2022)

(continued from previous page)

Platform (Provider)	Region	Resolution (sensor)	Citation
Planet (Planet Labs)	Global	3.7m (PlanetScope, near-daily), 50cm (SkySat); Forest Carbon Diligence 30m annual (2013–); Forest Carbon Monitoring 3m quarterly (2021–)	Planet Labs (2023, 2026)
Starling (Airbus, Earthworm Foundation)	25+ countries on five continents; eight commodities	10m (Sentinel-2) to 30cm (Airbus Pléiades Neo); AI-based change detection	Airbus and Earthworm Foundation (2023), Airbus Defence and Space (2026)
Satelligence	Global tropics	10m (Sentinel-1/2), 30m (Landsat); Planet imagery for reference data	Satelligence (2026)
EOSDA forestry solutions (EOS Data Analytics)	Global	Multi-sensor optical (10–30m: Sentinel-2, Landsat)	EOS Data Analytics (2026)
LiveEO TradeAware (LiveEO)	Global	Multi-source; effective resolution about 5m; machine-learning detection since the EUDR cut-off date	LiveEO (2026)
Forest Data Partnership commodity maps and Whisp (FAO, WRI, Google and partners)	Pantropical	10m machine-learning probability maps of oil palm, cocoa, rubber and coffee (2020–2024); plot-level "convergence of evidence" checks	Forest Data Partnership (2026), FAO Open Foris (2024)

Table EC.2: Forest Monitoring Tools

EC.5. Proofs of results in Section 3

EC.5.1. Proofs of results in §3.2 (The Non-Cooperative Game)

Proof of Lemma 1. Consider any $\pi \in \Pi_{\mathcal{L}}$. We prove the result separately for $\bar{\mathbb{D}}$ and $\bar{\mathbb{U}}$.

Proof for $\bar{\mathbb{D}}$. Consider an equilibrium $\big(d^\star, B^\star(d^\star)\big) \in \mathcal{Q}(\pi, \bar{\mathbb{D}})$. Because $\bar{\mathbb{D}}_\ell(d,B)$ is independent of B and blocking is costly, we have that $B^\star(d^\star) = 0$. (Otherwise, a coalition $S \in \pi$ with $B^\star_S(d^\star) \neq 0$ would strictly increase its net income by an amount $\eta \cdot \sum_{\ell \in S} \sum_{i \in \mathcal{L}} B^\star_{\ell i}$ by setting $B_S = 0$.) Assume for purposes of deriving a contradiction that $d^\star_\ell = 1$ and $d^\star_g = 0$ for some $\ell, g \in \mathcal{L}$. Because $d^\star_\ell = 1$, locals collectively do not comply with $\bar{\mathbb{D}}$ and $\bar{\mathbb{D}}_g\big(d^\star, B^\star(d^\star)\big) = \text{no}$. Therefore, every local earns his net income with deforestation, $V_\ell(1,0)$. But then, local g's coalition would be indifferent between setting $d_g = 0$ and setting $d_g = 1$ because it would earn the same income under either option; so according to our tie-breaking rule, it would set $d^\star_g = 1$, which provides a contradiction. This proves the desired result that $d^\star \in \{0,1\}$ and $B^\star(d^\star) \equiv 0$ under $\bar{\mathbb{D}}$.

Proof for $\bar{\mathbb{U}}$. We prove several intermediate results that lead us to the desired conclusions:

(i) Under subgame-perfect blocking decisions, every local is blocked by at most one local:

$$\sum_{g \in \mathcal{L}} B^\star_{g\ell}(d) \le 1, \forall \ell \in \mathcal{L}, \forall d \in \{0,1\}^{\mathcal{L}}. \tag{EC.1}$$

(ii) Under subgame-perfect blocking decisions, locals who do not deforest land should not be blocked:

$$d_\ell = 0 \;\;\Rightarrow\;\; B^\star_{g\ell}(d) = 0, \forall g \in \mathcal{L}. \tag{EC.2}$$

(iii) Under subgame-perfect blocking decisions, if some local who engages in deforestation is not blocked (i.e., locals do not comply with $\bar{\mathbb{U}}$), then no blocking occurs:

$$d_\ell \cdot \big(1 - \max_{g\in\mathcal{L}} B^\star_{g\ell}(d)\big) = 1 \text{ for some } \ell \in \mathcal{L} \quad \Rightarrow \quad B^\star(d) = 0. \tag{EC.3}$$

(iv) In any subgame-perfect equilibrium, all locals make the same deforestation decision:

$$\big(d^\star, B^\star(d^\star)\big) \in \mathcal{Q}(\pi, \bar{\mathbb{U}}) \quad \Rightarrow \quad d^\star \in \{0,1\}. \tag{EC.4}$$

(v) In any subgame-perfect equilibrium, no blocking occurs:

$$\big(d^\star, B^\star(d^\star)\big) \in \mathcal{Q}(\pi, \bar{\mathbb{U}}) \quad \Rightarrow \quad B^\star(d^\star) \equiv 0. \tag{EC.5}$$

We next prove each of the claims above.

(i) Suppose multiple locals block the same local, i.e., $\exists g, h \in \mathcal{L}$ with $g \neq h$ such that $B^\star_{g\ell}(d) = B^\star_{h\ell}(d) = 1$ for some $\ell \in \mathcal{L}$. Then, coalition S that contains g can set $B_{g\ell}(d) = 0$ and increase its net income by η, without affecting compliance, contradicting that $B^\star$ were subgame-perfect blocking decisions.

(ii) Consider d with $d_\ell = 0$ and assume for purposes of deriving a contradiction that ℓ is blocked, i.e., $\exists g$ such that $B^\star_{g\ell}(d) = 1$. But then, we claim that coalition S that contains g can increase its net income by setting $B_{g\ell}(d) = 0$. This would not affect compliance because $\bar{\mathbb{U}}_\ell\big(d, B^\star(d)\big) = \bar{\mathbb{U}}_\ell\big(d, [B_S, B^\star_{-S}(d)]\big) = \text{yes}$ because $d_\ell = 0$, and the deviation would increase S's net income by η (the cost of blocking ℓ), contradicting that $B^\star$ were subgame-perfect blocking decisions.

(iii) Consider $\big(d, B^\star(d)\big)$ so that there exists some local $\ell \in \mathcal{L}$ who is not blocked, i.e., $d_\ell \cdot \big(1 - \max_{g\in\mathcal{L}} B^\star_{g\ell}(d)\big) = 1$. To derive a contradiction, assume that some other local $i \in \mathcal{L}, i \neq \ell$ is blocked by some local g, i.e., $B^\star_{gi}(d) = 1$. But then, the coalition S containing g can set $B_{gi} = 0$; that would not affect compliance because local ℓ would still not be blocked, but it would increase coalition S's net income by η, contradicting that $B^\star$ were subgame-perfect blocking decisions.

(iv) Consider $\big(d^\star, B^\star(d^\star)\big) \in \mathcal{Q}(\pi, \bar{\mathbb{U}})$ and assume by contradiction that there exist coalitions $S_1, S_2 \in \pi$ (possibly with $S_1 = S_2$) and $g \in S_1$ and $h \in S_2$ with $d^\star_g = 0$ and $d^\star_h = 1$. We distinguish two cases, depending on whether locals collectively comply with $\bar{\mathbb{U}}$.

(a) Some local who deforests land is not blocked, i.e., $\exists \ell \in \mathcal{L} : d_\ell \cdot \big(1 - \max_{g\in\mathcal{L}} B^\star_{g\ell}(d)\big) = 1$. Note that this is equivalent to locals not complying with $\bar{\mathbb{U}}$. Then (EC.3) implies that $B^\star(d^\star) = 0$. We derive a contradiction by proving that coalition S_1 should have set $d^\star_g = 1$. Specifically, we prove that if $d_g = 1$, the same blocking decisions remain optimal under the new deforestation decisions $d' = [d_g = 1, d^\star_{-g}]$, i.e., $B^\star(d') = B^\star(d^\star) = 0$; this would imply that locals would still not comply with $\bar{\mathbb{U}}$ under $(d', B^\star(d'))$, so coalition S_1's net income would be the same with $d^\star_g = 0$ and with $d_g = 1$, which would contradict our tie-breaking rule that S_1 should have set $d^\star_g = 1$.

To complete this argument, it suffices to prove that $B^\star(d') = 0$. Assume otherwise; specifically, suppose that under d', some coalition $S \in \pi$ can strictly increase its net income by setting $B'_S \neq 0$. The new blocking decisions $B^\star(d') = [B'_S, B^\star_{-S}(d^\star)]$ must be such that all locals engaging in deforestation are blocked (so locals comply with $\bar{\mathbb{U}}$), because otherwise (EC.3) would imply that $B^\star(d') = 0$. Because $B^\star_{-S}(d^\star) = 0$ and a coalition cannot block its own members, this requires that no local in S engage in deforestation under d', i.e., $d^\star_\ell = 0$ for every $\ell \in S$ and $g \notin S$. Then, under d', it must be that the net income of coalition S with $B^\star_S(d^\star) = 0$ and no incentive is strictly smaller than its net income with blocking decisions B'_S and the incentive, i.e.,

$$v^{\text{sq}}(S) + D(S) < V_S\big(d', [B'_S, B^\star_{-S}(d^\star)]\big) = v^{\text{sq}}(S) + I(S) - \eta\left(1 + \sum_{i\in\mathcal{L}} d^\star_i\right). \tag{EC.6}$$

To understand the last equality, note that under d', coalition S must block all the locals who deforest land (which includes all locals who deforest land under $d^\star$ and local g) because $B^\star_{-S}(d^\star) = 0$; none of these locals belong to S, so coalition S incurs no deforestation costs. But recall that under $d^\star$, coalition S found it optimal to set $B^\star_S(d^\star) = 0$ and receive its net income with deforestation rather than block all the locals who engaged in deforestation and get the incentive, i.e.,

$$v^{\text{sq}}(S) + I(S) - \eta \sum_{i\in\mathcal{L}} d^\star_i < v^{\text{sq}}(S) + D(S). \tag{EC.7}$$

But then, note that (EC.6) and (EC.7) lead to a contradiction.

(b) All locals who deforest land are blocked, i.e., $d_\ell \cdot (1 - \max_{g\in\mathcal{L}} B_{g\ell}) = 0, \forall \ell \in \mathcal{L}$. Note that this is equivalent to locals complying with $\bar{\mathbb{U}}$. We claim that coalition S_2 (with $h \in S_2$) can benefit by deviating to set $d_{S_2} = 0$. This results in deforestation decisions $d' = [d_{S_2} = 0, d^\star_{-S_2}]$. We claim that the blocking decisions $B'(d')$ defined as:

$$B'_{g\ell}(d') = \begin{cases} 0 & \text{if } \ell \in S_2 \\ B^\star_{g\ell}(d^\star) & \text{otherwise} \end{cases}$$

would be a sub-game perfect response to d' and lead to a profitable deviation for S_2. Note that the blocking decisions B' are identical to $B^\star$ except that no locals from S_2 are blocked. With $[d', B'(d')]$, locals would continue to comply with $\bar{\mathbb{U}}$, so the net income of coalition S_2 under (d', B') would be strictly larger than under $\big(d^\star, B^\star(d^\star)\big)$ because S_2 would not incur the cost $\sum_{\ell\in S_2} d^\star_\ell c_\ell > 0$.

To complete the argument, it suffices then to prove that B' are subgame-perfect blocking decisions for d'. We claim that no coalition S would benefit by changing its blocking decisions away from B'. Not blocking any local from S_2 is optimal by (EC.2). Moreover, no coalition S would strictly benefit by changing its blocking decision concerning any local $\ell \notin S_2$: if $d'_\ell = 0$, then blocking ℓ is not optimal by (EC.2); if $d'_\ell = 1 (= d^\star_\ell)$ and S is not blocking ℓ under d', ℓ must nonetheless be blocked under d' (and $d^\star$) by some other coalition because locals are compliant with $\bar{\mathbb{U}}$, so S blocking ℓ would not be optimal by (EC.1); lastly, if $d'_\ell = d^\star_\ell = 1$ and S is the only coalition blocking ℓ under d' (and under $d^\star$), then if S benefited by not blocking ℓ under d', S would also benefit by not blocking ℓ under $d^\star$ (in both cases, S would save the same cost η and locals would not comply with $\bar{\mathbb{U}}$), which would contradict that $\big(d^\star, B^\star(d^\star)\big)$ is an equilibrium. This shows that B' are subgame-perfect decisions and proves our claim.

The argument in part (b) (applied iteratively) also proves that in every equilibrium where all locals are blocked, no coalition engages in deforestation:

$$\big(d^\star, B^\star(d^\star)\big) \in \mathcal{Q}(\pi, \bar{\mathbb{U}}) \text{ and } d^\star_\ell \cdot \big(1 - \max_{g \in \mathcal{L}} B^\star_{g\ell}(d^\star)\big) = 0, \forall \ell \in \mathcal{L} \quad \Rightarrow \quad d^\star = 0. \tag{EC.8}$$

This fact will be useful in our final proof.

(v) Consider $\big(d^\star, B^\star(d^\star)\big) \in \mathcal{Q}(\pi, \bar{\mathbb{U}})$. Under this equilibrium, if $d^\star_\ell \cdot \big(1 - \max_{g \in \mathcal{L}} B^\star_{g\ell}(d^\star)\big) = 1$ for some $\ell \in \mathcal{L}$, then (EC.3) implies that $B^\star(d^\star) = 0$, whereas if $d^\star_\ell \cdot \big(1 - \max_{g \in \mathcal{L}} B^\star_{g\ell}(d^\star)\big) = 0, \forall \ell \in \mathcal{L}$, then (EC.8) implies that $d^\star = 0$, which implies $B^\star(d^\star) = 0$ by (EC.2). □

Proof of Lemma 2. Consider any equilibrium $\big(d^\star, B^\star(d^\star)\big)$ and recall that we proved in Lemma 1 that $d^\star \in \{0, 1\}$ and $B^\star(d^\star) \equiv 0$. We prove the results separately, for each condition.

Proof for $\bar{\mathbb{D}}$. Recall from (1) the following expressions for the net income of coalition $S \in \pi$ under $\bar{\mathbb{D}}$:

$$V_S\big(d, B \equiv 0)\big) = \begin{cases} v^{\mathrm{sq}}(S) + I(S), & \text{if } d = 0 \\ v^{\mathrm{sq}}(S) + D(S), & \text{if } d_{S'} = 1 \text{ for any } S' \in \pi. \end{cases} \tag{EC.9}$$

We prove two simpler claims, which will imply the desired results. First, we prove that an equilibrium with $d^\star = 0$ exists under $\bar{\mathbb{D}}$ if and only if no coalition prefers deforestation:

$$0 \in T(\pi, \bar{\mathbb{D}}) \quad \Leftrightarrow \quad \nexists S \in \pi \text{ such that } I(S) \le D(S). \tag{EC.10}$$

Note that $0 \in T(\pi, \bar{\mathbb{D}})$ if and only if no coalition S would profitably deviate from $d^\star = 0$ by setting $d_S = 1$. By (EC.9), such a deviation is strictly profitable if and only if $I(S) < D(S)$. Moreover, if $I(S) = D(S)$, coalition S would prefer setting $d_S = 1$ by our tie-breaking assumption. This proves (EC.10).

We next prove that a deforestation equilibrium exists under $\bar{\mathbb{D}}$ if and only if there are at least two coalitions or there is a single (grand) coalition that does not prefer the incentive:

$$1 \in T(\pi, \bar{\mathbb{D}}) \quad \Leftrightarrow \quad (i)\ \pi \neq \{\mathcal{L}\} \text{ or } (ii)\ \pi = \{\mathcal{L}\} \text{ and } D(\mathcal{L}) \ge I(\mathcal{L}). \tag{EC.11}$$

Note that $1 \in T(\pi, \bar{\mathbb{D}})$ if and only if no coalition S would profitably deviate from $d^\star = 1$ by setting $d_S = 0$. If $\pi \neq \{\mathcal{L}\}$, such a deviation would entail no change in coalition S's net income because there would be at least one other coalition $S' \neq S$ with $d_{S'} = 1$, so even if $d_S = 0$, the locals would not comply with $\bar{\mathbb{D}}$ so coalition S's net income would be $v^{\text{sq}}(S) + D(S)$ under both configurations, by (EC.9). If $\pi = \{\mathcal{L}\}$, such a deviation would not be profitable if and only if $D(\mathcal{L}) \geq I(\mathcal{L})$, by (EC.9). This completes the proof of (EC.11).

These results in (EC.10) and (EC.11) immediately yield all the stated claims under $\bar{\mathbb{D}}$. Specifically:

- if $\pi = \{\mathcal{L}\}$ and $I(\mathcal{L}) > D(\mathcal{L})$: (EC.10) implies $0 \in T(\pi, \bar{\mathbb{D}})$, (EC.11) implies $1 \notin T(\pi, \bar{\mathbb{D}})$.
- if $\pi = \{\mathcal{L}\}$ and $I(\mathcal{L}) \leq D(\mathcal{L})$: (EC.10) implies $0 \notin T(\pi, \bar{\mathbb{D}})$, (EC.11) implies $1 \in T(\pi, \bar{\mathbb{D}})$.
- if $\pi \neq \{\mathcal{L}\}$ and $\exists S \in \pi$ with $I(S) \leq D(S)$: (EC.10) implies $0 \notin T(\pi, \bar{\mathbb{D}})$, (EC.11) implies $1 \in T(\pi, \bar{\mathbb{D}})$.
- if $\pi \neq \{\mathcal{L}\}$ and $I(S) > D(S)$, $\forall S \in \pi$: (EC.10) implies $0 \in T(\pi, \bar{\mathbb{D}})$, (EC.11) implies $1 \in T(\pi, \bar{\mathbb{D}})$.

Proof for $\bar{\mathbb{U}}$. Recall from (1) the following expressions for the net income of coalition $S \in \pi$ under $\bar{\mathbb{U}}$:

$$V_S\big(d, B\big) = \begin{cases} v^{\text{sq}}(S) + I(S), & \text{if } d = 0 \text{ and } B = 0 \\ v^{\text{sq}}(S) + I(S) - \eta \sum_{g \in S, \ell \in \mathcal{L}} B_{g\ell}, & \text{if } d_S = 0 \text{ and } \max_{i \in \mathcal{L}} B_{i\ell} = 1 \, \forall \ell \in \mathcal{L} \text{ with } d_\ell = 1 \\ v^{\text{sq}}(S) + D(S) - \eta \sum_{g \in S, \ell \in \mathcal{L}} B_{g\ell}, & \text{otherwise.} \end{cases} \tag{EC.12}$$

As with $\bar{\mathbb{D}}$, we prove two simpler claims. First, we prove that a deforestation equilibrium exists under $\bar{\mathbb{U}}$ if and only if the blocking cost η exceeds a certain threshold $\eta_1(\pi)$:

$$1 \in T(\pi, \bar{\mathbb{U}}) \quad \Leftrightarrow \quad \eta \geq \eta_1(\pi) := \sup\big\{\eta : \exists S \in \pi \text{ with } I(S) - D(S) > \eta \, |\mathcal{L} \setminus S|\big\}. \tag{EC.13}$$

Note that $1 \in T(\pi, \bar{\mathbb{U}})$ if and only if no coalition S would profitably deviate from $d^\star = 1, B^\star \equiv 0$ by setting $d_S = 0$. We consider two (sub)cases, depending on π.

If $\pi = \{\mathcal{L}\}$, such a deviation is profitable for the grand coalition if and only if its net income with $d = 0, B = 0$ strictly exceeds its net income with $d = 1, B = 0$. By (EC.12), this is equivalent to $I(\mathcal{L}) > D(\mathcal{L})$. Noting that $\eta_1(\{\mathcal{L}\}) = +\infty$ if $I(\mathcal{L}) > D(\mathcal{L})$ and $\eta_1(\{\mathcal{L}\}) = -\infty$ otherwise, we can immediately conclude that the deviation is profitable if and only if $\eta < \eta_1(\{\mathcal{L}\})$, as claimed in (EC.13).

Consider $\pi \neq \{\mathcal{L}\}$ and $\eta < \eta_1(\pi)$. Because every $S \in \pi$ is then a proper subset of $\mathcal{L}$, the definition of $\eta_1(\pi)$ implies that there exists $S \in \pi$ such that $I(S) - D(S) > \eta|\mathcal{L} \setminus S|$. We prove that coalition S has a profitable deviation from $d^\star = 1$. Suppose S sets $d_S = 0$ and sets B_S to block all locals outside of S, blocking each local exactly once (which must be the case by (EC.1)). Then, from (EC.12), the net income of coalition S would be $v^{\text{sq}}(S) + I(S) - \eta|\mathcal{L} \setminus S|$, and because $\eta < \eta_1(\pi)$, this is strictly larger than S's net income $v^{\text{sq}}(S) + D(S)$ in the initial equilibrium. This deviation implies that $1 \notin T(\pi, \bar{\mathbb{U}})$.

Consider $\pi \neq \{\mathcal{L}\}$ and $\eta \geq \eta_1(\pi)$. To derive a contradiction, assume that $\exists S \in \pi$ that would profitably deviate from $d^\star = 1$ (and $B^\star \equiv 0$) by setting $d_S = 0$, and let $d' = [d_S = 0, d^\star_{-S} = 1]$ denote the deforestation decisions. We claim that optimal blocking decisions $B^\star(d')$ exist whereby no coalitions engage in blocking, i.e., $B^\star(d') = 0$. To see this, consider first coalition S's blocking decisions. If S blocked each local from $\mathcal{L} \setminus S$ once – which is the case under subgame-perfect blocking, by (EC.1) – locals would collectively comply with $\bar{\mathbb{U}}$ and S's net income would be $v^{\text{sq}}(S) + I(S) - \eta \cdot |\mathcal{L} \setminus S|$, from (EC.12). If S blocked a number $k < |\mathcal{L} \setminus S|$ of locals from $\mathcal{L} \setminus S$, the locals would not comply with $\bar{\mathbb{U}}$ (by the pigeonhole principle) so S's net income would be $v^{\mathrm{sq}}(S) + D(S) - \eta \cdot k$, from (EC.12); because this expression is strictly decreasing in k, it would be optimal in that case for S to set $k = 0$, i.e., not block anyone. Comparing the two extreme options, S would strictly benefit from blocking everyone if and only if $I(S) - \eta|\mathcal{L} \setminus S| > D(S)$. This cannot hold when $\eta \geq \eta_1(\pi)$: otherwise, because $|\mathcal{L} \setminus S| > 0$, the same strict inequality would hold at some blocking cost strictly greater than η, contradicting the definition of $\eta_1(\pi)$ as a supremum. Consider now any other coalition $H \in \pi$, $H \neq S$, and recall that a coalition cannot block its own members. Because $H \subseteq \mathcal{L} \setminus S$, the locals in H deforest land under d', so when no other coalition engages in blocking, the locals in H remain unblocked regardless of H's own blocking decisions; locals then do not comply with $\bar{\mathbb{U}}$, so any blocking by H would only incur costs, and not blocking anyone is H's unique best response. This proves that $1 \in T(\pi, \bar{\mathbb{U}})$.

Next, we prove that an equilibrium with $d^\star = 0$ exists under $\bar{\mathbb{U}}$ if and only if the blocking cost η does not exceed another threshold $\eta_2(\pi)$:

$$0 \in T(\pi, \bar{\mathbb{U}}) \;\Leftrightarrow\; \eta \leq \eta_2(\pi) := \sup\left\{\eta : \sum_{S \in \pi : I(S) > D(S)} \left\lfloor \frac{I(S) - D(S)}{\eta} \right\rfloor \geq \max_{H \in \pi : I(H) \leq D(H)} |H| \right\}. \quad \text{(EC.14)}$$

Consider $\eta > \eta_2(\pi)$ and assume by contradiction that $0 \in T(\pi, \bar{\mathbb{U}})$. Because η is finite, it follows from the definition of $\eta_2(\pi)$ that $\exists H \in \pi$ with $I(H) \leq D(H)$. We prove that $H \in \arg\max_{S' \in \pi : I(S') \leq D(S')} |S'|$ has a profitable deviation by setting $d_H = 1$. One can use a similar argument to that in our earlier proof to show that optimal blocking decisions exist where no coalition engages in any blocking. This arises because no coalition $S \in \pi$ with $D(S) < I(S)$ would prefer to block, and even if all coalitions $S \in \pi$ with $I(S) \geq D(S)$ coordinated perfectly on blocking the locals in H, they would not profit from doing so. (Because $\eta > \eta_2(\pi)$, the pigeonhole principle would imply that some locals in H would not be blocked so locals would not comply with $\bar{\mathbb{U}}$; so coalition $S \neq H$ that would be blocking k locals would get net income $v^{\mathrm{sq}}(S) + D(S) - \eta \cdot k$, so it would prefer to set $k = 0$.) This implies $0 \notin T(\pi, \bar{\mathbb{U}})$.

Consider $\eta \leq \eta_2(\pi)$. A similar argument to the one above can be used to show that each coalition H that deviates by setting $d_H = 1$ would earn strictly lower net income because optimal subsequent blocking decisions exist wherein the coalitions $S \in \pi$ with $I(S) > D(S)$ block all the locals from coalition H. In that configuration, locals collectively comply with $\bar{\mathbb{U}}$ and because $\eta \leq \eta_2(\pi)$, no coalition S engaged in the blocking would profit from deviating. So coalition H would earn net income strictly below $v_H^{\mathrm{sq}} + I(H)$ because it incurs the deforestation cost $c(H)$. In turn, this implies that $0 \in T(\pi, \bar{\mathbb{U}})$.

Lastly, noting that $\eta_1(\pi) \leq \eta_2(\pi)$, (EC.13) and (EC.14) imply the desired results under $\bar{\mathbb{U}}$. $\square$

EC.5.2. Proofs of results in §3.3 (The Cooperative Game)

The proof of Proposition 1 follows as a corollary of a more comprehensive result in Lemma EC.1, which we state and prove below.

LEMMA EC.1. *Consider the cooperative game with transferable utility defined in §3.3. Every core outcome $(\pi, d^\star, a)$ must satisfy:*

$$a_g \geq v_g^{\text{sq}} + D_g, \forall g \in \mathcal{G} \tag{EC.15a}$$

$$a_\ell \geq v_\ell^{\text{sq}} + I_\ell, \forall \ell \in \mathcal{L} \setminus \mathcal{G}. \tag{EC.15b}$$

In particular, if $\mathcal{L} = \mathcal{G}$, then every core outcome must compensate all locals.

Proof of Lemma EC.1. Recalling the definition of $\mathcal{G}$, (EC.15a) and (EC.15b) are equivalent to:

$$a_\ell \geq \min\{v_\ell^{\text{sq}} + I_\ell, v_\ell^{\text{sq}} + D_\ell\} \text{ for all } \ell \in \mathcal{L} \text{ and for all } (\pi, d^\star, a) \in C(\mathcal{L}; \emptyset).$$

Assume by contradiction that a core outcome exists with $a_\ell < \min\{v_\ell^{\text{sq}} + I_\ell, v_\ell^{\text{sq}} + D_\ell\}$ for some $\ell \in \mathcal{L}$. Then, local ℓ can deviate and form a coalition $\{\ell\}$. By our assumption, $a_\ell < w(\{\ell\}; d^\star)$ for every $d^\star \in \{0, 1\}$, so the initial outcome is dominated and cannot be in the core. □

Proof of Proposition 1. Consider any outcome with $d^\star = 0$ in the core, denoted by $(\pi, 0, a)$. By Lemma EC.1 have that

$$a_\ell \geq \min\{v_\ell^{\text{sq}} + I_\ell, v_\ell^{\text{sq}} + D_\ell\} \geq v_\ell^{\text{sq}}$$

where the second inequality follows because the incentive is positive ($I_\ell \geq 0$) and every local benefits from deforestation ($D_\ell \geq 0$). □

Proof of Theorem 1.

We first establish three preliminary results.

(R1) If $I(F) > D(F)$ for all $F \in \bar{\pi}$, then the core must contain all outcomes with $d^\star = 0$ that compensate. Consider any such outcome: $(\pi, 0, a)$ with $0 \in T(\pi)$ and where $a_\ell \geq v_\ell^{\text{sq}} + D_\ell$ for all $\ell \in \mathcal{L}$. We prove that no coalition $S \subseteq \mathcal{L}$ would deviate from such an outcome. If $\bar{\pi} = \{\mathcal{L}\}$, the whole set $S = \mathcal{L}$ would not deviate, as $I(\mathcal{L}) > D(\mathcal{L})$ implies that $\sum_{\ell \in \mathcal{L}} a_\ell = w(\mathcal{L}; 0) > w(\mathcal{L}; 1)$. If $\bar{\pi} \neq \{\mathcal{L}\}$, consider a subset $S \subset \mathcal{L}$ that deviates and forms partition $\pi_S \in \Pi_S$, with $\pi_S \prec \bar{\pi}$. Lemma 2 implies that $1 \in T(\pi_S \cup \pi_{\mathcal{L} \setminus S})$ for any $\pi_{\mathcal{L} \setminus S}$, which implies that $1 \in A(\mathcal{L} \setminus S; \pi_S)$. But then, under pessimism, any (sub)coalition $S_i \in \pi_S$ would not prefer to deforest land because

$$\sum_{\ell \in S_i} a_\ell \geq \sum_{\ell \in S_i} (v_\ell^{\text{sq}} + D_\ell) = w(S_i; 1).$$

Therefore, the outcome $(\pi, 0, a)$ is not dominated and must be in the core.

(R2) An outcome with $d^\star\!=\!0$ on partition π is feasible only if $0 \in T(\pi, \bar{\mathbb{D}})$, which by Lemma 2 requires that either $\pi = \{\mathcal{L}\}$ with $I(\mathcal{L}) > D(\mathcal{L})$, or $\pi \neq \{\mathcal{L}\}$ and $I(S) > D(S)$ for every coalition $S \in \pi$.

(R3) An outcome with $d^\star\!=\!1$ $(\pi, 1, a)$ with a feasible partition π and $a_\ell = v_\ell^{\text{sq}} + D_\ell$ for all $\ell \in \mathcal{L}$ is in the core unless $\bar{\pi} = \{\mathcal{L}\}$ and $I(\mathcal{L}) > D(\mathcal{L})$. To see this, note first that no strict subset $S \subset \mathcal{L}$ can profitably deviate: if S deviates and forms $\pi_S \in \Pi_S$ with $\pi_S \prec \bar{\pi}$, the resulting partition $\pi_S \cup \pi_{\mathcal{L}\setminus S} \neq \{\mathcal{L}\}$, so Lemma 2 gives $1 \in T(\pi_S \cup \pi_{\mathcal{L}\setminus S})$ for any $\pi_{\mathcal{L}\setminus S}$ ($T(\cdot, \bar{\mathbb{D}}) = \{0\}$ only for the grand coalition $\{\mathcal{L}\}$), hence $1 \in A(\mathcal{L} \setminus S; \pi_S)$. Under pessimism, every (sub)coalition $S_i \in \pi_S$ then obtains at most

$$\min_{d^\star \in A(\mathcal{L}\setminus S;\pi_S)} w(S_i; d^\star) \leq w(S_i; 1) = \sum_{\ell \in S_i} a_\ell,$$

so the deviation is not profitable. Second, the grand coalition $S = \mathcal{L}$ can deviate only under full cooperation ($\bar{\pi} = \{\mathcal{L}\}$), since otherwise $\mathcal{L}$ cannot form as a single coalition ($\pi_{\mathcal{L}} \prec \bar{\pi} \neq \{\mathcal{L}\}$ is infeasible). When $\bar{\pi} = \{\mathcal{L}\}$, by Lemma 2 the grand coalition deviates profitably (forming $\{\mathcal{L}\}$ and not deforesting) if and only if $I(\mathcal{L}) > D(\mathcal{L})$, in which case, $\{0\} = T(\{\mathcal{L}\})$ and $w(\mathcal{L}; 0) > w(\mathcal{L}; 1)$.

We now prove the parts of the theorem.

Part A (nonemptiness and compensation). Nonemptiness follows from the constructions in Parts B and C below: in every parameter region we exhibit a core outcome (either an outcome with $d^\star\!=\!1$, by (R3), or an outcome with $d^\star\!=\!0$ that compensate, by (R1)). For the compensation claim, we prove the stronger claim that under $\bar{\mathbb{D}}$ every core outcome $(\pi, d^\star, a)$ satisfies $a_\ell \geq v_\ell^{\text{sq}} + D_\ell$ for all $\ell \in \mathcal{L}$; in particular every no-deforestation core outcome compensate. Suppose, for contradiction, that a core outcome has $a_\ell < v_\ell^{\text{sq}} + D_\ell$ for some ℓ; we show it is dominated by ℓ forming the singleton coalition $\{\ell\}$. If $I_\ell < D_\ell$, then Lemma 2 implies that $T(\{\{\ell\}\} \cup \pi_{\mathcal{L}\setminus\ell}) = \{1\}$ for every partition $\pi_{\mathcal{L}\setminus\ell}$ of the residual locals $\mathcal{L} \setminus \ell$, which yields ℓ a net income $v_\ell^{\text{sq}} + D_\ell > a_\ell$. If $I_\ell \geq D_\ell$, then by deviating and forming partition $\{\ell\}$, local ℓ profits regardless of whether or not deforestation occurs, because $v_\ell^{\text{sq}} + I_\ell \geq v_\ell^{\text{sq}} + D_\ell > a_\ell$. Either way, the outcome is dominated, a contradiction.

Part B (full cooperation, $\bar{\pi} = \{\mathcal{L}\}$). If $I(\mathcal{L}) > D(\mathcal{L})$, then (R1) implies that the core contains all outcomes with $d^\star\!=\!0$ that compensate, and (R3) implies that no outcome with $d^\star\!=\!1$ is in the core (the grand coalition dominates it). Hence the core contains *only* outcomes with $d^\star\!=\!0$. If instead $I(\mathcal{L}) \leq D(\mathcal{L})$, then for every partition π of $\mathcal{L}$ we have $\sum_{S\in\pi}(I(S) - D(S)) = I(\mathcal{L}) - D(\mathcal{L}) \leq 0$, so some coalition $S \in \pi$ satisfies $I(S) \leq D(S)$ and Lemma 2 gives $T(\pi) = \{1\}$. Thus $0 \notin T(\pi)$ for every π, so by (R2) every outcome with $d^\star\!=\!0$ is infeasible, while (R3) places an outcome with $d^\star\!=\!1$ in the core (so the core is non-empty). Hence the core contains *only* outcomes with $d^\star\!=\!1$.

Part C (limited cooperation, $\bar{\pi} \neq \{\mathcal{L}\}$). Take $\pi = \bar{\pi}$; since $\bar{\pi} \neq \{\mathcal{L}\}$, Lemma 2 gives $1 \in T(\bar{\pi})$, so the outcome with $d^\star\!=\!1$ $(\bar{\pi}, 1, a)$ with $a_\ell = v_\ell^{\text{sq}} + D_\ell$ for all $\ell \in \mathcal{L}$ is feasible, and by (R3) it lies in the core (the grand-coalition deviation is infeasible under limited cooperation). Hence, the core always contains outcomes with $d^{\star}=1$. It remains to show that the core contains outcomes with $d^{\star}=0$ if and only if $I(F)>D(F)$ for all $F\in\bar{\pi}$.

Suppose $I(F)>D(F)$ for all $F\in\bar{\pi}$; we show the core contains outcomes with $d^{\star}=0$. By (R1), every outcome with $d^{\star}=0$ that compensate lies in the core, so it suffices to show that there exists one. Consider the outcome $(\bar{\pi},0,a_\ell)$, with any $a_\ell\geq v_\ell^{\mathrm{sq}}+D_\ell$ for every $\ell\in\mathcal{L}$. Such an allocation is feasible because each family F allocates exactly its no-deforestation value $a(F)=w(F;0)=\sum_{\ell\in F}(v_\ell^{\mathrm{sq}}+I_\ell)$ among its members, which exceeds the compensation requirement $\sum_{\ell\in F}(v_\ell^{\mathrm{sq}}+D_\ell)$ by $I(F)-D(F)>0$; we may therefore choose an allocation with $a_\ell\geq v_\ell^{\mathrm{sq}}+D_\ell$ for every $\ell\in F$. This outcome is feasible, because $I(F)>D(F)$ for all $F\in\bar{\pi}$ so Lemma 2 gives $0\in T(\bar{\pi})$, and compensates all locals by definition. Therefore, by (R1) it lies in the core.

Suppose the core contains an outcome with $d^{\star}=0$ $(\pi,0,a)$. By (R2), $0\in T(\pi)$ requires $I(S)>D(S)$ for every coalition $S\in\pi$. Since $\pi\prec\bar{\pi}$, summing over all coalitions included in a family F gives $I(F)=\sum_{S\in\pi:S\subseteq F}I(S)>\sum_{S\in\pi:S\subseteq F}D(S)=D(F)$. Hence $I(F)>D(F)$ for every $F\in\bar{\pi}$, as claimed. □

Proof of Theorem 2. We prove Part A (nonemptiness, followed by its statements i–iii), Part B, and Part C in turn. Throughout, we write $\Delta_\ell:=I_\ell-D_\ell$ and $\Delta(S):=\sum_{\ell\in S}\Delta_\ell$, and we use Lemma 2 in the form of (EC.13) and (EC.14): for any partition π, $1\in T(\pi,\bar{\mathbb{U}})$ if and only if $\eta\geq\eta_1(\pi)$, and $0\in T(\pi,\bar{\mathbb{U}})$ if and only if $\eta\leq\eta_2(\pi)$. We also use that $\underline{\eta}_1\leq\overline{\eta}_1$, with equality when every family prefers the incentive.

Part A (nonemptiness). Let $\mathcal{G}\notin\{\emptyset,\mathcal{L}\}$. Suppose first $\eta\leq\eta_2$. Then $I(F)>D(F)$ for some $F\in\bar{\pi}$ (otherwise no $\eta>0$ satisfies the defining condition of η_2, and $\eta_2=-\infty$), so Lemma EC.2(a)-(ii) shows the core contains an outcome with $d^{\star}=0$. Suppose next $\eta\geq\overline{\eta}_1$. If $\bar{\pi}=\{\mathcal{L}\}$ with $I(\mathcal{L})>D(\mathcal{L})$, the single family prefers the incentive, so Lemma EC.5(i) places in the core the outcome $(\bar{\pi},0,a)$ with $a(\mathcal{L})=w(\mathcal{L};0)$ and $a_\ell\geq v_\ell^{\mathrm{sq}}+D_\ell$ for every $\ell\in\mathcal{L}$. Otherwise ($\bar{\pi}\neq\{\mathcal{L}\}$, or $\bar{\pi}=\{\mathcal{L}\}$ with $I(\mathcal{L})\leq D(\mathcal{L})$), Lemma EC.2(b)-(ii) shows the core contains an outcome with $d^{\star}=1$. In all of these cases, the core is nonempty. Finally, for $\eta\in(\eta_2,\overline{\eta}_1)$ the core can be empty. Lemma EC.3(i) exhibits instances with an empty core at every $\eta\in(\eta_2,\overline{\eta}_1)$; in those instances every family prefers the incentive, so $\underline{\eta}_1=\overline{\eta}_1$ and that interval is the entire range $(\eta_2,\overline{\eta}_1)$ – including, through its full-cooperation variant, instances with $\bar{\pi}=\{\mathcal{L}\}$. Lemma EC.3(ii) exhibits instances with $\underline{\eta}_1\leq\eta_2$, a configuration the former cannot produce; Lemma EC.3(iii) exhibits instances in the remaining configuration $\eta_2<\underline{\eta}_1<\overline{\eta}_1$, where the core is empty at blocking costs *above* $\underline{\eta}_1$; and Lemma EC.13(iv) exhibits instances in which no family prefers the incentive (there $\eta_2=-\infty$).

The remaining statements of Part A describe the core outcomes whenever the core is nonempty; accordingly, the arguments below do not restrict η to $\eta\notin(\eta_2,\overline{\eta}_1)$ unless explicitly stated.

Part A-i. Let $\bar{\pi} = \{\mathcal{L}\}$. If $I(\mathcal{L}) > D(\mathcal{L})$, Lemma EC.4(i) implies that all core outcomes have $d^\star = 0$. If $I(\mathcal{L}) < D(\mathcal{L})$, no core outcome has $d^\star = 0$: any outcome $(\sigma, 0, a)$ with $d^\star = 0$ has $a(\mathcal{L}) = v^{\text{sq}}_{\mathcal{L}} + I(\mathcal{L})$ and is dominated by the grand coalition $\mathcal{L}$ deviating and forming $\{\mathcal{L}\}$: by (5), $A(\emptyset; \{\mathcal{L}\}) = T(\{\mathcal{L}\}, \bar{\mathbb{U}}) = \{1\}$, because $\eta_1(\{\mathcal{L}\}) = -\infty$ and $\eta_2(\{\mathcal{L}\}) = -\infty$ when $I(\mathcal{L}) \leq D(\mathcal{L})$ (see the proof of Lemma 2), and $w(\mathcal{L}; 1) = v^{\text{sq}}_{\mathcal{L}} + D(\mathcal{L}) > a(\mathcal{L})$.

If $I(\mathcal{L}) = D(\mathcal{L})$, it is possible that no core outcome has $d^\star = 0$. Consider the instance of Lemma EC.4(ii) – $\mathcal{L} = \{g, b\}$, $\bar{\pi} = \{\mathcal{L}\}$, $\Delta_g = \Delta > 0$, $\Delta_b = -\Delta$ – at any blocking cost $\eta > \Delta$. Here $\eta_2 = -\infty$ (no family prefers the incentive) and $\overline{\eta}_1 = \Delta$, so $\eta \notin (\eta_2, \overline{\eta}_1)$ and, by Lemma EC.2(b)-(ii) (with $\bar{\pi} = \{\mathcal{L}\}$ and $I(\mathcal{L}) \leq D(\mathcal{L})$), the core is nonempty. Yet no outcome with $d^\star = 0$ exists: for the grand coalition, $\eta_2(\{\mathcal{L}\}) = -\infty$ as above; for the singleton partition, $0 \in T(\{\{g\}, \{b\}\})$ requires $\lfloor \Delta/\eta \rfloor \geq 1$, i.e., $\eta \leq \Delta$. Hence every core outcome has $d^\star = 1$. Conversely, Lemma EC.4(ii) shows (same instance, $\eta \leq \Delta$) that the core may also contain an outcome with $d^\star = 0$, so the qualifier "possible" cannot be strengthened in either direction.

Regarding compensation, suppose first $I(\mathcal{L}) > D(\mathcal{L})$; the unique family then prefers the incentive, so $\underline{\eta}_1 = \overline{\eta}_1$. If $\eta < \overline{\eta}_1 = \underline{\eta}_1$, Lemma EC.5(iii) implies that no core outcome with $d^\star = 0$ compensates. If $\eta \geq \overline{\eta}_1$, the outcome $(\bar{\pi}, 0, a)$ with $a(\mathcal{L}) = w(\mathcal{L}; 0)$ and $a_\ell \geq v^{\text{sq}}_\ell + D_\ell$ for every ℓ (which exists because $I(\mathcal{L}) > D(\mathcal{L})$) is in the core by Lemma EC.5(i). This establishes the equivalence. If moreover $\eta > \overline{\eta}$, Lemma EC.5(ii) implies that every core outcome with $d^\star = 0$ compensates. If instead $I(\mathcal{L}) = D(\mathcal{L})$, no outcome with $d^\star = 0$ $(\sigma, 0, a)$ that compensates exists: because $a(S) = w(S; 0) = v^{\text{sq}}_S + I(S)$ and $a_\ell \geq v^{\text{sq}}_\ell + D_\ell$ for every $\ell \in S$, every $S \in \sigma$ satisfies $\Delta(S) \geq 0$, and since these sum to $\Delta(\mathcal{L}) = 0$, every $S \in \sigma$ satisfies $\Delta(S) = 0$; then the sum on the left of the condition in (EC.14) is empty while the maximum on its right is at least 1, so $0 \notin T(\sigma, \bar{\mathbb{U}})$, contradicting the feasibility of the outcome.

Part A-ii. Let $\bar{\pi} \neq \{\mathcal{L}\}$ with $I(F) > D(F)$ for some $F \in \bar{\pi}$. If $\eta \leq \eta_2$, Lemma EC.2(a)-(ii) implies the core contains an outcome with $d^\star = 0$ (one that does not compensate). Otherwise ($\eta > \eta_2$), it is possible that no core outcome has $d^\star = 0$: in the instance of Lemma EC.7, $\max\{\eta_2, \underline{\eta}_1\} = \eta_2$, and for every $\eta > \eta_2$ the core is nonempty and contains only outcomes with $d^\star = 1$. Next, if $\eta < \underline{\eta}_1$, Lemma EC.6(i) implies all core outcomes have $d^\star = 0$, whereas if $\eta \geq \underline{\eta}_1$, Lemma EC.6(ii) implies the core is empty or contains an outcome with $d^\star = 1$; hence a nonempty core has at least one outcome with $d^\star = 1$.

Regarding compensation, suppose first that $I(F) > D(F)$ for every $F \in \bar{\pi}$ and $\eta \geq \overline{\eta}_1$. Then an outcome with $d^\star = 0$ that compensates exists: $0 \in T(\bar{\pi}, \bar{\mathbb{U}})$ by (EC.14), because every coalition $F \in \bar{\pi}$ satisfies $I(F) > D(F)$, and within each family F the surplus $I(F) - D(F) > 0$ allows an allocation with $a(F) = w(F; 0)$ and $a_\ell \geq v^{\text{sq}}_\ell + D_\ell$ for every $\ell \in F$; by Lemma EC.5(i), this outcome is in the core. If instead $I(F) < D(F)$ for some $F \in \bar{\pi}$, Lemma EC.14 implies that no core outcome with $d^\star = 0$ compensates, at any blocking cost. In the remaining circumstances – every family weakly prefers the incentive with $I(F) = D(F)$ for some $F \in \bar{\pi}$, or every family strictly prefers the incentive but

$\eta < \overline{\eta}_1$ – it is possible that no core outcome with $d^\star\!=\!0$ compensates: for the latter, Lemma EC.8 exhibits instances with every family strictly preferring the incentive whose core is nonempty at every $\eta < \overline{\eta}_1$ yet contains no outcome with $d^\star\!=\!0$ that compensates; for the former, Lemma EC.9 exhibits instances with one family exactly indifferent in which no core outcome with $d^\star\!=\!0$ compensates at *any* blocking cost $\eta > 0$.

Finally, the core does not contain only outcomes with $d^\star = 0$ that compensate. If $\eta \geq \underline{\eta}_1$, Lemma EC.6(ii) together with nonemptiness places in the core an outcome with $d^\star\!=\!1$. If $\eta < \underline{\eta}_1$, suppose for a contradiction that the core is nonempty and every core outcome is an outcome with $d^\star\!=\!0$ that compensates. In particular, the core contains an outcome with $d^\star\!=\!0$ that compensates, so $I(F) \geq D(F)$ must hold for every $F \in \bar{\pi}$ (otherwise Lemma EC.14 would preclude such an outcome from the core) and $\eta \geq \eta_1^{comp}$ must hold (otherwise Lemma EC.11 would do the same). Together with $\bar{\pi} \neq \{\mathcal{L}\}$, $\mathcal{G} \neq \mathcal{L}$, and $\eta_1^{comp} \leq \eta < \underline{\eta}_1$, these are the hypotheses of Lemma EC.12, whose part (d) places in the core an outcome with $d^\star\!=\!0$ that does not compensate – a contradiction.

Part A-iii. Let $\bar{\pi} \neq \{\mathcal{L}\}$ with $I(F) \leq D(F)$ for every $F \in \bar{\pi}$. It is possible that no core outcome has $d^\star\!=\!0$: if $\eta \leq \overline{\eta}$, Lemma EC.13(v) exhibits instances whose core is nonempty and contains only outcomes with $d^\star\!=\!1$; if $\eta > \overline{\eta}$, Lemma EC.13(ii) implies the core contains only outcomes with $d^\star\!=\!1$, and it is nonempty by Lemma EC.2(b)-(ii) because $\eta > \overline{\eta} \geq \overline{\eta}_1$ and $\bar{\pi} \neq \{\mathcal{L}\}$. (Lemma EC.13(iii) shows that for $\eta \leq \overline{\eta}$ the core may instead contain an outcome with $d^\star\!=\!0$, so the qualifier "possible" cannot be strengthened.) Lastly, if the core is nonempty, Lemma EC.13(i) implies it contains an outcome with $d^\star\!=\!1$.

Part B ($\mathcal{G} = \mathcal{L}$). Every local satisfies $\Delta_\ell > 0$, so every family satisfies $I(F) > D(F)$. The core contains the outcome $(\bar{\pi}, 0, a)$ with $a_\ell = v_\ell^{\text{sq}} + I_\ell$ for every $\ell \in \mathcal{L}$: it is feasible because no coalition of $\bar{\pi}$ satisfies $\Delta(F) \leq 0$, so the condition in (EC.14) holds vacuously and $0 \in T(\bar{\pi}, \bar{\mathbb{U}})$; and it is not dominated because for any deviating coalition H forming π_H, every $S \in \pi_H$ obtains at most $\max\{w(S;0), w(S;1)\} = w(S;0) = a(S)$ under any equilibrium indicator. Hence the core is nonempty and at least one core outcome has $d^\star\!=\!0$; moreover, Lemma EC.1 implies that every core outcome – in particular every core outcome with $d^\star\!=\!0$ – compensates. For the equivalence: if $\bar{\pi} = \{\mathcal{L}\}$, then $I(\mathcal{L}) > D(\mathcal{L})$ and Lemma EC.4(i) implies the core contains only outcomes with $d^\star\!=\!0$; if $\bar{\pi} \neq \{\mathcal{L}\}$ and $\eta < \underline{\eta}_1$, Lemma EC.6(i) implies the same. Conversely, if $\bar{\pi} \neq \{\mathcal{L}\}$ and $\eta \geq \underline{\eta}_1$, Lemma EC.6(ii) together with nonemptiness implies the core contains an outcome with $d^\star\!=\!1$.

Part C ($\mathcal{G} = \emptyset$). Every local satisfies $\Delta_\ell < 0$. The core contains the outcome $(\bar{\pi}, 1, a)$ with $a_\ell = v_\ell^{\text{sq}} + D_\ell$ for every $\ell \in \mathcal{L}$: it is feasible because every coalition S satisfies $\Delta(S) < 0 \leq \eta|\mathcal{L} \setminus S|$, so $1 \in T(\bar{\pi}, \bar{\mathbb{U}})$ by (EC.13); and it is not dominated because any deviating coalition S obtains at most $\max\{w(S;0), w(S;1)\} = w(S;1) = a(S)$. Moreover, no core outcome has $d^\star\!=\!0$: for every partition

$\pi \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$, no coalition of π satisfies $\Delta(S) > 0$, so the sum on the left of the condition in (EC.14) is empty and equals 0, whereas the maximum on its right is at least 1; hence $0 \notin T(\pi, \bar{\mathbb{U}})$ for every π, and no outcome with $d^\star = 0$ exists. □

EC.5.2.1. Lemmas for the Proof of Theorem 2 The proof of Theorem 2 relies on Lemma 2, on Lemma EC.1, and on the lemmas in this subsection. We group the latter according to the cooperation structure they concern, which is also how the theorem's statement is organized: conditions under which the core is nonempty, or may be empty (the opening claim of Part A); full cooperation, $\bar{\pi} = \{\mathcal{L}\}$ (Parts B and A-i); limited cooperation with at least one family that prefers the incentive (Parts B and A-ii); and limited cooperation with no family that prefers the incentive (Part A-iii). Within each group we separate the constructive results, which establish what the core must contain, from the instances showing that the core may be empty or may lack outcomes of a given type. The results are stated in terms of the blocking-cost thresholds, which satisfy $\underline{\eta}_1 \le \overline{\eta}_1 \le \overline{\eta}$, $\eta_2 \le \overline{\eta}$ when $\mathcal{G} \ne \mathcal{L}$, $\underline{\eta}_1 = \overline{\eta}_1$ when every family prefers the incentive, and $\eta_2 \le \underline{\eta}_1$ when $\bar{\pi} = \{\mathcal{L}\}$ and $\mathcal{G} \ne \mathcal{L}$.

Nonemptiness of the core. The next two lemmas establish the opening claim of Part A. Lemma EC.2 gives sufficient conditions for the core to be nonempty: for $\eta \le \eta_2$ it contains an outcome with $d^\star = 0$ sustained by blocking threats, and for $\eta \ge \overline{\eta}_1$ it contains an outcome with $d^\star = 1$; its statements about residual games are also used in the proofs of Parts A-i–iii and of Lemma EC.5. Lemma EC.3 shows, through explicit instances, that the core can be empty when $\emptyset \ne \mathcal{G} \ne \mathcal{L}$ and $\eta_2 < \eta < \overline{\eta}_1$.

Lemma EC.2. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3.*

(a) If $\mathcal{L} \ne \mathcal{G}$, $I(F) > D(F)$ for some $F \in \bar{\pi}$, and $\eta \le \eta_2$, then:

(i) for any residual set $R \subseteq \mathcal{L}$, $R \ne \emptyset$, and any partition $\pi_{\mathcal{L}\setminus R} \in \Pi_{\mathcal{L}\setminus R}^{\prec\bar{\pi}}$ satisfying

$$I(H) \le D(H) \text{ for every } H \in \pi_{\mathcal{L}\setminus R}, \tag{EC.16}$$

the residual core $C(R; \pi_{\mathcal{L}\setminus R})$ contains an outcome with $d^\star = 0$ that does not compensate the locals in $R \setminus \mathcal{G}$;

(ii) the core contains an outcome with $d^\star = 0$ that does not compensate.

(b) If $\eta \ge \overline{\eta}_1$, then:

(i) for every residual game with $|R| \le |\mathcal{L}| - 1$ and any $\pi_{\mathcal{L}\setminus R} \in \Pi_{\mathcal{L}\setminus R}^{\prec\bar{\pi}}$, $1 \in A(R; \pi_{\mathcal{L}\setminus R})$;

(ii) if $\bar{\pi} \ne \{\mathcal{L}\}$, or $\bar{\pi} = \{\mathcal{L}\}$ with $I(\mathcal{L}) \le D(\mathcal{L})$, the core contains an outcome with $d^\star = 1$.

Proof of Lemma EC.2. For this proof, we define and use $\Delta_\ell := I_\ell - D_\ell, \forall \ell \in \mathcal{L}$, which substantially simplifies notation. It also helps to recall the definitions of η_2 and $\overline{\eta}_1$ in (7) and (8), written in the Δ notation:

$$\eta_2 := \min_{F',H}\left\{\max_{\eta}\left\{\eta : \sum_{\substack{F\in\bar{\pi}:\\ \Delta(F\setminus H)>0}} \left\lfloor \frac{\Delta(F\setminus H)}{\eta}\right\rfloor \geq |H|\right\} : F'\in\bar{\pi},\, H\subseteq F',\, \Delta(H)\leq 0\right\},$$

$$\overline{\eta}_1 := \max_{F,S,\eta}\left\{\eta : \Delta(S)\geq \eta\,|\mathcal{L}\setminus S|,\, F\in\bar{\pi},\, S\subseteq F,\, S\neq\mathcal{L}\right\}.$$

Item (a)-(i). We prove by induction on $|R|$ that the residual core contains the following outcome:

$$\pi_R = \{S : \exists F\in\bar{\pi} \text{ with } S = \mathcal{G}\cap R\cap F\}\cup\{\{\ell\} : \ell\in R\setminus\mathcal{G}\}, \tag{EC.17a}$$

$$d^\star = 0, \tag{EC.17b}$$

$$a_\ell = v_\ell^{\text{sq}} + I_\ell, \qquad \forall \ell\in R. \tag{EC.17c}$$

We first show that this is a valid residual outcome. Let

$$\sigma := \pi_R \cup \pi_{\mathcal{L}\setminus R}$$

and let $\bar{H}$ be a largest coalition in $\pi_{\mathcal{L}\setminus R}\cup\{\{\ell\} : \ell\in R\setminus\mathcal{G}\}$. Every such coalition weakly prefers deforestation: this holds by (EC.16) for coalitions in $\pi_{\mathcal{L}\setminus R}$ and by the definition of $\mathcal{G}$ for the singleton coalitions in $R\setminus\mathcal{G}$. Moreover, these are exactly the coalitions in σ that weakly prefer deforestation, whereas every coalition $F\cap R\cap\mathcal{G}$ in π_R prefers the incentive. Hence

$$|\bar{H}| = \max_{H\in\sigma:\,\Delta(H)\leq 0} |H|. \tag{EC.18}$$

Because $\bar{H}$ lies within a family and $\Delta(\bar{H})\leq 0$, the hypothesis $\eta\leq\eta_2$ implies

$$\sum_{F\in\bar{\pi}:\,\Delta(F\setminus\bar{H})>0} \left\lfloor \frac{\Delta(F\setminus\bar{H})}{\eta}\right\rfloor \geq |\bar{H}|. \tag{EC.19}$$

For every family $F\in\bar{\pi}$,

$$\Delta(F\setminus\bar{H}) \leq \Delta(F\cap R\cap\mathcal{G}). \tag{EC.20}$$

To see this, decompose F into $F\cap R\cap\mathcal{G}$, $F\cap(R\setminus\mathcal{G})$, and $F\cap(\mathcal{L}\setminus R)$. The second set has negative total Δ, and the third is partitioned by coalitions in $\pi_{\mathcal{L}\setminus R}$, each with weakly negative total Δ. If $\bar{H}$ is contained in F, removing it leaves the union of the same incentive-preferring residual locals and the remaining weakly negative components; if $\bar{H}$ is not contained in F, the same conclusion follows without removing any component. This proves (EC.20).

Equations (EC.19)–(EC.20) imply

$$\sum_{F\in\bar{\pi}} \left\lfloor \frac{\Delta(F\cap R\cap\mathcal{G})}{\eta}\right\rfloor \geq |\bar{H}| = \max_{H\in\sigma:\,\Delta(H)\leq 0} |H|.$$

The left-hand side is precisely the total blocking capacity of the coalitions in σ that prefer the incentive. Therefore, $\eta \le \eta_2(\sigma)$, and Lemma 2 gives $0 \in T(\sigma)$. Thus, (EC.17a)–(EC.17c) define a valid residual outcome.

For $|R| = 1$, the outcome belongs to the residual core by its definition. Suppose now that the claim holds for every residual game with at most k residual locals, and consider $|R| = k+1$. To show that the outcome is not dominated, consider a deviating coalition $S \subseteq R$ that forms a partition $\pi_S \in \Pi_S^{\prec\bar{\pi}}$. If some $S_i \in \pi_S$ satisfies $\Delta(S_i) \ge 0$, then

$$w(S_i; d^\star) \le w(S_i; 0) = \sum_{\ell \in S_i} (v_\ell^{\text{sq}} + I_\ell) = a(S_i), \qquad \forall d^\star \in \{0,1\},$$

so the deviation is not profitable.

Otherwise, $\Delta(S_i) < 0$ for every $S_i \in \pi_S$. Together with (EC.16), this means that every coalition in the fixed partition $\pi_S \cup \pi_{\mathcal{L}\setminus R}$ weakly prefers deforestation. The existence of a family that prefers the incentive implies $R \setminus S \neq \emptyset$ in this case: if $S = R$, the coalitions in $\pi_S \cup \pi_{\mathcal{L}\setminus R}$ would partition every family into coalitions that weakly prefer deforestation. Hence $|R \setminus S| \le k$, and the inductive hypothesis gives

$$0 \in A(R \setminus S; \pi_S \cup \pi_{\mathcal{L}\setminus R}).$$

Consequently, for every $S_i \in \pi_S$,

$$\min_{d^\star \in A(R\setminus S; \pi_S \cup \pi_{\mathcal{L}\setminus R})} w(S_i; d^\star) \le w(S_i; 0) = a(S_i),$$

so the deviation again fails under pessimism. This completes the induction. Finally, every $\ell \in R \setminus \mathcal{G}$ receives $a_\ell = v_\ell^{\text{sq}} + I_\ell < v_\ell^{\text{sq}} + D_\ell$, so the outcome does not compensate those locals.

Item (a)-(ii). Apply part (i) to $R = \mathcal{L}$ and $\pi_{\mathcal{L}\setminus R} = \emptyset$. The core, therefore, contains the outcome

$$\pi = \{S : \exists F \in \bar{\pi} \text{ with } S = \mathcal{G} \cap F\} \cup \{\{\ell\} : \ell \in \mathcal{L} \setminus \mathcal{G}\}, \qquad d^\star = 0, \qquad a_\ell = v_\ell^{\text{sq}} + I_\ell,$$

which does not compensate the locals in $\mathcal{L} \setminus \mathcal{G}$.

Item (b)-(i). We prove the result by induction on $|R|$. First, consider $R = \{h\}$. Because $\mathcal{L} \setminus \{h\} \neq \emptyset$, the partition $\{\{h\}\} \cup \pi_{\mathcal{L}\setminus\{h\}}$ is not the grand-coalition partition. By the definitions of $\overline{\eta}_1$ and $\eta_1(\cdot)$,

$$\eta \ge \overline{\eta}_1 \ge \eta_1(\pi), \qquad \forall \pi \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}, \ \pi \neq \{\mathcal{L}\}. \tag{EC.21}$$

Hence Lemma 2 gives $1 \in T(\{\{h\}\} \cup \pi_{\mathcal{L}\setminus\{h\}})$, so the singleton residual core contains

$$(\{\{h\}\}, 1, a_h), \qquad a_h = v_h^{\text{sq}} + D_h.$$

Suppose inductively that $1 \in A(R'; \pi_{\mathcal{L}\setminus R'})$ for every residual set R' with $1 \le |R'| \le k$ and every feasible fixed partition. Consider a residual set R with $|R| = k+1 \le |\mathcal{L}| - 1$. Define

$$\pi_R := \{S : \exists F \in \bar{\pi} \text{ with } S = R \cap F\}, \qquad a_\ell := v_\ell^{\text{sq}} + D_\ell, \quad \ell \in R.$$

Because $\pi_R \cup \pi_{\mathcal{L}\setminus R} \neq \{\mathcal{L}\}$, (EC.21) and Lemma 2 imply $1 \in T(\pi_R \cup \pi_{\mathcal{L}\setminus R})$, so $(\pi_R, 1, a)$ is a valid residual outcome.

Consider a deviating coalition $H \subseteq R$ forming $\pi_H \in \Pi_H^{\prec \bar{\pi}}$. If $H \subsetneq R$, the inductive hypothesis gives $1 \in A(R \setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R})$. If $H = R$, then $\pi_H \cup \pi_{\mathcal{L}\setminus R} \neq \{\mathcal{L}\}$, so (EC.21) and Lemma 2 give

$$1 \in A(\emptyset; \pi_H \cup \pi_{\mathcal{L}\setminus R}) = T(\pi_H \cup \pi_{\mathcal{L}\setminus R}).$$

Thus, in either case, for every $H_i \in \pi_H$,

$$\min_{d^\star \in A(R\setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R})} w(H_i; d^\star) \le w(H_i; 1) = \sum_{\ell \in H_i} (v_\ell^{\text{sq}} + D_\ell) = a(H_i).$$

No deviation is profitable, so $(\pi_R, 1, a)$ belongs to the residual core. This completes the induction.

Item (b)-(ii). Consider the outcome $(\bar{\pi}, 1, a)$ with $a_\ell = v_\ell^{\text{sq}} + D_\ell$ for every $\ell \in \mathcal{L}$. It is feasible: if $\bar{\pi} \neq \{\mathcal{L}\}$, then (EC.21) and Lemma 2 give $1 \in T(\bar{\pi})$; if $\bar{\pi} = \{\mathcal{L}\}$, feasibility follows from $I(\mathcal{L}) \le D(\mathcal{L})$.

Suppose a coalition $H \subseteq \mathcal{L}$ forms $\pi_H \in \Pi_H^{\prec \bar{\pi}}$. If $H \subsetneq \mathcal{L}$, part (i) gives $1 \in A(\mathcal{L} \setminus H; \pi_H)$, so for every $S \in \pi_H$,

$$\min_{d^\star \in A(\mathcal{L}\setminus H; \pi_H)} w(S; d^\star) \le w(S; 1) = a(S).$$

Hence the deviation is not profitable.

The only remaining possibility is $H = \mathcal{L}$, which requires $\bar{\pi} = \{\mathcal{L}\}$. If $\pi_H = \{\mathcal{L}\}$, then

$$\max\{w(\mathcal{L}; 0), w(\mathcal{L}; 1)\} = w(\mathcal{L}; 1) = a(\mathcal{L}),$$

because $I(\mathcal{L}) \le D(\mathcal{L})$. If $\pi_H \neq \{\mathcal{L}\}$, then (EC.21) and Lemma 2 imply $1 \in A(\emptyset; \pi_H) = T(\pi_H)$. Under this indicator, $w(S; 1) = a(S)$ for every $S \in \pi_H$. Thus the grand coalition cannot profitably deviate either, and $(\bar{\pi}, 1, a)$ belongs to the core. □

Lemma EC.3. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3, and suppose $I(F) > D(F)$ for some $F \in \bar{\pi}$. In each of the following cases, the core may be empty:*

(i) if $\eta \in (\eta_2, \overline{\eta}_1)$;

(ii) if $\underline{\eta}_1 \le \eta_2 < \overline{\eta}_1$ and $\eta \in (\eta_2, \overline{\eta}_1)$;

(iii) if $\eta_2 < \underline{\eta}_1 < \overline{\eta}_1$ and $\eta \in (\underline{\eta}_1, \overline{\eta}_1)$.

Proof of Lemma EC.3. To simplify notation, we define $\Delta_\ell := I_\ell - D_\ell, \forall \ell \in \mathcal{L}$.

Item (i). Consider the following example with limited cooperation:

$$\mathcal{L} = \{\ell, g, h, e\}, \qquad \bar{\pi} = \{\{\ell, g, h\}, \{e\}\} \tag{EC.22a}$$

$$\Delta_\ell = \Delta_g = \Delta > 0, \qquad \Delta_h < 0, \qquad 0 < \Delta_e < \frac{\Delta}{2}, \qquad \Delta(\mathcal{L}) > 0, \tag{EC.22b}$$

$$\Delta_h + \Delta < 0, \qquad \Delta(\{\ell, g, h\}) > 0. \tag{EC.22c}$$

For this instance, we calculate the relevant thresholds. Recall the definitions (7) and (9), in the Δ notation:

$$\eta_2 := \min_{F',H} \left\{ \max_\eta \left\{ \eta : \sum_{\substack{F \in \bar{\pi}: \\ \Delta(F \setminus H) > 0}} \left\lfloor \frac{\Delta(F \setminus H)}{\eta} \right\rfloor \ge |H| \right\} : F' \in \bar{\pi},\, H \subseteq F',\, \Delta(H) \le 0 \right\},$$

$$\underline{\eta}_1 := \max_{F,S,\eta} \left\{ \eta : \Delta(S) \ge \eta \, |\mathcal{L} \setminus S|,\, F \in \bar{\pi},\, S \subseteq F,\, S \ne \mathcal{L},\, \Delta(F) > 0 \right\},$$

$$\overline{\eta}_1 := \max_{F,S,\eta} \left\{ \eta : \Delta(S) \ge \eta \, |\mathcal{L} \setminus S|,\, F \in \bar{\pi},\, S \subseteq F,\, S \ne \mathcal{L} \right\}.$$

For η_2, note that the coalition $H \subseteq F$ with $\Delta(H) \le 0$ that is optimal in the outer minimization is $\{\ell, h\}$ (or equivalently, $\{g, h\}$), so that η must satisfy:

$$\left\lfloor \frac{\Delta}{\eta} \right\rfloor + \left\lfloor \frac{\Delta_e}{\eta} \right\rfloor \ge 2 \quad \Rightarrow \quad \eta \le \frac{\Delta}{2},$$

where the inequality follows because $\Delta_e < \Delta/2$. We conclude that $\eta_2 = \frac{\Delta_\ell}{2}$. Similarly, because all families F satisfy $\Delta(F) > 0$,

$$\underline{\eta}_1 = \overline{\eta}_1 = \max\left(\frac{\Delta}{3}, \quad \frac{2\Delta}{2}, \quad \Delta(\{\ell, g, h\}), \quad \frac{\Delta_e}{3} \right) = \Delta,$$

where each candidate is the largest η feasible for the corresponding pair (F, S) in (9), and the second equality follows because $\Delta(\{\ell, g, h\}) < \Delta$ and $\Delta_e < \Delta$. Note that both families prefer the incentive here ($\Delta(\{\ell, g, h\}) > 0$ and $\Delta_e > 0$), so $\overline{\eta}_1 = \underline{\eta}_1 = \Delta$ and the blocking costs considered below span the entire interval $(\eta_2, \overline{\eta}_1)$; the same is true of the full-cooperation variant at the end of this proof, whose single family prefers the incentive. Thus, we consider η satisfying:

$$\eta_2 = \frac{\Delta}{2} < \eta < \underline{\eta}_1 = \overline{\eta}_1 = \Delta. \tag{EC.23}$$

Let us assume by contradiction that there is an outcome in $C(\mathcal{L}; \emptyset)$ with allocation a. We show that a would have to satisfy the following infeasible system of inequalities:

$$a_\ell + a_g + a_h = \sum_{j \in \{\ell, g, h\}} \left(v_j^{\text{sq}} + I_j \right), \tag{EC.24a}$$

$$a_\ell + a_g \ge \sum_{j \in \{\ell, g\}} \left(v_j^{\text{sq}} + I_j \right), \tag{EC.24b}$$

$$a_h \geq v_h^{\mathrm{sq}} + I_h, \tag{EC.24c}$$

$$a_\ell + a_h \geq \sum_{j\in\{\ell,h\}} \big(v_j^{\mathrm{sq}} + D_j\big), \tag{EC.24d}$$

$$a_g + a_h \geq \sum_{j\in\{g,h\}} \big(v_j^{\mathrm{sq}} + D_j\big). \tag{EC.24e}$$

First, we show that the above system is indeed infeasible. Equations (EC.24a) to (EC.24c) imply

$$a_h = v_h^{\mathrm{sq}} + I_h, \qquad a_\ell + a_g = \sum_{j\in\{\ell,g\}} \big(v_j^{\mathrm{sq}} + I_j\big).$$

Together with (EC.24d), this gives

$$\begin{aligned} a_\ell &\geq v_\ell^{\mathrm{sq}} + D_\ell + v_h^{\mathrm{sq}} + D_h - \big(v_h^{\mathrm{sq}} + I_h\big) \\ &= v_\ell^{\mathrm{sq}} + D_\ell - \Delta_h \\ &= v_\ell^{\mathrm{sq}} + I_\ell - (\Delta_\ell + \Delta_h) > v_\ell^{\mathrm{sq}} + I_\ell, \end{aligned}$$

where the strict inequality follows from (EC.22c). By an analogous argument, we also have that $a_g > v_g^{\mathrm{sq}} + I_g$, which implies a contradiction for (EC.24a) because

$$a_\ell + a_g + a_h > \sum_{j\in\{\ell,g,h\}} (v_j^{\mathrm{sq}} + I_j).$$

We now show that the system (EC.24a)-(EC.24e) must hold. The equality in (EC.24a) comes from Item (i), which applies because $\eta < \underline{\eta}_1$, and states that all outcomes in the core should be outcomes with $d^\star\!=\!0$. Thus, $a_\ell + a_g + a_h$ should equal $w(\{\ell,g,h\};0) = \sum_{j\in\{\ell,g,h\}}\big(v_j^{\mathrm{sq}} + I_j\big)$.

Inequality (EC.24b) comes from the plausible deviation of ℓ and g, where they cooperate to block any production by h or e. This applies because the only feasible partition $\pi_{\{e,h\}}$ finer than π is $\pi_{\{e,h\}} = \{\{h\},\{e\}\}$, and

$$\eta_1(\{\{\ell,g\},\{h\},\{e\}\}) = \Delta_g = \Delta, \tag{EC.25}$$

which together with (EC.23) implies that $\eta < \eta_1(\{\{\ell,g\},\{h\},\{e\}\})$. By Lemma 2, this implies

$$T(\{\{h\},\{\ell,g\},\{e\}\}) = A(\{h,e\};\{\{\ell,g\}\}) = \{0\}.$$

Thus, if ℓ and g were to form the coalition $\{\ell,g\}$, they could block h and e and ensure an outcome with $d^\star\!=\!0$, where they would obtain $\sum_{j\in\{\ell,g\}}\big(v_j^{\mathrm{sq}} + I_j\big)$. Therefore, $a_\ell + a_g$ must satisfy (EC.24b).

Inequality (EC.24c) comes from Lemma EC.1 and $h \in \mathcal{L}\setminus\mathcal{G}$.

The last two inequalities (EC.24d)–(EC.24e) come from two plausible deviations of $\{\ell,h\}$ and $\{g,h\}$, respectively. Because ℓ and g are symmetric in our example, we show this for $\{\ell,h\}$. Note that the only feasible partition of $\{g,e\}$ is as singletons, $\{\{g\},\{e\}\}$. We have

$$\eta_2\big(\{\{g\},\{\ell,h\},\{e\}\}\big) = \frac{\Delta_g}{2} = \frac{\Delta}{2}, \tag{EC.26}$$

which holds because $\Delta_e < \Delta_g = \Delta$ and $\Delta_h + \Delta < 0$. This together with (EC.23) implies that $\eta > \eta_2(\{\{g\},\{\ell,h\},\{e\}\})$, which by Lemma 2, implies that

$$\{1\} = T(\{\{g\},\{\ell,h\},\{e\}\}) = A(\{g,e\};\{\{\ell,h\}\}).$$

Thus, $\{\ell,h\}$ could deviate to an outcome with $d^\star = 1$ and obtain $\sum_{j\in\{\ell,h\}}\big(v_j^{\mathrm{sq}} + D_j\big)$, so the core outcome must satisfy (EC.24d). A symmetric argument for $\{g,h\}$ implies that any outcome in the core $C(\mathcal{L};\emptyset)$ must satisfy (EC.24d)–(EC.24e). This proves the lemma for the case when $\bar{\pi} \neq \{\mathcal{L}\}$.

Finally, we note that the same instance with $\mathcal{L} = \{\ell, g, h\}$ and $\bar{\pi} = \{\mathcal{L}\}$ would also lead to an empty core, as long as η satisfies

$$\eta_2 = \min_{\substack{H\subseteq\mathcal{L}:\\ \Delta(H)\le 0}} \frac{\Delta(\mathcal{L}\setminus H)}{|H|} = \frac{\Delta_\ell}{2} = \frac{\Delta_g}{2} \quad < \quad \eta \quad < \quad \underline{\eta}_1 = \Delta(\mathcal{G}) = 2\Delta_\ell = 2\Delta_g.$$

The proof is identical, except that $\eta_1(\{\{\ell,g\},\{h\}\})$ takes value $\Delta(\mathcal{G})$ in (EC.25).

Item (ii). Because $\underline{\eta}_1 \le \eta_2 < \overline{\eta}_1$, there must be limited cooperation. Consider the following instance:

$$\mathcal{L} = \{g, b_1, b_2, e\}, \qquad \bar{\pi} = \{F_1, F_2\}, \qquad F_1 := \{g, b_1, b_2\}, \quad F_2 := \{e\}, \tag{EC.27a}$$

$$\Delta_g > 0, \qquad \Delta_{b_1}, \Delta_{b_2} < 0, \qquad \Delta_g + \Delta_{b_i} < 0 \;\; (i = 1,2), \qquad 0 < \Delta_e < \tfrac{2}{3}\Delta_g \tag{EC.27b}$$

(for example, $(\Delta_g, \Delta_{b_1}, \Delta_{b_2}, \Delta_e) = (5, -6, -7, 1)$). Then $\Delta(F_1) < 0 < \Delta(F_2)$, and within F_1 the only coalition with $\Delta(S) > 0$ is $\{g\}$, because every $S \ni g$ with $S \neq \{g\}$ satisfies $\Delta(S) \le \Delta_g + \max_i \Delta_{b_i} < 0$.

We first compute the thresholds. For $\overline{\eta}_1$, the candidate values in (8) are $\Delta_g/3$ and $\Delta_e/3$, so $\overline{\eta}_1 = \Delta_g/3$. For $\underline{\eta}_1$, the only family with $\Delta(F) > 0$ is F_2, so $\underline{\eta}_1 = \Delta_e/3$. For η_2, the coalitions H within a family with $\Delta(H) \le 0$ are $\{b_i\}$, $\{g, b_i\}$, $\{b_1, b_2\}$, and F_1; for the first two, the only family F with $\Delta(F \setminus H) > 0$ is F_2, and their conditions in the definition of η_2 (7) hold if and only if $\eta \le \Delta_e$ and $\eta \le \Delta_e/2$, respectively; for $\{b_1, b_2\}$ the condition holds if and only if $\eta \le \max\{\Delta_g/2, \Delta_e\}$; and for $H = F_1$ it reads $\lfloor \Delta_e/\eta \rfloor \ge 3$, i.e., $\eta \le \Delta_e/3$. Hence $\underline{\eta}_1 = \eta_2 = \Delta_e/3 < \overline{\eta}_1 = \Delta_g/3$. Fix any

$$\eta \in \left(\frac{\Delta_e}{2}, \frac{\Delta_g}{3}\right) \subset (\eta_2, \overline{\eta}_1), \tag{EC.28}$$

a non-empty interval because $\Delta_e < \frac{2}{3}\Delta_g$. (As $\Delta_e/\Delta_g \to 0$, the relative position of the left endpoint within $(\eta_2, \overline{\eta}_1)$, which equals $\Delta_e/\big(2(\Delta_g - \Delta_e)\big)$, vanishes; instances of this form therefore exhibit an empty core at every relative position of η within $(\eta_2, \overline{\eta}_1)$.)

The partitions in $\Pi_{\mathcal{L}}^{\preceq\bar{\pi}}$ are $\pi_A := \{F_1, \{e\}\}$, $\pi_B^i := \{\{g, b_i\}, \{b_j\}, \{e\}\}$ (for $\{i,j\} = \{1,2\}$), $\pi_C := \{\{b_1, b_2\}, \{g\}, \{e\}\}$, and $\pi_D := \{\{g\}, \{b_1\}, \{b_2\}, \{e\}\}$. By Lemma 2:

$$T(\pi_A) = T(\pi_B^i) = \{1\}, \qquad T(\pi_C) = T(\pi_D) = \{0\}. \tag{EC.29}$$

Indeed, $1 \in T(\pi)$ if and only if $\{g\} \notin \pi$: the only coalitions with $\Delta(S) > 0$ are $\{g\}$ and $\{e\}$, and (EC.28) gives $\Delta_g > 3\eta$ and $\Delta_e < 2\eta < 3\eta$. And $0 \in T(\pi)$ if and only if $\{g\} \in \pi$: if $\{g\} \in \pi$, then $\lfloor \Delta_g/\eta \rfloor \geq 3 \geq \max_{H\in\pi:\Delta(H)\leq 0} |H|$; if $\{g\} \notin \pi$, the only coalition with $\Delta(S) > 0$ is $\{e\}$, with $\lfloor \Delta_e/\eta \rfloor \leq 1$ by $\eta > \Delta_e/2$, whereas π_A and π_B^i contain a coalition with $\Delta(H) \leq 0$ of size 3 and 2, respectively.

Assume by contradiction that the core $C(\mathcal{L};\emptyset)$ contains an outcome.

Every outcome with $d^\star\!=\!0$ is dominated. By (EC.29), its partition is π_C or π_D; in either case the coalitions containing F_1's members partition F_1, so $a(F_1) = v^{\mathrm{sq}}_{F_1} + I(F_1) = w(F_1;0)$. Let F_1 deviate forming $\{F_1\}$: the residual $\{e\}$ admits the unique partition $\{\{e\}\}$, so $A(\{e\};\{F_1\}) = T(\pi_A) = \{1\}$ by (5), and the deviation secures $w(F_1;1) = w(F_1;0) - \Delta(F_1) > a(F_1)$ because $\Delta(F_1) < 0$.

Every outcome with $d^\star\!=\!1$ is dominated. By (EC.29), its partition is π_A or π_B^i, so $a(F_1) = w(F_1;1)$. First, $a_g \geq v^{\mathrm{sq}}_g + I_g$ must hold: every partition of $\mathcal{L}$ containing $\{g\}$ is π_C or π_D, so (EC.29) and (5) give $A(\mathcal{L}\setminus\{g\};\{\{g\}\}) = \{0\}$ (whether or not the residual core is empty), and $\{g\}$ deviating secures $w(\{g\};0) = v^{\mathrm{sq}}_g + I_g$; if $a_g < v^{\mathrm{sq}}_g + I_g$, the outcome is dominated. But then

$$a_{b_1} + a_{b_2} \;=\; w(F_1;1) - a_g \;\leq\; \sum_{i=1,2} \big(v^{\mathrm{sq}}_{b_i} + D_{b_i}\big) - \Delta_g \;<\; \sum_{i=1,2} \big(v^{\mathrm{sq}}_{b_i} + D_{b_i}\big),$$

so $a_b < v^{\mathrm{sq}}_b + D_b$ for some $b \in \{b_1, b_2\}$; write b' for the other, so that $\pi_B := \{\{g,b'\},\{b\},\{e\}\}$. We show that $A(\mathcal{L}\setminus\{b\};\{\{b\}\}) = \{1\}$; then $\{b\}$ deviating secures $w(\{b\};1) = v^{\mathrm{sq}}_b + D_b > a_b$, and the outcome is dominated. The residual game for $R = \{g,b',e\}$ facing $\{\{b\}\}$ admits the two partitions $\sigma_1 = \{\{g,b'\},\{e\}\}$ and $\sigma_2 = \{\{g\},\{b'\},\{e\}\}$, whose resulting partitions of $\mathcal{L}$ are π_B and π_D.

First, no outcome with $d^\star\!=\!0$ lies in $C(R;\{\{b\}\})$: by (EC.29), such an outcome uses σ_2, with allocation pinned at $a'_\ell = w(\{\ell\};0)$ for every $\ell \in R$. The coalition $\{g,b'\}$ deviates forming $\{\{g,b'\}\}$: the remaining local e admits the unique partition $\{\{e\}\}$, so $A(\{e\};\{\{g,b'\},\{b\}\}) = T(\pi_B) = \{1\}$, and the deviation secures $w(\{g,b'\};1) = a'(\{g,b'\}) - \Delta(\{g,b'\}) > a'(\{g,b'\})$, because $\Delta(\{g,b'\}) = \Delta_g + \Delta_{b'} < 0$.

Second, an outcome with $d^\star\!=\!1$ lies in $C(R;\{\{b\}\})$: take $(\sigma_1, 1, a')$ with

$$a'_g = v^{\mathrm{sq}}_g + I_g, \qquad a'_{b'} = v^{\mathrm{sq}}_{b'} + D_{b'} - \Delta_g, \qquad a'_e = v^{\mathrm{sq}}_e + D_e,$$

which is feasible because $a'_g + a'_{b'} = w(\{g,b'\};1)$, $a'_e = w(\{e\};1)$, and $1 \in T(\pi_B)$; note that $a'_{b'} - \big(v^{\mathrm{sq}}_{b'} + I_{b'}\big) = -(\Delta_g + \Delta_{b'}) > 0$. No deviation is profitable. $\{g\}$ faces $A(\{b',e\};\{\{g\},\{b\}\}) = T(\pi_D) = \{0\}$ (the only partition of $\{b',e\}$ is into singletons) and secures $w(\{g\};0) = a'_g$. $\{b'\}$ faces $A(\{g,e\};\{\{b'\},\{b\}\}) = T(\pi_D) = \{0\}$ and secures $w(\{b'\};0) = v^{\mathrm{sq}}_{b'} + I_{b'} < a'_{b'}$. The pair $\{g,b'\}$ secures at most $\max\{w(\{g,b'\};0), w(\{g,b'\};1)\} = w(\{g,b'\};1) = a'(\{g,b'\})$, because $\Delta(\{g,b'\}) < 0$. Lastly, consider $\{e\}$, which faces $A(\{g,b'\};\{\{e\},\{b\}\})$. The residual game for $\{g,b'\}$ facing $\{\{e\},\{b\}\}$ admits the partitions $\{\{g,b'\}\}$ and $\{\{g\},\{b'\}\}$, whose resulting partitions of $\mathcal{L}$ are π_B and π_D; its outcomes with $d^\star\!=\!0$ (on $\{\{g\},\{b'\}\}$, pinned at $w(\cdot;0)$) are dominated by $\{g,b'\}$ re-forming as one coalition, which faces $A(\emptyset;\{\{g,b'\},\{e\},\{b\}\}) = T(\pi_B) = \{1\}$ and secures $w(\{g,b'\};1) > w(\{g,b'\};0)$; and the outcome with $d^\star\!=\!1$ $\big(\{\{g,b'\}\},1,a''\big)$ with $a''_g = v_g^{\text{sq}} + I_g$ and $a''_{b'} = w(\{g,b'\};1) - a''_g$ lies in its core, by the same checks as for $(\sigma_1,1,a')$ above. Hence every equilibrium indicator in $A(\{g,b'\};\{\{e\},\{b\}\})$ equals 1, so $\{e\}$ secures $\min_{d^\star} w(\{e\};d^\star) = w(\{e\};1) = a'_e$. Therefore $(\sigma_1,1,a') \in C(R;\{\{b\}\})$.

The two observations give $A(\mathcal{L}\setminus\{b\};\{\{b\}\}) = \{1\}$, as claimed. Every outcome is therefore dominated, so $C(\mathcal{L};\emptyset) = \emptyset$.

Item (iii). Consider the following instance with limited cooperation:

$$\mathcal{L} = \{\ell,g,h,r,e\}, \qquad \bar\pi = \{F,F'\}, \qquad F := \{\ell,g,h\}, \quad F' := \{r,e\}, \tag{EC.30a}$$

$$\Delta_g > 0, \qquad \Delta_h > 2\Delta_g, \qquad \Delta(F) < 0 < \Delta(F) + \Delta_g, \tag{EC.30b}$$

$$\Delta_r < 0 < \Delta(F'), \qquad \Delta_e < 4|\Delta_r|, \qquad 3\Delta_e < 2(\Delta_g + \Delta_h), \tag{EC.30c}$$

(for example, $(\Delta_\ell,\Delta_g,\Delta_h,\Delta_r,\Delta_e) = (-14,\,4,\,9,\,-2,\,7)$), together with any blocking cost

$$\max\Big\{\Delta_g,\ \tfrac{1}{2}\Delta_e\Big\} \;<\; \eta \;<\; \tfrac{1}{3}\big(\Delta_g + \Delta_h\big), \tag{EC.31}$$

a non-empty range because $\Delta_h > 2\Delta_g$ and $3\Delta_e < 2(\Delta_g + \Delta_h)$.

We first locate the thresholds. The only family that prefers the incentive is F', whose coalitions $\{e\}$, $\{r\}$ and F' contribute the values $\Delta_e/4$, a negative value, and $\Delta(F')/3$ to (9); because $\Delta_e < 4|\Delta_r|$ is equivalent to $\Delta_e/4 > \Delta(F')/3$, we obtain $\underline{\eta}_1 = \Delta_e/4$. In (8), the coalitions of F contribute $\Delta_g/4$, $\Delta_h/4$ and $(\Delta_g+\Delta_h)/3$ – every coalition containing ℓ has $\Delta \le 0$, because $\Delta(F) < 0$ and $\Delta_g, \Delta_h > 0$ imply $\Delta_\ell + \Delta_g < 0$ and $\Delta_\ell + \Delta_h < 0$ – and the largest candidate overall is $(\Delta_g+\Delta_h)/3$, so $\overline{\eta}_1 = (\Delta_g+\Delta_h)/3$. Finally, $H = F$ is admissible in (7) because $\Delta(F) < 0$, and the only family F'' with $\Delta(F''\setminus F) > 0$ is F', so the condition in (7) reads $\lfloor \Delta(F')/\eta \rfloor \ge |F| = 3$ and yields $\eta_2 \le \Delta(F')/3 < \Delta_e/4 = \underline{\eta}_1$. Hence $\eta_2 < \underline{\eta}_1 < \overline{\eta}_1$ and, by (EC.31), $\eta \in (\underline{\eta}_1, \overline{\eta}_1)$ because $\eta > \Delta_e/2 > \Delta_e/4 = \underline{\eta}_1$.

We show that every outcome is dominated, using throughout that $w(S;0) = w(S;1) + \Delta(S)$ for every coalition S.

Every outcome with $d^\star\!=\!0$ is dominated. Let $(\pi,0,a)$ be such an outcome. Its coalitions containing members of F partition F, so $a(F) = w(F;0) = w(F;1) + \Delta(F) < w(F;1)$ by (EC.30b). Let F deviate as the single coalition $\{F\}$. The residual locals F' admit only the partitions $\{F'\}$ and $\{\{r\},\{e\}\}$, and for both resulting partitions of $\mathcal{L}$ we have $1 \in T(\cdot,\bar{\mathrm{U}})$ and $0 \notin T(\cdot,\bar{\mathrm{U}})$: no coalition present satisfies $\Delta(S) > \eta|\mathcal{L}\setminus S|$, because $\Delta(F) < 0$, $\Delta_r < 0$, $\Delta(F') < \Delta_e < 2\eta < 3\eta$ and $\Delta_e < 2\eta < 4\eta$ by (EC.31), which gives $1 \in T$ by (EC.13); and the only coalitions with $\Delta > 0$ are $\{e\}$ and F', whose blocking capacity $\lfloor \Delta_e/\eta \rfloor \le 1$, respectively $\lfloor \Delta(F')/\eta \rfloor \le 1$, falls short of $|F| = 3$, so $0 \notin T$ by (EC.14). Hence $A(F';\{F\}) = \{1\}$ by (5), and the deviation secures $w(F;1) > a(F)$.

Every outcome with $d^\star\!=\!1$ is dominated. Let $(\pi,1,a)$ be such an outcome, so that $a(F) = w(F;1)$. We record what four deviating coalitions secure; each is contained in F and deviates as a single coalition.

(a) $\{g\}$ secures at least $w(\{g\};1)$, because $\Delta_g > 0$ gives $w(\{g\};0) > w(\{g\};1)$; hence $a_g \ge w(\{g\};1)$.

(b) $\{\ell,h\}$ secures $w(\{\ell,h\};1)$; hence $a_\ell + a_h \ge w(\{\ell,h\};1)$. Indeed, the residual locals $\{g\}\cup F'$ admit the partitions $\{\{g\},F'\}$ and $\{\{g\},\{r\},\{e\}\}$; in both, $1\in T$ (no coalition present blocks all other locals, as above, and $\Delta(\{\ell,h\})<0$), whereas $0\notin T$, because the coalitions with $\Delta>0$ are $\{g\}$ and either $\{e\}$ or F', with total blocking capacity at most $\lfloor \Delta_g/\eta\rfloor + \lfloor \Delta_e/\eta\rfloor \le 0+1$ by $\eta>\Delta_g$ and $\eta > \Delta_e/2$, short of $|\{\ell,h\}| = 2$. Hence $A(\mathcal{L}\setminus\{\ell,h\};\{\{\ell,h\}\}) = \{1\}$.

(c) $\{g,h\}$ secures $w(\{g,h\};0) = w(\{g,h\};1)+\Delta_g+\Delta_h$; hence $a_g + a_h \ge w(\{g,h\};1)+\Delta_g+\Delta_h$. Indeed, every partition containing $\{g,h\}$ has $1\notin T$ by (EC.13), because $\Delta_g+\Delta_h > 3\eta = \eta|\mathcal{L}\setminus\{g,h\}|$ by (EC.31), and has $0\in T$ by (EC.14), because $\lfloor(\Delta_g+\Delta_h)/\eta\rfloor \ge 3$ while every coalition with $\Delta\le 0$ is a singleton. Hence $A(\mathcal{L}\setminus\{g,h\};\{\{g,h\}\}) = \{0\}$.

(d) $\{\ell,g\}$ secures at least $w(\{\ell,g\};1)+\Delta_\ell+\Delta_g$, the smaller of $w(\{\ell,g\};0)$ and $w(\{\ell,g\};1)$ because $\Delta_\ell+\Delta_g<0$; hence $a_\ell + a_g \ge w(\{\ell,g\};1)+\Delta_\ell+\Delta_g$.

Combining, $a(F) = w(F;1)$ and (b) give $a_g \le w(F;1) - w(\{\ell,h\};1) = w(\{g\};1)$, so $a_g = w(\{g\};1)$ by (a). Then (c) gives $a_h \ge w(\{h\};1)+\Delta_g+\Delta_h$, and therefore

$$a_\ell \;=\; w(F;1) - a_g - a_h \;\le\; w(\{\ell\};1) - \big(\Delta_g+\Delta_h\big),$$

whereas (d) with $a_g = w(\{g\};1)$ gives $a_\ell \ge w(\{\ell\};1)+\Delta_\ell+\Delta_g$. These are incompatible, because $\Delta_\ell+\Delta_g > -(\Delta_g+\Delta_h)$ is exactly the condition $\Delta(F)+\Delta_g>0$ assumed in (EC.30b).

As every outcome is dominated, $C(\mathcal{L};\emptyset)=\emptyset$ for every η satisfying (EC.31). Finally, no part of $(\underline{\eta}_1,\overline{\eta}_1)$ escapes this conclusion: holding $\Delta_g,\Delta_r,\Delta_e$ fixed and increasing Δ_h (adjusting Δ_ℓ so that (EC.30b) continues to hold) leaves $\underline{\eta}_1 = \Delta_e/4$ and the left endpoint of (EC.31) unchanged while $\overline{\eta}_1 = (\Delta_g+\Delta_h)/3$ grows without bound, so that for every $\theta\in(0,1)$ the parameters can be chosen so that the blocking cost $\underline{\eta}_1 + \theta\,(\overline{\eta}_1 - \underline{\eta}_1)$ satisfies (EC.31). Multiplying every Δ_ℓ by a common factor $\lambda>0$ rescales all the thresholds and the range (EC.31) by λ. □

Full cooperation. Suppose $\bar\pi = \{\mathcal{L}\}$. Lemma EC.4 shows that the sign of $I(\mathcal{L}) - D(\mathcal{L})$ determines whether the core contains outcomes with $d^\star{=}0$, outcomes with $d^\star{=}1$, or both (Parts B and A-i), and Lemma EC.5 characterizes when the core outcomes with $d^\star{=}0$ compensate (Part A-i). Parts (i) and (ii) of the latter hold for any family partition in which every family prefers the incentive, and part (i) is also used in the nonemptiness claim of Part A and in Part A-ii.

Lemma EC.4. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3, and suppose $\bar\pi = \{\mathcal{L}\}$.*

(i) If $I(\mathcal{L}) > D(\mathcal{L})$, the core contains only outcomes with $d^\star{=}0$.

(ii) If $I(\mathcal{L}) = D(\mathcal{L})$, the core may contain both an outcome with $d^\star{=}1$ and an outcome with $d^\star{=}0$.

Proof of Lemma EC.4.

Item (i). Assume by contradiction that the core contains an outcome with $d^\star = 1$ ($d^\star = 1$). But then, the trivial deviation of $\mathcal{L}$ forming the grand coalition dominates this outcome:

$$a(\mathcal{L}) = v^{\text{sq}}_{\mathcal{L}} + D(\mathcal{L}) < w(\mathcal{L};0) = v^{\text{sq}}_{\mathcal{L}} + I(\mathcal{L}). \tag{EC.32}$$

Item (ii). To simplify notation, we define $\Delta_\ell := I_\ell - D_\ell, \forall \ell \in \mathcal{L}$. Consider the following instance with full cooperation:

$$\mathcal{L} = \{g, b\}, \qquad \bar{\pi} = \{\mathcal{L}\}, \qquad \Delta_g = \Delta > 0, \qquad \Delta_b = -\Delta,$$

so that $\Delta(\mathcal{L}) = I(\mathcal{L}) - D(\mathcal{L}) = 0$, and consider any blocking cost $\eta \le \Delta$.

Only two partitions refine $\bar{\pi}$: the grand coalition $\{\mathcal{L}\}$ and the singletons $\pi := \{\{g\}, \{b\}\}$. The grand coalition has no other local to block and, because $I(\mathcal{L}) = D(\mathcal{L})$, is indifferent between the incentive and deforestation; by the tie-breaking convention in §3.3 it deforests, so $T(\{\mathcal{L}\}) = \{1\}$. For the singletons, coalition $\{g\}$ can block the single local b at cost η and gains $\Delta_g - \eta = \Delta - \eta \ge 0$ from doing so; hence $\eta \le \eta_2(\pi)$ and Lemma 2 gives $0 \in T(\pi)$. Precisely, $\eta_1(\pi) = \eta_2(\pi) = \Delta$, so $T(\pi) = \{0\}$ if $\eta < \Delta$ and $T(\pi) = \{0, 1\}$ if $\eta = \Delta$.

We exhibit one outcome of each type, both with the allocation

$$a_\ell = v^{\text{sq}}_\ell + I_\ell, \qquad \ell \in \{g, b\},$$

whose total $v^{\text{sq}}_{\mathcal{L}} + I(\mathcal{L}) = v^{\text{sq}}_{\mathcal{L}} + D(\mathcal{L})$ equals both $w(\mathcal{L};0)$ and $w(\mathcal{L};1)$.

An outcome with $d^\star = 0$ in the core. Let $O_N = (\pi, d^\star = 0, a)$. The allocation is feasible because each singleton $\{\ell\}$ receives $w(\{\ell\};0) = v^{\text{sq}}_\ell + I_\ell$, and $0 \in T(\pi)$ as shown. No coalition deviates profitably: g already receives its maximal attainable net income $v^{\text{sq}}_g + I_g$; b, deviating alone, faces $A(\{g\};\{\{b\}\}) = T(\pi)$, which contains 0, under which it obtains $w(\{b\};0) = a_b$; and the grand coalition, upon reforming, obtains $w(\mathcal{L};1) = v^{\text{sq}}_{\mathcal{L}} + D(\mathcal{L}) = a_g + a_b$, so it cannot make both members strictly better off. Hence O_N is in the core.

An outcome with $d^\star = 1$ in the core. Let $O_D = (\{\mathcal{L}\}, d^\star = 1, a)$. The allocation is feasible because $a_g + a_b = v^{\text{sq}}_{\mathcal{L}} + I(\mathcal{L}) = w(\mathcal{L};1)$, and $1 \in T(\{\mathcal{L}\})$ as shown. Consider any deviation. If $\{g\}$ deviates, the residual faces $\{g\}$ with resulting partition π and $T(\pi)$ contains 0, so $0 \in A(\mathcal{L} \setminus \{g\}; \{\{g\}\})$ and under it g obtains $w(\{g\};0) = v^{\text{sq}}_g + I_g = a_g$. If $\{b\}$ deviates, the residual is again π; because $\{g\}$ can profitably block b ($\Delta_g \ge \eta$), Lemma 2 gives $0 \in T(\pi)$, so under that indicator b cannot use deforested land and obtains $w(\{b\};0) = v^{\text{sq}}_b + I_b = a_b$. The grand coalition, being the coalition of O_D, cannot exceed $w(\mathcal{L};1) = a_g + a_b$. Hence no coalition strictly improves and O_D is in the core.

Both outcomes lie in the core precisely when $\eta \le \Delta = \Delta(\{g\})/|\mathcal{L} \setminus \{g\}| = \overline{\eta}_1$: for $\eta > \Delta$ the threat to block is not credible, $T(\pi) = \{1\}$, and only outcomes with $d^\star = 1$ remain. In particular, at $\eta = \Delta = \overline{\eta}_1$ the instance lies in the range $\eta \notin (\eta_2, \overline{\eta}_1)$ (here $\eta_2 = -\infty$, as no family prefers the incentive) and its core contains an outcome with $d^\star = 0$. □

Lemma EC.5. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3.*

(i) If $I(F) > D(F)$ for every $F \in \bar{\pi}$ and $\eta \geq \overline{\eta}_1$, the core contains all outcomes with $d^\star = 0$ that compensate. In particular, it contains the outcome $(\bar{\pi}, 0, a)$, for every allocation a with $a(F) = w(F;0)$ for every $F \in \bar{\pi}$ and $a_\ell \geq v_\ell^{\mathrm{sq}} + D_\ell$ for every $\ell \in \mathcal{L}$.

(ii) If $\eta > \overline{\eta}$, every outcome with $d^\star = 0$ in the core compensate.

(iii) If $\bar{\pi} = \{\mathcal{L}\}$, $\mathcal{G} \neq \mathcal{L}$, $I(\mathcal{L}) > D(\mathcal{L})$, and $\eta < \underline{\eta}_1$, then no outcome with $d^\star = 0$ in the core compensates.

Proof of Lemma EC.5.

Item (i). Consider any outcome with $d^\star = 0$ $(\sigma, 0, a)$ that compensates, i.e., $\sigma \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$, $0 \in T(\sigma)$, $a(S) = w(S;0)$ for all $S \in \sigma$, and $a_\ell \geq v_\ell^{\mathrm{sq}} + D_\ell$ for all $\ell \in \mathcal{L}$. Since $(\sigma, 0, a)$ is an arbitrary outcome, showing it lies in the core proves that the core contains all outcomes with $d^\star = 0$ that compensate. We have assumed it is feasible, so it suffices to show it is not dominated. Recall that a deviating coalition S must satisfy $S \subseteq F$ for some $F \in \bar{\pi}$.

Consider first any deviating coalition with $S \subsetneq \mathcal{L}$, forming $\pi_S \in \Pi_S^{\prec\bar{\pi}}$. Because $\mathcal{L} \setminus S \neq \emptyset$, Lemma EC.2(b)-(i) implies $1 \in A(\mathcal{L} \setminus S; \pi_S)$. Hence, for every $S_i \in \pi_S$,

$$\min_{d' \in A(\mathcal{L} \setminus S; \pi_S)} w(S_i; d') \;\leq\; w(S_i; 1) \;=\; \sum_{\ell \in S_i} \left(v_\ell^{\mathrm{sq}} + D_\ell\right) \;\leq\; \sum_{\ell \in S_i} a_\ell \;=\; a(S_i),$$

where the last inequality uses that $(\sigma, 0, a)$ compensates. Thus S cannot derive strictly larger welfare under pessimism.

The only deviation with $S = \mathcal{L}$ is feasible only if $\bar{\pi} = \{\mathcal{L}\}$, in which case $I(\mathcal{L}) > D(\mathcal{L})$ by hypothesis. Then $\mathcal{L}$ forms some $\hat{\pi}_{\mathcal{L}} \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$. Aggregating $a(S) = w(S;0)$ over $S \in \sigma$ gives $a(\mathcal{L}) = v_{\mathcal{L}}^{\mathrm{sq}} + I(\mathcal{L})$, and $\sum_{S_i \in \hat{\pi}_{\mathcal{L}}} w(S_i; d') \leq v_{\mathcal{L}}^{\mathrm{sq}} + I(\mathcal{L}) = a(\mathcal{L})$ for both $d' \in \{0,1\}$; hence $\mathcal{L}$'s secured total is at most $a(\mathcal{L})$ and the deviation cannot leave every coalition S_i strictly better off. Therefore, $(\sigma, 0, a)$ is not dominated and belongs to the core.

Finally, consider any allocation a with $a(F) = w(F;0)$ for every $F \in \bar{\pi}$ and $a_\ell \geq v_\ell^{\mathrm{sq}} + D_\ell$ for every $\ell \in \mathcal{L}$. Such allocations exist because $w(F;0) = \sum_{\ell \in F}(v_\ell^{\mathrm{sq}} + I_\ell) > \sum_{\ell \in F}(v_\ell^{\mathrm{sq}} + D_\ell)$ for every $F \in \bar{\pi}$; for instance, $a_\ell = v_\ell^{\mathrm{sq}} + D_\ell + (I(F) - D(F))/|F|$ for every $\ell \in F$ and $F \in \bar{\pi}$. Moreover, $\bar{\pi} \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$, and $0 \in T(\bar{\pi}, \bar{\mathbb{U}})$ by (EC.14) because every coalition $F \in \bar{\pi}$ satisfies $I(F) > D(F)$ (so that $\eta_2(\bar{\pi}) = +\infty$). Hence $(\bar{\pi}, 0, a)$ is an outcome with $d^\star = 0$ that compensates, and by the argument above it is in the core.

Item (ii).

When $\mathcal{L} = \mathcal{G}$, Lemma EC.1 implies that every outcome in the core must compensate all locals, and so the results hold trivially. When $\mathcal{L} \neq \mathcal{G}$, assume by contradiction that there is an outcome with $d^\star = 0$ in the core with $a_\ell < v_\ell^{\mathrm{sq}} + D_\ell$, for some ℓ. By Lemma EC.1, it must be that $\ell \in \mathcal{L} \setminus \mathcal{G}$. Then,

ℓ can deviate towards a deforestation equilibrium all by himself. This is because, for any feasible partition of the remaining locals $\pi_{\mathcal{L}\setminus\{\ell\}}$,

$$\eta_2(\pi_{\mathcal{L}\setminus\{\ell\}} \cup \{\{\ell\}\}) \leq \overline{\eta} < \eta,$$

where the first inequality holds because every coalition S of such a partition lies within a family, so $I(S) - D(S) \leq \overline{\eta}$ by (6): for $\eta > \overline{\eta}$, every floor term in the definition of η_2 in (EC.14) vanishes, while the coalition $\{\ell\}$ satisfies $I(\{\ell\}) \leq D(\{\ell\})$; by Lemma 2, this implies that $\{1\} = T(\pi_{\mathcal{L}\setminus\{\ell\}} \cup \{\{\ell\}\})$ and thus $\{1\} = A(\mathcal{L} \setminus \{\ell\}; \{\{\ell\}\})$ and

$$w(\{\ell\}; d^\star) > a_\ell, \text{ for every } d^\star \in A(\mathcal{L} \setminus \{\ell\}; \{\{\ell\}\}).$$

So $\{\ell\}$ can profitably deviate, contradicting the assumption that the outcome was in the core.

Item (iii). Write $\Delta_\ell := I_\ell - D_\ell$ and $\Delta(S) := \sum_{\ell \in S} \Delta_\ell$. Because $\bar{\pi} = \{\mathcal{L}\}$ and $I(\mathcal{L}) > D(\mathcal{L})$, the definition of $\underline{\eta}_1$ in (9) reads $\underline{\eta}_1 = \max_{S,\eta}\{\eta : \Delta(S) \geq \eta\,|\mathcal{L} \setminus S|,\ \emptyset \neq S \subsetneq \mathcal{L}\}$; let S^* be a coalition attaining the maximum, so that $\Delta(S^*) = \underline{\eta}_1\,|\mathcal{L} \setminus S^*|$. Because $\mathcal{G} \neq \emptyset$, any singleton $\{g\}$ with $g \in \mathcal{G}$ is feasible with $\eta = \Delta_g/(|\mathcal{L}| - 1) > 0$, so $\underline{\eta}_1 > 0$ and $\Delta(S^*) > 0$. We claim that $\Delta(\mathcal{L} \setminus S^*) < 0$. Otherwise $\mathcal{L} \setminus S^*$ contains a local p with $\Delta_p > 0$; if $\mathcal{L} \setminus S^* \neq \{p\}$, the coalition $S^* \cup \{p\} \neq \mathcal{L}$ has ratio $\frac{\Delta(S^*)+\Delta_p}{|\mathcal{L}\setminus S^*|-1} > \frac{\Delta(S^*)}{|\mathcal{L}\setminus S^*|}$, and if $\mathcal{L} \setminus S^* = \{p\}$, then S^* contains a local ℓ with $\Delta_\ell < 0$ (because $\mathcal{G} \neq \mathcal{L}$) and the coalition $\mathcal{L} \setminus \{\ell\}$ has ratio $\Delta(\mathcal{L}) - \Delta_\ell > \Delta(\mathcal{L}) - \Delta_p = \Delta(S^*)$; either way a feasible value $\eta > \underline{\eta}_1$ would exist in (9), contradicting the definition of $\underline{\eta}_1$.

Now assume by contradiction that the core contains an outcome with $d^\star$=0 $(\pi, d^\star = 0, a)$ that compensates. Because every coalition of π lies within the single family $\mathcal{L}$, $a(\mathcal{L}) = w(\mathcal{L}; 0)$, and because the outcome compensates,

$$a(S^*) = w(\mathcal{L}; 0) - a(\mathcal{L} \setminus S^*) \leq w(\mathcal{L}; 0) - \sum_{\ell \in \mathcal{L}\setminus S^*} \left(v_\ell^{\mathrm{sq}} + D_\ell\right) = w(S^*; 0) + \Delta(\mathcal{L} \setminus S^*) < w(S^*; 0).$$

Moreover, for every partition $\pi_{\mathcal{L}\setminus S^*} \in \Pi^{\prec\bar{\pi}}_{\mathcal{L}\setminus S^*}$, the coalition S^* satisfies $\Delta(S^*) = \underline{\eta}_1|\mathcal{L} \setminus S^*| > \eta|\mathcal{L} \setminus S^*|$, so $1 \notin T(\{S^*\} \cup \pi_{\mathcal{L}\setminus S^*}, \bar{\mathbb{U}})$ by (EC.13), and Lemma 2 gives $T(\{S^*\} \cup \pi_{\mathcal{L}\setminus S^*}, \bar{\mathbb{U}}) = \{0\}$; hence $A(\mathcal{L} \setminus S^*; \{S^*\}) = \{0\}$ by (5). Therefore $a(S^*) < w(S^*; d^\star)$ for every $d^\star \in A(\mathcal{L} \setminus S^*; \{S^*\})$, so the coalition S^* forming $\{S^*\}$ dominates $(\pi, 0, a)$, a contradiction. □

Limited cooperation with a family that prefers the incentive. Suppose $\bar{\pi} \neq \{\mathcal{L}\}$ and $I(F) > D(F)$ for some $F \in \bar{\pi}$. Lemma EC.6 shows that the core contains only outcomes with $d^\star$=0 if and only if $\eta < \underline{\eta}_1$ (Parts B and A-ii). The three instances that follow show that the remaining claims of Part A-ii cannot be strengthened: for every $\eta > \max\{\eta_2, \underline{\eta}_1\}$ the core may be nonempty and contain only outcomes with $d^\star$=1 (Lemma EC.7); even when every family prefers the incentive, for every $\eta < \overline{\eta}_1$ the core may contain no outcome with $d^\star$=0 that compensates (Lemma EC.8); and when one family is exactly indifferent, the core may contain no outcome with $d^\star\!=\!0$ that compensates at any blocking cost (Lemma EC.9). The last three results establish the final claim of Part A-ii: below the threshold η_1^{comp}, no core outcome with $d^\star\!=\!0$ compensates (Lemma EC.11), and at or above it, whenever the core contains a compensating outcome with $d^\star\!=\!0$, it also contains one that does not compensate (Lemma EC.12, which relies on Lemma EC.10).

LEMMA EC.6. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3, and suppose $\bar{\pi} \neq \{\mathcal{L}\}$ and $I(F) > D(F)$ for some $F \in \bar{\pi}$.*

(i) If $\eta < \underline{\eta}_1$, the core contains only outcomes with $d^\star\!=\!0$.

(ii) If $\eta \geq \underline{\eta}_1$, then either the core is empty or it contains outcomes with $d^\star\!=\!1$.

Proof of Lemma EC.6. To facilitate following the proof, recall the definition of $\underline{\eta}_1$ in (9):

$$\underline{\eta}_1 := \max_{F,S,\eta}\left\{\eta : I(S) - D(S) \geq \eta|\mathcal{L} \setminus S|,\, F \in \bar{\pi},\, S \subseteq F,\, S \neq \mathcal{L},\, I(F) > D(F)\right\}.$$

Item (i). We show that every outcome with $d^\star = 1$ is dominated. The assumption on the value of η implies that there exists a coalition $S \subseteq F$ such that

$$I(S) - D(S) > \eta|\mathcal{L} \setminus S| \geq 0. \tag{EC.33}$$

Moreover, because including in S locals from $\mathcal{G} \setminus S$ only expands $I(S) - D(S)$ and reduces $|\mathcal{L} \setminus S|$ in (EC.33), we consider S that also satisfies $\mathcal{G} \cap F \subseteq S$, or equivalently, $F \setminus S \subseteq \mathcal{L} \setminus \mathcal{G}$.

Consider any outcome with $d^\star = 1$. We show that coalition S satisfying (EC.33) and $\mathcal{G} \cap F \subseteq S$ can profitably deviate towards an outcome with $d^\star = 0$. Because $I(F) > D(F)$ and $d^\star = 1$, we have:

$$\sum_{\ell \in S} a_\ell + \sum_{\ell \in F \setminus S} a_\ell = \sum_{\ell \in F}\left(v_\ell^{\mathrm{sq}} + D_\ell\right) < \sum_{\ell \in F}(v_\ell^{\mathrm{sq}} + I_\ell) = \sum_{\ell \in S}(v_\ell^{\mathrm{sq}} + I_\ell) + \sum_{\ell \in F \setminus S}(v_\ell^{\mathrm{sq}} + I_\ell).$$

But note that $F \setminus S \notin \mathcal{G}$ and Lemma EC.1 imply that $\sum_{\ell \in F \setminus S} a_\ell \geq \sum_{\ell \in F \setminus S}(v_\ell^{\mathrm{sq}} + I_\ell)$. Therefore, $\sum_{\ell \in S} a_\ell < \sum_{\ell \in S}(v_\ell^{\mathrm{sq}} + I_\ell)$. But then, consider any partition $\pi_{\mathcal{L}\setminus S} \in \Pi_{\mathcal{L}\setminus S}^{\prec\bar{\pi}}$. Condition (EC.33) implies that $\eta < \eta_1(\pi_{\mathcal{L}\setminus S} \cup \{S\})$, which by Lemma 2 implies that $\{0\} = T(\pi_{\mathcal{L}\setminus S} \cup \{S\})$. Because this holds for every partition $\pi_{\mathcal{L}\setminus S}$, then $\{0\} = A(\mathcal{L} \setminus S; \{S\})$, which implies that $\sum_{\ell \in S} a_\ell < w(S; d^\star)$ for every $d^\star \in A(\mathcal{L} \setminus S; \{S\})$, proving that the outcome with $d^\star = 1$ is dominated by coalition S deviating and guaranteeing higher welfare with an outcome with $d^\star\!=\!0$.

Item (ii).

For this proof, we define and use $\Delta_\ell := I_\ell - D_\ell, \forall \ell \in \mathcal{L}$, which substantially simplifies notation. For the proof, it helps to recall the definition of $\underline{\eta}_1$ from (9), in the Δ notation:

$$\underline{\eta}_1 := \max_{F,S,\eta}\left\{\eta : \Delta(S) \geq \eta\,|\mathcal{L} \setminus S|,\, F \in \bar{\pi},\, S \subseteq F,\, S \neq \mathcal{L},\, \Delta(F) > 0\right\}.$$

We prove the following more general claim by induction, which implies the result by taking $R = \mathcal{L}$:

$$\begin{gathered} 0 \in A(R;\pi_{\mathcal{L}\setminus R}) \quad \Rightarrow \quad 1 \in A(R;\pi_{\mathcal{L}\setminus R}), \qquad \forall R \subseteq \mathcal{L}, \quad \forall \pi_{\mathcal{L}\setminus R} \in \Sigma, \quad \text{where} \\ \Sigma := \Big\{\sigma \in \Pi_{\mathcal{L}\setminus R} : S \in \sigma \Rightarrow \exists F \in \bar{\pi} \text{ with } \Delta(F) > 0 \text{ and } S \subseteq F\Big\}. \end{gathered} \tag{EC.34}$$

Thus, every coalition in the fixed partition belongs to a family that prefers the incentive.

We first prove the claim for $|R| = 1$. Let $R = \{\ell\}$. If the family containing ℓ prefers the incentive, then $\Delta(R)/|\mathcal{L} \setminus R| \le \underline{\eta}_1$. Otherwise, the definition of Σ requires that this entire family be contained in R, so $\Delta(R) \le 0$. Every coalition in $\pi_{\mathcal{L}\setminus R}$ belongs to a family that prefers the incentive and therefore also satisfies $\Delta(S)/|\mathcal{L} \setminus S| \le \underline{\eta}_1$ whenever $\Delta(S) > 0$. Hence

$$\eta_1(\{R\} \cup \pi_{\mathcal{L}\setminus R}) \le \underline{\eta}_1 \le \eta.$$

By Lemma 2, $1 \in T(\{R\} \cup \pi_{\mathcal{L}\setminus R})$. Because the residual set is a singleton, $A(R;\pi_{\mathcal{L}\setminus R}) = T(\{R\} \cup \pi_{\mathcal{L}\setminus R})$, so $1 \in A(R;\pi_{\mathcal{L}\setminus R})$.

Suppose now that the claim holds for every smaller residual set, and consider $|R| > 1$. Assume that $(\pi'_R, 0, a') \in C(R;\pi_{\mathcal{L}\setminus R})$. Define

$$\pi_R := \{H : \exists F \in \bar{\pi} \text{ with } H = R \cap F\}.$$

For every family F with $\Delta(F) > 0$, set

$$a_\ell := v_\ell^{\text{sq}} + D_\ell, \qquad \ell \in R \cap F.$$

If $\Delta(F) \le 0$, the definition of Σ implies $F \subseteq R$. Since $w(F;1) \ge w(F;0) = a'(F)$, allocate $w(F;1)$ among the members of F so that

$$a_\ell \ge a'_\ell, \qquad \ell \in F. \tag{EC.35}$$

This defines a feasible allocation for $(\pi_R, 1, a)$. Moreover, every coalition in $\pi_R \cup \pi_{\mathcal{L}\setminus R}$ with positive Δ belongs to a family that prefers the incentive. Therefore,

$$\eta_1(\pi_R \cup \pi_{\mathcal{L}\setminus R}) \le \underline{\eta}_1 \le \eta,$$

and Lemma 2 gives $1 \in T(\pi_R \cup \pi_{\mathcal{L}\setminus R})$. Thus $(\pi_R, 1, a)$ is a valid residual outcome.

It remains to prove that the outcome is not dominated. Let $H \subseteq R$ form a partition $\pi_H \in \Pi_H^{\prec\bar{\pi}}$, and let $F \in \bar{\pi}$ contain H.

If $\Delta(F) \le 0$, then $a_\ell \ge a'_\ell$ for every $\ell \in F$. Hence, if (H, π_H) dominated $(\pi_R, 1, a)$, then for every $H_i \in \pi_H$,

$$a'(H_i) \le a(H_i) < \min_{d^\star \in A(R\setminus H;\pi_H \cup \pi_{\mathcal{L}\setminus R})} w(H_i; d^\star),$$

so the same deviation would dominate $(\pi'_R, 0, a')$, a contradiction.

Suppose instead that $\Delta(F) > 0$. If $H = R$, then every coalition in $\pi_H \cup \pi_{\mathcal{L}\setminus R}$ with positive Δ belongs to a family that prefers the incentive, so $\eta_1(\pi_H \cup \pi_{\mathcal{L}\setminus R}) \le \underline{\eta}_1 \le \eta$. Hence $1 \in A(\emptyset; \pi_H \cup \pi_{\mathcal{L}\setminus R}) = T(\pi_H \cup \pi_{\mathcal{L}\setminus R})$ by Lemma 2. Suppose now that $H \subsetneq R$. If the residual core $C(R \setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R})$ is nonempty, then either its assumption set already excludes 0, in which case it contains 1, or it contains 0, in which case the inductive hypothesis gives $1 \in A(R \setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R})$. If that residual core is empty, consider

$$\widetilde{\pi} := \{S : \exists F' \in \bar{\pi} \text{ with } S = (R \setminus H) \cap F'\} \cup \pi_H \cup \pi_{\mathcal{L}\setminus R}.$$

Every coalition in $\widetilde{\pi}$ with positive Δ belongs to a family that prefers the incentive, so $\eta_1(\widetilde{\pi}) \le \underline{\eta}_1 \le \eta$. Hence $1 \in T(\widetilde{\pi})$ by Lemma 2, and the empty-core definition of the assumption set gives $1 \in A(R \setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R})$.

Thus, in every case involving a deviation from a family that prefers the incentive, $1 \in A(R \setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R})$. For every $H_i \in \pi_H$,

$$\min_{d^\star \in A(R\setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R})} w(H_i; d^\star) \le w(H_i; 1) = a(H_i),$$

so (H, π_H) is not profitable. Therefore $(\pi_R, 1, a)$ belongs to the residual core, completing the induction. Taking $R = \mathcal{L}$ proves that whenever the core is nonempty it contains an outcome with $d^\star = 1$. □

Lemma EC.7. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3. There exist instances with $\bar{\pi} \ne \{\mathcal{L}\}$ and $I(F) > D(F)$ for some $F \in \bar{\pi}$ for which, for every $\eta > \max\{\eta_2, \underline{\eta}_1\}$, the core is non-empty and contains only outcomes with $d^\star = 1$.*

Proof of Lemma EC.7. For this proof, we use $\Delta_\ell := I_\ell - D_\ell, \forall \ell \in \mathcal{L}$, which simplifies notation. Consider the instance:

$$\mathcal{L} = \{g, h, \ell\}, \qquad \bar{\pi} = \{F_1, F_2\}, \qquad F_1 := \{g\}, \quad F_2 := \{h, \ell\},$$
$$\Delta_g > 0, \qquad \Delta_h > 0, \qquad \Delta_\ell < -\Delta_h, \qquad \text{so } \Delta(F_1) = \Delta_g > 0 \text{ and } \Delta(F_2) = \Delta_h + \Delta_\ell < 0$$

(for example, $\Delta_g = 1$, $\Delta_h = 2$, $\Delta_\ell = -7$). The only family with $\Delta(F) > 0$ is F_1, whose only nonempty coalition is $\{g\}$, so $\underline{\eta}_1 = \Delta_g / |\mathcal{L} \setminus \{g\}| = \Delta_g/2$. For η_2, the coalitions H within a family with $\Delta(H) \le 0$ are $\{\ell\}$, whose condition $\lfloor \Delta_g/\eta \rfloor + \lfloor \Delta_h/\eta \rfloor \ge 1$ holds if and only if $\eta \le \max\{\Delta_g, \Delta_h\}$, and F_2, whose condition $\lfloor \Delta_g/\eta \rfloor \ge 2$ holds if and only if $\eta \le \Delta_g/2$; hence $\eta_2 = \Delta_g/2$ and $\max\{\eta_2, \underline{\eta}_1\} = \Delta_g/2$. Fix any $\eta > \Delta_g/2$.

The only two partitions in $\Pi_{\mathcal{L}}^{\prec \bar{\pi}}$ are $\pi^* := \{\{g\}, F_2\}$ and $\pi^0 := \{\{g\}, \{h\}, \{\ell\}\}$, so the partition $\pi_R \cup \pi_{\mathcal{L}\setminus R}$ of every residual game equals π^* or π^0. By Lemma 2:

$$T(\pi^*) = \{1\} \text{ for every } \eta > \Delta_g/2, \tag{EC.36}$$

$$T(\pi^0) = \begin{cases} \{0\} & \text{if } \Delta_g/2 < \eta < \max\{\Delta_g, \Delta_h\}/2 \\ \{0,1\} & \text{if } \max\{\Delta_g, \Delta_h\}/2 \le \eta \le \max\{\Delta_g, \Delta_h\} \\ \{1\} & \text{if } \eta > \max\{\Delta_g, \Delta_h\}. \end{cases} \tag{EC.37}$$

Indeed, $0 \in T(\pi^*)$ requires $\lfloor \Delta_g/\eta \rfloor \geq |F_2| = 2$, i.e., $\eta \leq \Delta_g/2$, whereas $1 \in T(\pi^*)$ holds because $\Delta_g \leq 2\eta$ and $\Delta(F_2) < 0$; similarly, $0 \in T(\pi^0)$ if and only if $\lfloor \Delta_g/\eta \rfloor + \lfloor \Delta_h/\eta \rfloor \geq |\{\ell\}| = 1$, i.e., $\eta \leq \max\{\Delta_g, \Delta_h\}$, and $1 \in T(\pi^0)$ if and only if both $\Delta_g \leq 2\eta$ and $\Delta_h \leq 2\eta$, i.e., $\eta \geq \max\{\Delta_g, \Delta_h\}/2$.

In particular, both sets in (EC.36)–(EC.37) are non-empty. We claim that

$$A\big(R; \pi_{\mathcal{L}\setminus R}\big) = T\big(\{\{i\} : i \in R\} \cup \pi_{\mathcal{L}\setminus R}\big) \qquad \text{for every } R \subsetneq \mathcal{L} \text{ with } R \neq F_2 \text{ and every } \pi_{\mathcal{L}\setminus R} \in \Pi^{\prec \bar{\pi}}_{\mathcal{L}\setminus R}. \tag{EC.38}$$

For $R = \emptyset$, (EC.38) holds by (5), and for $|R| = 1$ it holds because, by Definition 2, the core of a one-local residual game consists of *all* of its outcomes. The remaining sets are $R \in \{\{g,h\}, \{g,\ell\}\}$, whose two locals belong to different families: the only residual partition is $\{\{i\} : i \in R\}$, the resulting partition of $\mathcal{L}$ is π^0, and every residual outcome $\big(\{\{i\} : i \in R\}, d^\star, a\big)$, $d^\star \in T(\pi^0)$, $a_i = w(\{i\}; d^\star)$, is not dominated: a deviation by a singleton $\{i\} \subseteq R$ faces $A\big(R \setminus \{i\}; \{\{i\}\} \cup \pi_{\mathcal{L}\setminus R}\big) = T(\pi^0)$ by the preceding cases, and $\min_{d' \in T(\pi^0)} w(\{i\}; d') \leq w(\{i\}; d^\star) = a_i$ because $d^\star \in T(\pi^0)$. Hence $C(R; \pi_{\mathcal{L}\setminus R}) \neq \emptyset$ and (EC.38) holds.

Step 1: no outcome with $d^\star\!=\!0$ is in the core, for any $\eta > \Delta_g/2$. Consider any outcome with $d^\star\!=\!0$ $(\pi, d^\star = 0, a)$. Because the coalitions containing F_2's members partition F_2 and $d^\star = 0$, feasibility implies $a(F_2) = v^{\text{sq}}_{F_2} + I(F_2)$. Let coalition F_2 deviate forming the partition $\{F_2\}$: by (EC.38) and (EC.36), $A(\{g\}; \{F_2\}) = T(\pi^*) = \{1\}$, so the deviation secures $w(F_2; 1) = v^{\text{sq}}_{F_2} + D(F_2) > v^{\text{sq}}_{F_2} + I(F_2) = a(F_2)$, where the inequality is $\Delta(F_2) < 0$. Every outcome with $d^\star\!=\!0$ is therefore dominated.

Step 2: the core contains an outcome with $d^\star\!=\!1$, for any $\eta > \Delta_g/2$. We distinguish two cases. *Case* $\Delta_g/2 < \eta \leq \max\{\Delta_g, \Delta_h\}$. We show that the core contains the outcome with $d^\star\!=\!1$ $(\pi^*, d^\star = 1, a)$ with

$$a_g = v^{\text{sq}}_g + D_g, \qquad a_h = v^{\text{sq}}_h + I_h, \qquad a_\ell = v^{\text{sq}}_\ell + D_\ell - \Delta_h.$$

The allocation is feasible because $a_h + a_\ell = v^{\text{sq}}_{F_2} + D(F_2) = w(F_2; 1)$, and $1 \in T(\pi^*)$ by (EC.36). By Definition 2, a deviating group lies within one family, so $H \in \big\{\{g\}, \{h\}, \{\ell\}, F_2\big\}$. No profitable deviation (H, π_H) uses $\{h\}$ or F_2 as a coalition of π_H: $a_h = \max\{w(\{h\}; 0), w(\{h\}; 1)\}$ because $\Delta_h > 0$, and $a(F_2) = \max\{w(F_2; 0), w(F_2; 1)\}$ because $\Delta(F_2) < 0$, so neither ever strictly improves. This rules out $H = \{h\}$ and $H = F_2$, leaving $H = \{\ell\}$ and $H = \{g\}$.

If $H = \{\ell\}$: by (EC.38), $A(\{g,h\}; \{\{\ell\}\}) = T(\pi^0)$, and $0 \in T(\pi^0)$ because $\eta \leq \max\{\Delta_g, \Delta_h\}$, per (EC.37); so $\{\ell\}$ secures at most $w(\{\ell\}; 0) = a_\ell + \Delta(F_2) < a_\ell$ and does not strictly improve.

If $H = \{g\}$: we show $1 \in A(F_2; \{\{g\}\})$, so that $\{g\}$ secures at most $w(\{g\}; 1) = a_g$ and does not strictly improve. The residual outcome $\big(\{F_2\}, d^\star = 1, (a_h, a_\ell)\big)$ is feasible, as above. Within F_2, the deviating groups are $\{h\}$, $\{\ell\}$, and F_2: deviations using $\{h\}$ or F_2 are unprofitable exactly as above, and $\{\ell\}$ secures at most $w(\{\ell\}; 0) < a_\ell$ because $A(\{h\}; \{\{g\}, \{\ell\}\}) = T(\pi^0)$ by (EC.38), with $0 \in T(\pi^0)$. Hence $\big(\{F_2\}, 1, (a_h, a_\ell)\big) \in C(F_2; \{\{g\}\})$ and $1 \in A(F_2; \{\{g\}\})$.

Case $\eta > \max\{\Delta_g, \Delta_h\}$. Now $\lfloor \Delta(S)/\eta \rfloor = 0$ for every coalition S with $\Delta(S) > 0$, and both π^* and π^0 contain a coalition with $\Delta(\cdot) \leq 0$ (F_2, respectively $\{\ell\}$), so $0 \notin T(\pi^*) \cup T(\pi^0)$; moreover $\Delta(S) \leq \max\{\Delta_g, \Delta_h\} < \eta \leq \eta\,|\mathcal{L} \setminus S|$ for every coalition S, so Lemma 2 implies $T(\pi^*) = T(\pi^0) = \{1\}$. Hence $A(R; \pi_{\mathcal{L}\setminus R}) = \{1\}$ for every residual game: for $R \neq F_2$ this is (EC.38), and for $R = F_2$ every outcome of the residual game has its indicator in $T(\pi^*) \cup T(\pi^0) = \{1\}$, so (5) gives $\emptyset \neq A(F_2; \{\{g\}\}) \subseteq \{1\}$. Consider the outcome with $d^\star{=}1$ $(\pi^*, d^\star = 1, a)$ with $a_i = v_i^{\text{sq}} + D_i$ for every $i \in \mathcal{L}$: in any deviation (H, π_H), every coalition $S \in \pi_H$ secures exactly $w(S; 1) = v_S^{\text{sq}} + D(S) = a(S)$, so no coalition strictly improves and the outcome is in the core.

Combining Steps 1 and 2: for every $\eta > \Delta_g/2 = \max\{\eta_2, \underline{\eta}_1\}$, the core is non-empty and contains only outcomes with $d^\star{=}1$. □

Lemma EC.8. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3. There exist instances with $\bar\pi \neq \{\mathcal{L}\}$, $\mathcal{G} \neq \mathcal{L}$, and $I(F) > D(F)$ for every $F \in \bar\pi$, in which, at every blocking cost $\eta < \overline{\eta}_1$, the core is non-empty yet contains no outcome with $d^\star{=}0$ that compensates.*

Proof of Lemma EC.8. To simplify notation, we define $\Delta_\ell := I_\ell - D_\ell$, $\forall \ell \in \mathcal{L}$. Consider the instance

$$\mathcal{L} = \{e, g, h, r\}, \qquad \bar\pi = \{\{e, g\}, \{h, r\}\},$$

$$\Delta_e > 0 > \Delta_g, \qquad \Delta_h > 0 > \Delta_r, \qquad \Delta(\{e, g\}) > 0, \qquad \Delta(\{h, r\}) > 0,$$
$$\frac{\Delta(\{h, r\})}{2} \;\geq\; \max\left\{ \frac{\Delta_h}{3}, \frac{\Delta_e}{3}, \frac{\Delta(\{e, g\})}{2} \right\},$$

for example $(\Delta_e, \Delta_g, \Delta_h, \Delta_r) = (2, -1, 5, -1)$. Then $\bar\pi \neq \{\mathcal{L}\}$, $\mathcal{G} \neq \mathcal{L}$ because $\Delta_g < 0$, and both families strictly prefer the incentive, so the hypotheses hold. By (8), the candidate values of $\overline{\eta}_1$ are $\Delta(\{h, r\})/2$, $\Delta_h/3$, $\Delta_e/3$, and $\Delta(\{e, g\})/2$ (every other coalition S within a family has $\Delta(S) < 0$), so the displayed condition gives $\overline{\eta}_1 = \Delta(\{h, r\})/2$. Fix any blocking cost $\eta < \overline{\eta}_1 = \Delta(\{h, r\})/2$.

The core contains no outcome with $d^\star{=}0$ that compensates. Suppose, for the purpose of deriving a contradiction, that the core contains an outcome with $d^\star{=}0$ $(\pi, 0, a)$ that compensates. Because $a(S) = w(S; 0) = \sum_{\ell \in S} (v_\ell^{\text{sq}} + I_\ell)$ for every $S \in \pi$ and compensation requires $a_\ell \geq v_\ell^{\text{sq}} + D_\ell$ for every $\ell \in \mathcal{L}$, every coalition $S \in \pi$ must satisfy $\Delta(S) \geq 0$. Since $\Delta_g < 0$ and $\Delta_r < 0$, the only partitions of $\{e, g\}$ and $\{h, r\}$ into coalitions with $\Delta(S) \geq 0$ are the families themselves, so $\pi = \bar\pi$. Moreover,

$$a_e \;=\; a(\{e, g\}) - a_g \;\leq\; v_e^{\text{sq}} + I_e + \Delta_g \;<\; v_e^{\text{sq}} + I_e,$$

where the first inequality uses $a(\{e, g\}) = w(\{e, g\}; 0)$ and $a_g \geq v_g^{\text{sq}} + D_g$, and the second uses $\Delta_g < 0$.

We prove in three steps that every equilibrium indicator in $A(\{g,h,r\};\{\{e\}\})$ equals 0, which yields a contradiction: the deviation $H=\{e\}$ with $\pi_H=\{\{e\}\}$ then satisfies (4), because

$$w(\{e\};d^\star)=w(\{e\};0)=v_e^{\mathrm{sq}}+I_e>a_e \qquad \forall\, d^\star\in A(\{g,h,r\};\{\{e\}\}),$$

so $(\bar\pi,0,a)$ is dominated.

Step 1: at $\eta<\Delta(\{h,r\})/2$, $0\in T(\pi,\bar{\mathbb{U}})$ for every partition $\pi\in\Pi_{\mathcal{L}}^{\prec\bar\pi}$. In any partition π, the coalition S containing local h (either $\{h\}$ or $\{h,r\}$) satisfies $\lfloor\Delta(S)/\eta\rfloor\ge 2$, because $\min\{\Delta_h,\Delta(\{h,r\})\}=\Delta(\{h,r\})>2\eta$; whereas every coalition $S\in\pi$ with $\Delta(S)\le 0$ is a singleton (namely $\{g\}$ or $\{r\}$, because $\Delta(\{e,g\})>0$ and $\Delta(\{h,r\})>0$). The condition in (EC.14) therefore holds for π.

Step 2: at $\eta<\Delta(\{h,r\})/2$, for every non-empty $Z\subseteq\mathcal{L}$ and every $\pi_{\mathcal{L}\setminus Z}\in\Pi_{\mathcal{L}\setminus Z}^{\prec\bar\pi}$, the outcome with $d^\star{=}0$ $\big(\{\{\ell\}:\ell\in Z\},\,0,\,\tilde a\big)$ with $\tilde a_\ell=v_\ell^{\mathrm{sq}}+I_\ell$, $\forall\ell\in Z$, is in $C(Z;\pi_{\mathcal{L}\setminus Z})$; in particular, $0\in A(Z;\pi_{\mathcal{L}\setminus Z})$. By Step 1, this is an outcome of the residual game. We prove by strong induction on $|Z|$ that it is not dominated. For $|Z|=1$ there is nothing to prove, because by Definition 2 the core then consists of all outcomes. Let $|Z|\ge 2$ and consider any deviating group $H\subseteq F\cap Z$ for some $F\in\bar\pi$, with partition $\pi_H\in\Pi_H^{\prec\bar\pi}$. If $H=Z$, then $A(\emptyset;\pi_H\cup\pi_{\mathcal{L}\setminus Z})=T(\pi_H\cup\pi_{\mathcal{L}\setminus Z})$, which contains 0 by Step 1; if $H\subsetneq Z$, the induction hypothesis applied to $Z\setminus H$ gives $0\in A(Z\setminus H;\pi_H\cup\pi_{\mathcal{L}\setminus Z})$. In either case, every coalition $S\in\pi_H$ satisfies

$$\min_{d^\star\in A(Z\setminus H;\,\pi_H\cup\pi_{\mathcal{L}\setminus Z})} w(S;d^\star)\;\le\; w(S;0)\;=\;\sum_{\ell\in S}\big(v_\ell^{\mathrm{sq}}+I_\ell\big)\;=\;\tilde a(S),$$

so condition (4) fails and the outcome is not dominated.

Step 3: at $\eta<\Delta(\{h,r\})/2$, the residual core $C(\{g,h,r\};\{\{e\}\})$ contains no outcome with $d^\star{=}1$. An outcome with $d^\star{=}1$ of this residual game requires $1\in T(\pi_R\cup\{\{e\}\},\bar{\mathbb{U}})$ for some $\pi_R\in\Pi_{\{g,h,r\}}^{\prec\bar\pi}$. Because $\Delta(\{h,r\})>2\eta$, the family $\{h,r\}$ cannot form a single coalition in such a partition by (EC.13), so $\pi_R=\{\{g\},\{h\},\{r\}\}$, for which (EC.13) gives $1\in T$ if and only if $\eta\ge\max\{\Delta_e/3,\,\Delta_h/3\}$. Consider such an outcome $(\pi_R,1,\bar a)$, which has $\bar a_\ell=v_\ell^{\mathrm{sq}}+D_\ell$ for every $\ell\in\{g,h,r\}$, and the deviation $H'=\{h,r\}$ with $\pi_{H'}=\{\{h,r\}\}$. The only partition of the remaining local g joined with $\{\{e\},\{h,r\}\}$ is $\{\{e\},\{g\},\{h,r\}\}$, which contains the coalition $\{h,r\}$ with $\Delta(\{h,r\})>2\eta$, so (EC.13) gives $1\notin T$, and by (5) every equilibrium indicator in $A(\{g\};\{\{e\},\{h,r\}\})$ equals 0. Hence

$$w(\{h,r\};d^\star)=w(\{h,r\};0)=\bar a(\{h,r\})+\Delta(\{h,r\})>\bar a(\{h,r\}) \qquad \forall\, d^\star\in A(\{g\};\{\{e\},\{h,r\}\}),$$

so condition (4) holds for $(H',\pi_{H'})$ and the outcome with $d^\star{=}1$ is dominated.

By Step 2 (applied with $Z=\{g,h,r\}$ and $\pi_{\mathcal{L}\setminus Z}=\{\{e\}\}$), the residual core $C(\{g,h,r\};\{\{e\}\})$ is non-empty, so by (5) the set $A(\{g,h,r\};\{\{e\}\})$ consists of the equilibrium indicators of its core outcomes; by Step 3, none of these is an outcome with $d^\star{=}1$, so every indicator in $A(\{g,h,r\};\{\{e\}\})$

equals 0 – the contradiction announced above. Hence the core contains no outcome with $d^{\star}=0$ that compensates, at any blocking cost $\eta < \overline{\eta}_1$.

The core is non-empty. The coalitions H within a family with $I(H) \leq D(H)$ are $\{g\}$ and $\{r\}$, whose respective conditions in the definition of η_2 hold if and only if $\eta \leq \max\{\Delta_e, \Delta(\{h,r\})\}$ and $\eta \leq \max\{\Delta(\{e,g\}), \Delta_h\}$; both bounds are at least $\Delta(\{h,r\})$, so $\eta_2 \geq \Delta(\{h,r\}) > \Delta(\{h,r\})/2 > \eta$. Since $\mathcal{G} \neq \mathcal{L}$ and $I(F) > D(F)$ for some $F \in \bar{\pi}$, Lemma EC.2(a)-(ii) implies the core contains an outcome with $d^{\star}=0$; in particular, it is non-empty.

Therefore, at every blocking cost $\eta < \overline{\eta}_1$, the core is non-empty yet contains no outcome with $d^{\star}=0$ that compensates. □

LEMMA EC.9. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3. There exist instances with $\bar{\pi} \neq \{\mathcal{L}\}$, $\mathcal{G} \notin \{\emptyset, \mathcal{L}\}$, $I(F) \geq D(F)$ for every $F \in \bar{\pi}$, and $I(F) = D(F)$ for some $F \in \bar{\pi}$, in which, at every blocking cost $\eta > 0$, the core contains no outcome with $d^{\star}=0$ that compensates; and the core is nonempty at every $\eta \notin (\eta_2, \overline{\eta}_1)$.*

Proof of Lemma EC.9. To simplify notation, we define $\Delta_\ell := I_\ell - D_\ell$, $\forall \ell \in \mathcal{L}$. Consider the instance

$$\mathcal{L} = \{g, b_1, b_2, e\}, \qquad \bar{\pi} = \{F_1, F_2\}, \qquad F_1 := \{g, b_1, b_2\}, \quad F_2 := \{e\},$$

$$\Delta_g > 0, \qquad \Delta_{b_1} = \Delta_{b_2} = -\frac{\Delta_g}{2}, \qquad 0 < \Delta_e < \Delta_g,$$

for example $(\Delta_g, \Delta_{b_1}, \Delta_{b_2}, \Delta_e) = (4, -2, -2, 3)$. Then $\Delta(F_1) = 0$ and $\Delta(F_2) = \Delta_e > 0$, so the hypotheses on the families hold, and $\mathcal{G} = \{g, e\} \notin \{\emptyset, \mathcal{L}\}$.

We first compute the thresholds. For $\overline{\eta}_1$, the candidate values in (8) are $\Delta_g/3$, $\Delta(\{g, b_i\})/2 = \Delta_g/4$, $\Delta(F_1)/1 = 0$, and $\Delta_e/3$ (every other coalition within a family has $\Delta(S) < 0$); because $\Delta_e < \Delta_g$, we get $\overline{\eta}_1 = \Delta_g/3$. For η_2, the coalitions H within a family with $\Delta(H) \leq 0$ are $\{b_1\}$, $\{b_2\}$, $\{b_1, b_2\}$, and F_1; their conditions in the definition of η_2 (7) hold, respectively, if and only if $\eta \leq \max\{\Delta_g/2, \Delta_e\}$ (for each $\{b_i\}$), $\eta \leq \max\{\Delta_g/2, \Delta_e\}$, and, for $H = F_1$ (where the only family F with $\Delta(F \setminus H) > 0$ is F_2), $\lfloor \Delta_e/\eta \rfloor \geq |F_1| = 3$, i.e., $\eta \leq \Delta_e/3$. Hence $\eta_2 = \Delta_e/3 < \Delta_g/3 = \overline{\eta}_1$.

The core contains no outcome with $d^{\star}=0$ that compensates, at any $\eta > 0$. Suppose, for the purpose of deriving a contradiction, that the core contains an outcome with $d^{\star}=0$ $(\sigma, 0, a)$ that compensates. Because $a(S) = w(S; 0)$ for every $S \in \sigma$ and compensation requires $a_\ell \geq v_\ell^{\text{sq}} + D_\ell$ for every $\ell \in \mathcal{L}$, every coalition $S \in \sigma$ satisfies $\Delta(S) \geq 0$. Within F_1, any coalition containing b_1 or b_2 but not g has $\Delta(S) < 0$, and $\{g, b_i\}$ leaves $\{b_j\}$, $j \neq i$, with $\Delta < 0$; hence the only partition of F_1 into coalitions with $\Delta(S) \geq 0$ is $\{F_1\}$, so $F_1 \in \sigma$. Because $\Delta(F_1) = 0$, the constraints $a(F_1) = w(F_1; 0)$ and $a_\ell \geq v_\ell^{\text{sq}} + D_\ell$ pin down $a_\ell = v_\ell^{\text{sq}} + D_\ell$ for every $\ell \in F_1$; in particular $a_g = v_g^{\text{sq}} + I_g - \Delta_g$.

Feasibility requires $0 \in T(\sigma, \bar{\mathbb{U}})$. The coalition $F_1 \in \sigma$ has $\Delta(F_1) \le 0$ and $|F_1| = 3$, while the only coalition of σ with $\Delta(S) > 0$ is $F_2 = \{e\}$; the condition in (EC.14) therefore reads $\lfloor \Delta_e/\eta \rfloor \ge 3$, i.e., $\eta \le \Delta_e/3$. Hence no such outcome exists for $\eta > \Delta_e/3$.

For $\eta \le \Delta_e/3$, the outcome is dominated by $\{g\}$ deviating and forming $\{\{g\}\}$. Indeed, every partition $\pi' \in \Pi_{\mathcal{L}}^{\prec \bar{\pi}}$ with $\{g\} \in \pi'$ satisfies $\Delta(\{g\}) = \Delta_g > 3\eta = \eta|\mathcal{L} \setminus \{g\}|$ (because $\eta \le \Delta_e/3 < \Delta_g/3$), so $1 \notin T(\pi', \bar{\mathbb{U}})$ by (EC.13); and $0 \in T(\pi', \bar{\mathbb{U}})$ by (EC.14), because $\lfloor \Delta_g/\eta \rfloor \ge 3 \ge \max_{H \in \pi' : \Delta(H) \le 0} |H|$. Hence $T(\pi', \bar{\mathbb{U}}) = \{0\}$ for every such π', and by (5) every equilibrium indicator in $A(\mathcal{L} \setminus \{g\}; \{\{g\}\})$ equals 0, whether or not the residual core is empty. The deviation therefore secures

$$w(\{g\}; d^\star) = w(\{g\}; 0) = v_g^{\mathrm{sq}} + I_g > a_g \qquad \forall\, d^\star \in A(\mathcal{L} \setminus \{g\}; \{\{g\}\}),$$

so $(\sigma, 0, a)$ is dominated, a contradiction. Hence the core contains no outcome with $d^\star = 0$ that compensates, at any blocking cost $\eta > 0$.

The core is nonempty at every $\eta \notin (\eta_2, \overline{\eta}_1)$. If $\eta \le \eta_2$, this follows from Lemma EC.2(a)-(ii), because $\mathcal{G} \ne \mathcal{L}$ and $\Delta(F_2) > 0$; if $\eta \ge \overline{\eta}_1$, it follows from Lemma EC.2(b)-(ii), because $\bar{\pi} \ne \{\mathcal{L}\}$. □

LEMMA EC.10. *If the core $C(\mathcal{L}; \emptyset)$ contains an outcome with deforestation decisions $d^\star$ and allocation a, then it contains* every *outcome with the same allocation and deforestation decisions, as long as the deforestation decision is feasible, i.e., every $(\sigma, d^\star, a)$ with $\sigma \in \Pi_{\mathcal{L}}^{\prec \bar{\pi}}$, $d^\star \in T(\sigma)$, and $a(S) = w(S; d^\star)$ for all $S \in \sigma$. In particular, $(\bar{\pi}, d^\star, a) \in C(\mathcal{L}; \emptyset)$ whenever $d^\star \in T(\bar{\pi})$.*

Proof of Lemma EC.10. Assume $(\pi, d^\star, a) \in C(\mathcal{L}; \emptyset)$ and consider any σ as in the statement. Because $d^\star \in T(\sigma)$ and $a(S) = w(S; d^\star)$ for all $S \in \sigma$, the triple $(\sigma, d^\star, a)$ is a valid outcome. It remains to show it is not dominated. Suppose a coalition $S \subseteq \mathcal{L}$ could profitably deviate and form $\hat{\pi}_S \in \Pi_S^{\prec \bar{\pi}}$, so that $a(S_i) < w(S_i; d')$ for every $S_i \in \hat{\pi}_S$ and every $d' \in A(\mathcal{L} \setminus S; \hat{\pi}_S)$. The deviation condition involves only a, $\hat{\pi}_S$, and $A(\mathcal{L} \setminus S; \hat{\pi}_S)$, none of which depends on the partition; hence the same $(S, \hat{\pi}_S)$ would dominate $(\pi, d^\star, a)$, contradicting $(\pi, d^\star, a) \in C(\mathcal{L}; \emptyset)$. □

LEMMA EC.11. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3. Define*

$$\eta_1^{comp} := \max_{F,S} \left\{ \min_{\pi_{\mathcal{L} \setminus S} \in \Pi_{\mathcal{L} \setminus S}^{\prec \bar{\pi}}} \eta_1(\{S\} \cup \pi_{\mathcal{L} \setminus S}) \ : \ F \in \bar{\pi},\ \emptyset \ne S \subset F,\ I(F \setminus S) < D(F \setminus S) \right\}.$$

If $\eta < \eta_1^{comp}$, then the core does not contain outcomes with $d^\star = 0$ that compensate.

Proof of Lemma EC.11. Assume by contradiction that the core contains an outcome with $d^\star = 0$ $(\pi, d^\star = 0, a)$ that compensates. We prove that this outcome is dominated.

Because $\eta < \eta_1^{comp}$, there exist $F \in \bar{\pi}$ and $\emptyset \neq S \subset F$ such that

$$I(F \setminus S) < D(F \setminus S) \tag{EC.39a}$$

$$\eta < \eta_1(\{S\} \cup \pi_{\mathcal{L}\setminus S}), \forall \pi_{\mathcal{L}\setminus S} \in \Pi_{\mathcal{L}\setminus S}^{\prec\bar{\pi}}. \tag{EC.39b}$$

We will show that S can deviate from the outcome $(\pi, 0, a)$. Because $d^\star = 0$, the aggregate allocation to all locals in family F is $\sum_{\ell \in F} a_\ell = w(F; 0)$, which implies that:

$$\sum_{\ell \in S} a_\ell = w(F;0) - \sum_{\ell \in F \setminus S} a_\ell \leq w(F;0) - \sum_{\ell \in F \setminus S} \left(v_\ell^{\text{sq}} + D_\ell\right) = w(S;0) + \big(I(F \setminus S) - D(F \setminus S)\big) < w(S;0),$$

where the first inequality follows because the outcome compensates, and the strict inequality follows from (EC.39a).

Because (EC.39b) holds, Lemma 2 implies that $T(\{S\} \cup \pi_{\mathcal{L}\setminus S}) = \{0\}$ holds for every feasible partition $\pi_{\mathcal{L}\setminus S} \in \Pi_{\mathcal{L}\setminus S}^{\prec\bar{\pi}}$, so we must also have $A(\mathcal{L} \setminus S; \{S\}) = \{0\}$. In turn, this, together with the inequality on allocations a implies that

$$\sum_{\ell \in S} a_\ell < w(S; d^\star), \qquad \forall d^\star \in A(\mathcal{L} \setminus S; \{S\}).$$

Therefore, coalition S with partition $\{S\}$ has a profitable deviation, which contradicts the assumption that the outcome $(\pi, d^\star = 0, a)$ belongs to the core. $\square$

Lemma EC.12. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3. Suppose that*

$$\bar{\pi} \neq \{\mathcal{L}\}, \qquad \mathcal{L} \neq \mathcal{G}, \qquad I_\ell < D_\ell \ \textit{ for some } \ell \in \mathcal{L}, \qquad I(F) \geq D(F)\ \forall F \in \bar{\pi}, \qquad \eta_1^{comp} \leq \eta < \underline{\eta}_1,$$

and that the core $C(\mathcal{L}; \emptyset)$ contains an outcome with $d^\star = 0$ that compensates. Then:

(a) (Structural result on $\bar{\pi}$.) There is a unique family $F^* \in \bar{\pi}$ with $\frac{I(F^*) - D(F^*)}{|\mathcal{L}\setminus F^*|} > \eta$, and this family contains a local $\ell \in F^*$ with $I_\ell < D_\ell$. Every other family $F \in \bar{\pi} \setminus \{F^*\}$ satisfies $\frac{I(F)-D(F)}{|\mathcal{L}\setminus F|} \leq \eta$.

(b) (Bounds on residual games.) Let F^* be the family from (a). Then:

(i) For any $F \in \bar{\pi}$ and any $S \subseteq F$ with $S \neq F^*$, we have $\frac{\Delta(S)}{|\mathcal{L}\setminus S|} \leq \eta$.

(ii) For every non-empty $H \subsetneq F^*$ and every partition $\pi_H \in \Pi_H^{\prec\bar{\pi}}$, $1 \in A(\mathcal{L} \setminus H; \pi_H)$.

(iii) For every $\ell \in F^*$ with $I_\ell < D_\ell$, $0 \in A(\mathcal{L} \setminus \{\ell\}; \{\{\ell\}\})$.

(c) (Specific core outcome.) Let F^* be the family from (a). There exist no-deforestation core outcomes $(\bar{\pi}, 0, a), (\bar{\pi}, 0, a^*) \in C(\mathcal{L}; \emptyset)$, each of which compensates all locals, such that

$$a_\ell^* = \begin{cases} v_\ell^{\text{sq}} + D_\ell, & \textit{if } \ell \in F^* \textit{ and } I_\ell < D_\ell, \\ v_\ell^{\text{sq}} + D_\ell + \dfrac{I(F^*) - D(F^*)}{|\{i \in F^* : I_i \geq D_i\}|}, & \textit{if } \ell \in F^* \textit{ and } I_\ell \geq D_\ell, \\ a_\ell, & \textit{if } \ell \notin F^*. \end{cases}$$

(d) *(Outcome without compensation.) Consider the outcome $(\bar{\pi}, 0, a^*)$ from (c). There exists $(\bar{\pi}, 0, a^\lambda) \in C(\mathcal{L}; \emptyset)$ such that $a_\ell^\lambda = a_\ell^* - \lambda$ for some $\lambda > 0$ and some $\ell \in \mathcal{L}$ with $I_\ell < D_\ell$. In particular, the core $C(\mathcal{L}; \emptyset)$ contains an outcome that does* not *compensate all locals.*

Proof of Lemma EC.12. For the proof, define $\Delta_\ell := I_\ell - D_\ell \ \forall \ell \in \mathcal{L}$ and $\Delta(S) := \sum_{\ell \in S} \Delta_\ell \ \forall S \subseteq \mathcal{L}$, so that $w(S;0) - w(S;1) = \Delta(S)$ and $w(S;1) = \sum_{i \in S} w(\{i\};1)$. For each $F \in \bar{\pi}$, define $F_- := \{\ell \in F : \Delta_\ell < 0\}$ and $F_+ := F \setminus F_-$, and call F a *blocker* if $\Delta(F)/|\mathcal{L} \setminus F| > \eta$. Recall that

$$\eta_1^{comp} = \max_{F,S} \left\{ \min_{\pi_{\mathcal{L}\setminus S} \in \Pi^{\prec \bar{\pi}}_{\mathcal{L}\setminus S}} \eta_1(\{S\} \cup \pi_{\mathcal{L}\setminus S}) \ : \ F \in \bar{\pi}, \ \emptyset \neq S \subsetneq F, \ \Delta(F \setminus S) < 0 \right\},$$
$$\underline{\eta}_1 = \max_{F,S,\eta} \left\{ \eta : \Delta(S) \geq \eta \, |\mathcal{L} \setminus S|, \, F \in \bar{\pi}, \, S \subseteq F, \, S \neq \mathcal{L}, \, \Delta(F) > 0 \right\}.$$

The hypothesis that $I_\ell < D_\ell$ for some $\ell \in \mathcal{L}$ ensures the existence of a local with $\Delta_\ell < 0$, hence of a family $F \in \bar{\pi}$ with $F_- \neq \emptyset$. Since $\bar{\pi} \neq \{\mathcal{L}\}$, every family $F' \in \bar{\pi}$ is a proper subset of $\mathcal{L}$, so $|\mathcal{L} \setminus F'| \geq 1$.

A premise of the lemma is that the core contains an outcome with $d^\star = 0$ that compensates; let a denote its allocation. Because it is an outcome with $d^\star = 0$, $a(F') = w(F';0)$ for every $F' \in \bar{\pi}$ (summing $a(S) = w(S;0) = v_S^{\mathrm{sq}} + I(S)$ over the coalitions S contained in F'), and because it compensates, $a_\ell \geq w(\{\ell\};1) = v_\ell^{\mathrm{sq}} + D_\ell$ for every $\ell \in \mathcal{L}$.

We prove the four sub-results in turn.

Proof of (a). We proceed in three steps to prove: the existence of a blocker (Step 1), that every blocker has $F_- \neq \emptyset$ (Step 2), and that the blocker is unique (Step 3).

Step 1: a blocker exists. The hypothesis $\eta < \underline{\eta}_1$ yields (F^*, S^*) with $F^* \in \bar{\pi}$, $S^* \subseteq F^*$, $S^* \neq \mathcal{L}$, $\Delta(F^*) > 0$, and $\Delta(S^*)/|\mathcal{L} \setminus S^*| > \eta$. If $S^* = F^*$, then F^* is a blocker. Otherwise $S^* \subsetneq F^*$. If $\Delta(F^* \setminus S^*) < 0$, then (F^*, S^*) is a feasible candidate for η_1^{comp}, giving $\eta_1^{comp} \geq \Delta(S^*)/|\mathcal{L} \setminus S^*| > \eta$, contradicting $\eta \geq \eta_1^{comp}$. Hence $\Delta(F^* \setminus S^*) \geq 0$, so $\Delta(F^*) \geq \Delta(S^*) > \eta \, |\mathcal{L} \setminus S^*| > \eta \, |\mathcal{L} \setminus F^*|$, and F^* is a blocker.

Because F^* satisfies $\Delta(F^*)/|\mathcal{L} \setminus F^*| > \eta$ and $\bar{\pi} \neq \{\mathcal{L}\}$, we have $\eta < \eta_1(\bar{\pi})$, so Lemma 2 gives $0 \in T(\bar{\pi})$. By Lemma EC.10, the no-deforestation compensating outcome can therefore be taken with partition $\bar{\pi}$, i.e., $(\bar{\pi}, 0, a) \in C(\mathcal{L}; \emptyset)$. The proofs for Steps 2-3 rely on Claim EC.1 below, which we prove after proving Steps 2-3.

Claim EC.1. *Under the premises of Lemma EC.12, consider any blocker F° satisfying:*

$$|S| \leq |\mathcal{L} \setminus F^\circ| \quad \textit{for every } S \subseteq F^\circ \textit{ with } \Delta(S) < 0. \tag{EC.40}$$

Then, the following results hold:

(A) *Let $H \subseteq \mathcal{L}$ be a coalition so that $H \cap F^\circ = \emptyset$, $\Delta(H) \geq 0$, and $w(H;0) > a(H)$. Then $A(\mathcal{L} \setminus H; \{H\}) = \{0\}$, so H has a profitable deviation from $(\bar{\pi}, 0, a)$ via $\pi_H = \{H\}$.*

(B) *If at least two blockers exist, then at least one of them satisfies* (EC.40).

Proof of Step 2: every blocker has $F_- \neq \emptyset$. Suppose for contradiction that $F^\dagger \in \bar{\pi}$ is a blocker with $F^\dagger_- = \emptyset$; then $F^\dagger$ has no sub-coalition S with $\Delta(S) < 0$, and thus $F^\dagger$ trivially satisfies (EC.40). By the hypothesis that $I_\ell < D_\ell$ for some $\ell \in \mathcal{L}$, there is a local with $\Delta_{\ell^\circ} < 0$; let $\hat{F} \in \bar{\pi}$ be the family containing it, so $\hat{F}_- \neq \emptyset$ and $\hat{F} \neq F^\dagger$ (because $F^\dagger_- = \emptyset$). Setting $H := \hat{F}_+$, we have:

$$\Delta(H) = \Delta(\hat{F}) - \Delta(\hat{F}_-) > 0, \tag{EC.41a}$$

$$a(H) = w(\hat{F}; 0) - a(\hat{F}_-) \; < \; w(\hat{F}; 0) - w(\hat{F}_-; 0) \; = \; w(H; 0). \tag{EC.41b}$$

In (EC.41a), the inequality follows because $\Delta(\hat{F}_-) < 0$ (by definition of $\hat{F}_-$) and $\Delta(\hat{F}) \geq 0$ (by the Lemma's hypothesis), which together give $\Delta(H) = \Delta(\hat{F}) - \Delta(\hat{F}_-) > 0$. The inequality in (EC.41b) follows because $a(\hat{F}_-) \geq w(\hat{F}_-; 1) > w(\hat{F}_-; 0)$ because $(\bar{\pi}, 0, a)$ is an outcome that compensates and $\Delta(\hat{F}_-) < 0$.

Because $H \subseteq \hat{F}$ and $\hat{F} \neq F^\dagger$, eqs. (EC.41a) and (EC.41b) imply that Claim EC.1-(A) applies with $F^\circ = F^\dagger$ and H, so H has a profitable deviation from $(\bar{\pi}, 0, a)$, which contradicts $(\bar{\pi}, 0, a) \in C(\mathcal{L}; \emptyset)$. Hence, every blocker has $F_- \neq \emptyset$; in particular, the blocker F^* of Step 1 has $F^*_- \neq \emptyset$.

Step 3: the blocker is unique. Suppose for contradiction that several blockers exist. By Claim EC.1-(B), assume without loss of generality that F^* is a blocker that satisfies (EC.40) and let $\hat{F} \in \bar{\pi} \setminus \{F^*\}$ be another blocker. By Step 2, $\hat{F}_- \neq \emptyset$. We can follow the arguments in Step 2 (with $\hat{F}$ playing the same role) to argue that $H := \hat{F}_+$ has a profitable deviation from $(\bar{\pi}, 0, a)$, which is a contradiction.

Combining Steps 1–3, we conclude that part (a) of Lemma EC.12 holds: F^* is the unique blocker, it has $F^*_- \neq \emptyset$ (so it contains a local ℓ with $\Delta_\ell < 0$), and $\Delta(F)/|\mathcal{L} \setminus F| \leq \eta$, $\forall F \in \bar{\pi} \setminus \{F^*\}$.

Proof of Claim EC.1. The proof relies on an intermediate result that we state as Claim EC.2.

Claim EC.2. *Under the premises of Lemma EC.12, consider any blocker F° satisfying* (EC.40) *and define $m := |\mathcal{L} \setminus F^\circ|$. Also, consider $Z \subseteq \mathcal{L}$ and $\tau \in \Pi^{\prec \bar{\pi}}_{\mathcal{L} \setminus Z}$ satisfying:*

$$|S| \leq m \quad \textit{for every } S \in \tau \textit{ with } \Delta(S) \leq 0, \tag{EC.42}$$

and suppose that either (i) $F^\circ \cap Z \neq \emptyset$ and $\Delta(F^\circ \cap Z) \geq \Delta(F^\circ)$, or (ii) $F^\circ \in \tau$. Then, $A(\emptyset; \tau) = \{0\}$. Moreover, if $Z \neq \emptyset$, then $C(Z; \tau)$ is nonempty and contains the residual outcome $(\pi^*, 0, a^\flat)$, where $a^\flat_i := w(\{i\}; 0), \forall i \in Z$ and $\pi^* \in \Pi^{\prec \bar{\pi}}_Z$ is defined as:

$$\pi^* := \begin{cases} \{F^\circ \cap Z\} \cup \big\{\{i\} : i \in Z \setminus F^\circ\big\} & \textit{in case (i),} \\ \big\{\{i\} : i \in Z\big\} & \textit{in case (ii).} \end{cases} \tag{EC.43}$$

Proof of Claim EC.2. Note that $m := |\mathcal{L} \setminus F^\circ|$ satisfies $m \geq 1$ and $\Delta(F^\circ) > \eta m$ by our standing assumptions (because $F^\circ \neq \mathcal{L}$ and F° is a blocker). Define G as $G := F^\circ \cap Z$ in case (i) and $G := F^\circ$ in case (ii). We claim that:

$$G \in \pi^* \cup \tau \qquad \text{and} \qquad \lfloor \Delta(G)/\eta \rfloor \geq m. \tag{EC.44}$$

That $G \in \pi^* \cup \tau$ follows because π^* contains G in case (i) and $G = F^\circ \in \tau$ in case (ii). In either case, we have $G \subseteq F^\circ$ and $\Delta(G) \geq \Delta(F^\circ) > \eta m$, where the first inequality follows from the definitions of cases (i)-(ii), and the last inequality follows because F° is a blocker. So $\lfloor \Delta(G)/\eta \rfloor \geq m$. We next claim that:

$$|S| \leq m, \qquad \text{for every } S \in \pi^* \cup \tau \text{ with } \Delta(S) \leq 0. \tag{EC.45}$$

Every $S \in \pi^*$ with $\Delta(S) \leq 0$ must be a singleton, so $|S| = 1 \leq m$. (In case (i), if $S = F^\circ \cap Z$, then $S = G$ and $\Delta(S) > 0$ by (EC.44).) Every $S \in \tau$ with $\Delta(S) \leq 0$ satisfies $|S| \leq m$ by (EC.42). We claim that $0 \in T(\pi^* \cup \tau)$. Recalling the set G defined above, (EC.44) and (EC.45) imply that:

$$\max_{S \in \pi^* \cup \tau :\, \Delta(S) \leq 0} |S| \;\leq\; m \;\leq\; \left\lfloor \tfrac{\Delta(G)}{\eta} \right\rfloor \;\leq \sum_{S \in \pi^* \cup \tau :\, \Delta(S) > 0} \left\lfloor \tfrac{\Delta(S)}{\eta} \right\rfloor,$$

which implies that $\eta \leq \eta_2(\pi^* \cup \tau)$, and by Lemma 2, this implies that $0 \in T(\pi^* \cup \tau)$.

Consider first the base case that $Z = \emptyset$. Then case (ii) must hold, so $F^\circ \in \tau$. Because F° is a blocker, we readily have $\eta < \eta_1(\tau)$, which by Lemma 2 implies that $\{0\} = T(\tau) = A(\emptyset; \tau)$.

Assume the stated claim holds for any sets Z' satisfying the premises and with $|Z'| \leq k$, and consider a set Z with $|Z| = k + 1 \geq 1$. We show $(\pi^*, 0, a^\flat)$ is not dominated. Let (K, π_K) be a deviation with $\emptyset \neq K \subseteq Z$ and $K \subseteq F_k$ for some $F_k \in \bar{\pi}$. If some $S \in \pi_K$ has $\Delta(S) \geq 0$, then $a^\flat(S) = \sum_{i \in S} w(\{i\}; 0) = w(S; 0) \geq w(S; d^\star)$ for every $d^\star \in \{0, 1\}$, so K would not deviate under pessimism. So we must have $\Delta(S) < 0, \forall S \in \pi_K$. Then, consider $Z' := Z \setminus K$ and the partition $\tau' := \pi_K \cup \tau$. It suffices to argue that $0 \in A(Z'; \tau')$, because that would imply that $a^\flat(S) = w(S; 0) \geq \min_{d^\star \in A(Z'; \tau')} w(S; d^\star), \forall S \in \pi_K$, so K would not deviate under pessimism. To prove that $0 \in A(Z'; \tau')$, we use the induction hypothesis, by verifying that (EC.42) and either (i) or (ii) hold for Z', τ'. To see that (EC.42) holds: if $F_k \neq F^\circ$, then $|F_k| \leq |\mathcal{L} \setminus F^\circ| = m$, so any $S \in \pi_K$ satisfies $|S| \leq |F_k| \leq m$, whereas if $F_k = F^\circ$, any $S \in \pi_K$ must satisfy $|S| \leq |\mathcal{L} \setminus F^\circ| \leq m$ by (EC.40), because $\Delta(S) < 0$; in both cases, $\tau' = \pi_K \cup \tau$ satisfies (EC.42). To see that either (i) or (ii) holds for Z', τ', consider Z, τ, which satisfy these premises. If case (ii) holds for Z, τ, then $F^\circ \in \pi_K \cup \tau = \tau'$, so (ii) also holds for Z', τ'. If case (i) holds for Z, τ, we distinguish two sub-cases. If $F_k \neq F^\circ$, then $K \cap F^\circ = \emptyset$, so $F^\circ \cap Z' = F^\circ \cap Z$ and (i) also holds for Z', τ'. If $F_k = F^\circ$, then $K \subseteq F^\circ \cap Z = G$ and $F^\circ \cap Z' = G \setminus K$ satisfies:

$$\Delta(G \setminus K) = \Delta(G) - \Delta(K) = \Delta(G) + |\Delta(K)| > \Delta(G) \geq \Delta(F^\circ).$$

This implies that $K \neq G$, so $G \setminus K \neq \emptyset$, so again (i) holds for Z', τ'. Because (Z', τ') satisfies (EC.42) and one of (i),(ii), the inductive hypothesis gives $0 \in A(Z \setminus K; \pi_K \cup \tau)$.

That the allocation is feasible follows from the definition of $a^\flat$ and $0 \in T(\pi^* \cup \tau)$, as argued above.

We conclude that $(\pi^*, 0, a^\flat) \in C(Z; \tau)$ and $0 \in A(Z; \tau)$, which proves Claim EC.2.

Proof of Claim EC.1-(A). Because $H \cap F^\circ = \emptyset$, we have $F^\circ \subseteq \mathcal{L} \setminus H$. Case (i) of Claim EC.2 is applicable with $Z = \mathcal{L} \setminus H$ and $\tau = \{H\}$: (i) holds readily because $F^\circ \cap Z = F^\circ$ holds, and (EC.42) holds because $H \subseteq \mathcal{L} \setminus F^\circ$, so $|H| \le |\mathcal{L} \setminus F^\circ| = m$). Claim EC.2 then implies that $0 \in A(\mathcal{L} \setminus H; \{H\})$.

We claim that $1 \notin A(\mathcal{L} \setminus H; \{H\})$. For a contradiction, suppose $(\pi, 1, a') \in C(\mathcal{L} \setminus H; \{H\})$. We claim that F° can profitably deviate by forming $\{F^\circ\}$. First, we prove that $A(\mathcal{L} \setminus (H \cup F^\circ); \{H, F^\circ\}) = \{0\}$. That $0 \in A(\mathcal{L} \setminus (H \cup F^\circ); \{H, F^\circ\})$ follows from case (ii) of Claim EC.2 with $Z = \mathcal{L} \setminus (H \cup F^\circ)$ and $\tau' = \tau \cup \{F^\circ\}$, which satisfies (EC.42) because $\Delta(F^\circ) > 0$ and τ satisfies (EC.42). Moreover, $1 \notin T(\{H, F^\circ\} \cup \pi)$ for any $\pi \in \Pi^{\prec \bar{\pi}}_{\mathcal{L} \setminus (H \cup F^\circ)}$ because $\eta_1(\{H, F^\circ\} \cup \pi) \ge \Delta(F^\circ)/|\mathcal{L} \setminus F^\circ| > \eta$ holds because F° is a blocker, so Lemma 2 implies $1 \notin A(\mathcal{L} \setminus (H \cup F^\circ); \{H, F^\circ\})$. So F° can deviate because:

$$a'(F^\circ) = \sum_{S \in \pi : S \subseteq F^\circ} a'(S) = \sum_{S \in \pi : S \subseteq F^\circ} w(S; 1) = w(F^\circ; 1) < w(F^\circ; 0) = \min_{d^\star \in A(\mathcal{L} \setminus (H \cup F^\circ); \{H, F^\circ\})} w(F^\circ; d^\star),$$

This yields a contradiction. Thus, all outcomes in $C(\mathcal{L} \setminus H; \{H\})$ have $d^\star = 0$.

That $\{0\} = A(\mathcal{L} \setminus H; \{H\})$ and $w(H; 0) > a(H)$ imply that H has a profitable deviation $\pi_H = \{H\}$.

Proof of Claim EC.1-(B). Note that if any blocker F satisfies $|F| \le |\mathcal{L} \setminus F|$, it would readily satisfy (EC.40) because $|S| \le |F| \le |\mathcal{L} \setminus F|$. Assume that all blockers F satisfy $|F| > |\mathcal{L} \setminus F|$; this leads to a contradiction because at most one family $F \in \bar{\pi}$ can have this property.

Proof of (b). Let F^* denote the unique blocker from (a), which has $F^*_- \neq \emptyset$ and $\Delta(F^*)/|\mathcal{L} \setminus F^*| > \eta$.

Proof of (b)(i). The result trivially holds if $\Delta(S) \le 0$ because $\eta > 0$, so assume that $\Delta(S) > 0$.

Consider first the case $F = F^*$ and fix a non-empty $S \subsetneq F^*$ with $\Delta(S) > 0$. We construct a set $S^* \neq \emptyset$ with $S^* \subsetneq F^*$, $\Delta(F^* \setminus S^*) < 0$, and $\frac{\Delta(S^*)}{|\mathcal{L} \setminus S^*|} \ge \frac{\Delta(S)}{|\mathcal{L} \setminus S|}$. This would complete the proof because the pair (F^*, S^*) would be a feasible candidate for the maximization defining η_1^{comp}, implying that:

$$\eta \;\ge\; \eta_1^{comp} \;\ge\; \min_{\pi} \eta_1(\{S^*\} \cup \pi) \;\ge\; \frac{\Delta(S^*)}{|\mathcal{L} \setminus S^*|} \;\ge\; \frac{\Delta(S)}{|\mathcal{L} \setminus S|}.$$

If $F^*_- \nsubseteq S$, set $S^* := S \cup F^*_+$. Then, $S^* \subsetneq F^*$ (because $F^*_- \neq \emptyset$). Moreover, $\Delta(S^*) \ge \Delta(S)$ and $|\mathcal{L} \setminus S^*| \le |\mathcal{L} \setminus S|$ because $S \subseteq S^*$, so $\frac{\Delta(S^*)}{|\mathcal{L} \setminus S^*|} \ge \frac{\Delta(S)}{|\mathcal{L} \setminus S|}$. Lastly, $\Delta(F^* \setminus S^*) < 0$ because $F^* \setminus S^* = F^*_- \setminus S^* \neq \emptyset$.

If $F^*_- \subseteq S$, pick any $b \in F^*_-$ and set $S^* := F^* \setminus \{b\}$. Clearly, $\Delta(F^* \setminus S^*) = \Delta_b < 0$ and $S^* \subsetneq F^*$. Because $F^* \setminus S \subseteq F^*_+$, we have $\Delta(F^* \setminus S) \ge 0$, so $\Delta(F^*) \ge \Delta(S)$ and $\Delta(S^*) = \Delta(F^*) + |\Delta_b| > \Delta(S)$. Also, $|\mathcal{L} \setminus S^*| = |\mathcal{L} \setminus F^*| + 1 \le |\mathcal{L} \setminus S|$ (because $S \subsetneq F^*$ implies $|S| \le |F^*| - 1$). Therefore, $\frac{\Delta(S^*)}{|\mathcal{L} \setminus S^*|} > \frac{\Delta(S)}{|\mathcal{L} \setminus S|}$.

Finally, consider the case $F \neq F^*$. If $\Delta(F \setminus S) < 0$, then $S \neq F$ and (F, S) is a feasible candidate for η_1^{comp}, so $\frac{\Delta(S)}{|\mathcal{L} \setminus S|} \le \eta_1^{comp} \le \eta$, proving (b)-(i). If $\Delta(F \setminus S) \ge 0$, then $\Delta(S) \le \Delta(F)$, and because $|\mathcal{L} \setminus S| \ge |\mathcal{L} \setminus F|$, we obtain: $\frac{\Delta(S)}{|\mathcal{L} \setminus S|} \le \frac{\Delta(F)}{|\mathcal{L} \setminus F|} \le \eta$, where the last inequality is due to (a) of the Lemma.

Proof of (b)(ii). The proof relies on the following inductive claim.

CLAIM EC.3 **(Inductive claim)**. *For every $R \subseteq \mathcal{L}$ with $F^* \nsubseteq R$ and every $\sigma \in \Pi^{\prec \bar{\pi}}_{\mathcal{L} \setminus R}$, if there exists $\pi^*_R \in \Pi^{\prec \bar{\pi}}_R$ with $\eta_1(\pi^*_R \cup \sigma) \le \eta$, then $1 \in A(R; \sigma)$.*

We first prove that (b)(ii) follows from Claim EC.3. Consider any $\emptyset \neq H \subsetneq F^*$, any $\pi_H \in \Pi_H$, and any $\pi_{\mathcal{L}\setminus H} \in \Pi_{\mathcal{L}\setminus H}^{\prec\bar{\pi}}$. Because $\emptyset \neq H \subsetneq F^*$, we have $F^* \nsubseteq \mathcal{L} \setminus H$. Moreover, because $H \subsetneq F^*$, result (b)-(i) implies that any coalition $S \in \pi_H \cup \pi_{\mathcal{L}\setminus H}$ satisfies $\frac{\Delta(S)}{|\mathcal{L}\setminus S|} \leq \eta$. This implies that $\eta_1(\pi_{\mathcal{L}\setminus H} \cup \pi_H) \leq \eta$. But then, the inductive claim applies with $R = \mathcal{L} \setminus H$, $\sigma = \pi_H$, $\pi_R^* = \pi_{\mathcal{L}\setminus H}$, implying that $1 \in A(\mathcal{L} \setminus H; \pi_H)$.

Proof of Claim EC.3. We prove by induction on the size of the residual set, $|R|$.

Base case $|R| = 0$. In this case the hypothesis is that $\eta_1(\sigma) \leq \eta$. Because $A(\emptyset;\sigma) = T(\sigma)$, then $\eta_1(\sigma) \leq \eta$ and Lemma 2 implies that $1 \in T(\sigma)$, and therefore $1 \in A(\emptyset;\sigma)$.

Inductive step. Suppose the claim holds for residuals R with $|R| \leq k$. Consider any $R \subseteq \mathcal{L}$ with $|R| = k+1$ and $F^* \nsubseteq R$, and any $\sigma \in \Pi_{\mathcal{L}\setminus R}^{\prec\bar{\pi}}$. The inductive claim premises imply that:

$$\exists \pi_R^* \in \Pi_R^{\prec\bar{\pi}} \;:\; \eta_1(\pi_R^* \cup \sigma) \leq \eta \quad \Longrightarrow \quad F^* \notin \pi_R^* \cup \sigma \quad \text{and (by Lemma 2)} \quad 1 \in T(\pi_R^* \cup \sigma). \tag{EC.46}$$

If $C(R;\sigma) = \emptyset$, then $A(R;\sigma) = \{d : \exists \pi_R,\, d \in T(\pi_R \cup \sigma)\}$, and the result follows from $1 \in T(\pi_R^* \cup \sigma)$.

If $C(R;\sigma) \neq \emptyset$, we show that the outcome $(\pi_R^*, 1, a^{(1)}) \in C(R;\sigma)$, where $a_\ell^{(1)} := w(\{\ell\};1), \forall \ell \in R$. This is a valid outcome with $d^\star = 1$. We show that it is not dominated. Let (H, π_H) be a candidate deviation with $\emptyset \neq H \subseteq R$ and consider the residual game for $R \setminus H$ facing partition $\pi_H \cup \sigma$. Let $\pi^{**} := \{S \setminus H \;:\; S \in \pi_R^*,\, S \setminus H \neq \emptyset\} \in \Pi_{R\setminus H}^{\prec\bar{\pi}}$. We claim that $F^* \notin \pi^{**} \cup \pi_H \cup \sigma$: this follows because $F^* \nsubseteq R$ so $F^* \nsubseteq H$ and no coalition in π^{**} can contain F^*, and (EC.46) implies $F^* \notin \sigma$. Therefore, by result (b)-(i), this implies that $\frac{\Delta(S)}{|\mathcal{L}\setminus S|} \leq \eta$ for all $S \in \pi^{**} \cup \pi_H \cup \sigma$, and therefore $\eta_1(\pi^{**} \cup \pi_H \cup \sigma) \leq \eta$. This implies that $1 \in A(R \setminus H; \pi_H \cup \sigma)$: if $H \subsetneq R$, this follows from the inductive hypothesis; if $H = R$, then $A(\emptyset; \pi_H \cup \sigma) = T(\pi_H \cup \sigma)$ and $1 \in T(\pi_H \cup \sigma)$ by $\eta_1(\pi_H \cup \sigma) \leq \eta$ and Lemma 2. But then, $1 \in A(R \setminus H; \pi_H \cup \sigma)$ implies that (H, π_H) cannot be a deviation because $a^{(1)}(S) = w(S;1) \geq \min_{d^\star \in A(R\setminus H;\pi_H\cup\sigma)} w(S;d^\star)$ for all $S \in \pi_H$. Therefore, $(\pi_R^*, 1, a^{(1)}) \in C(R;\sigma)$ and $1 \in A(R;\sigma)$, completing the induction and the claim.

Proof of (b)(iii). Fix $\ell \in F_-^*$ and set $\hat{F} := F^* \setminus \{\ell\}$, $R := \mathcal{L} \setminus \{\ell\}$, $\sigma := \{\{\ell\}\}$. By (a), $\Delta(F^*) > \eta\,|\mathcal{L} \setminus F^*|$, and because $\ell \in F_-^*$, we have:

$$\Delta(\hat{F}) = \Delta(F^*) + |\Delta_\ell| > \eta\,|\mathcal{L} \setminus F^*|, \qquad \text{and therefore also} \qquad \lfloor \Delta(\hat{F})/\eta \rfloor \;\geq\; |\mathcal{L} \setminus F^*| \geq 1. \tag{EC.47}$$

For the residual game for R faced with σ, we define an outcome with $d^\star = 0$ that compensates.

Compensation. Let $\pi_R^* := \{\hat{F}\} \cup (\bar{\pi} \setminus \{F^*\})$ and note that $\pi_R^* \in \Pi_R^{\prec\bar{\pi}}$, that $\Delta(\hat{F}) > 0$ (by (EC.47)), and that $\Delta(F) \geq 0$ for every $F \in \bar{\pi} \setminus \{F^*\}$ (by the Lemma's hypothesis); hence $\Delta(S) \geq 0, \forall S \in \pi_R^*$. Define $a^{(0)} \in \mathbb{R}^{\mathcal{L}}$ so that for any $S \in \pi_R^*$, $a^{(0)}(S) = w(S;0)$ and $a_i^{(0)} \geq w(\{i\};1), \forall i \in S$, which is possible because $w(S;0) - w(S;1) = \Delta(S) \geq 0$. Then we readily have:

$$a^{(0)}(S') \;\geq\; w(S';1) \qquad \text{for every } S' \subseteq R. \tag{EC.48}$$

No-deforestation. Recall from Lemma 2 that $0 \in T(\pi_R^* \cup \sigma)$ if and only if $\eta \le \eta_2(\pi_R^* \cup \sigma)$, i.e.,

$$\sum_{S \in \pi_R^* \cup \sigma:\, \Delta(S)>0} \lfloor \Delta(S)/\eta \rfloor \;\ge\; \max_{S \in \pi_R^* \cup \sigma:\, \Delta(S)\le 0} |S|. \tag{EC.49}$$

The coalitions $S \in \pi_R^* \cup \sigma$ with $\Delta(S) \le 0$ are $\{\ell\}$ together with any family $F \in \bar{\pi} \setminus \{F^*\}$ satisfying $\Delta(F) = 0$; since each such F satisfies $F \subseteq \mathcal{L} \setminus F^*$ and hence $|F| \le |\mathcal{L} \setminus F^*|$, the right-hand side of (EC.49) is at most $|\mathcal{L} \setminus F^*|$. The left-hand side is at least $\lfloor \Delta(\hat{F})/\eta \rfloor \ge |\mathcal{L} \setminus F^*|$ by (EC.47). Hence (EC.49) holds and $0 \in T(\pi_R^* \cup \sigma)$.

Non-domination. If $C(R;\sigma) = \emptyset$, then $0 \in T(\pi_R^* \cup \sigma)$ already implies $0 \in A(R;\sigma)$, proving (b)-(iii). If $C(R;\sigma) \ne \emptyset$, we prove that $(\pi_R^*, 0, a^{(0)}) \in C(R;\sigma)$. Let (H, π_H) be a deviation with $\emptyset \ne H \subseteq R$. We claim that $1 \in A(R \setminus H; \pi_H \cup \sigma)$, which implies that (H, π_H) cannot be a deviation because $a^{(0)}$ satisfies (EC.48) for every $S \in \pi_H$. To establish this claim, we exhibit a partition $\pi' \in \Pi_{R\setminus H}^{\prec\bar{\pi}}$ with $\eta_1(\pi' \cup \pi_H \cup \sigma) \le \eta$ and invoke Claim EC.3 from the proof of (b)(ii). Consider $\pi' = \{(R \setminus H) \cap F : F \in \bar{\pi},\, (R \setminus H) \cap F \ne \emptyset\}$. Because $\sigma = \{\{\ell\}\}$ and $\ell \in F^*$, any coalition $S \in \pi' \cup \pi_H \cup \sigma$ satisfies $S \ne F^*$ and thus $\frac{\Delta(S)}{|\mathcal{L}\setminus S|} \le \eta$ by result (b)(i), which implies that $\eta_1(\pi' \cup \pi_H \cup \sigma) \le \eta$. Because $\ell \in F^*$ and $\ell \notin R \setminus H$, we have $F^* \not\subseteq R \setminus H$, so Claim EC.3 of (b)(ii) is applicable. This implies that $1 \in A(R \setminus H; \pi_H \cup \sigma)$. Because (H, π_H) was arbitrary, this implies $(\pi_R^*, 0, a^{(0)}) \in C(R;\sigma)$, so $0 \in A(\mathcal{L} \setminus \{\ell\}; \{\{\ell\}\})$. This proves (b)(iii).

Proof of (c). Recall the outcome $(\bar{\pi}, 0, a) \in C(\mathcal{L};\emptyset)$ that compensates all locals, which exists by the premises of Lemma EC.12. With F^* denoting the family blocker from (a), recall that $F_-^* \ne \emptyset$ and $F_+^* \ne \emptyset$. Define the allocation a^* as:

$$a_i^* := \begin{cases} w(\{i\};1), & i \in F_-^*, \\ w(\{i\};1) + \dfrac{\Delta(F^*)}{|F_+^*|}, & i \in F_+^*, \\ a_i, & i \notin F^*. \end{cases} \tag{EC.50}$$

We verify that a^* satisfies all the stated properties. For $\ell \in F_-^*$, $a_\ell^* = w(\{\ell\};1) = v_\ell^{\rm sq} + D_\ell$. For $\ell \in F_+^*$ (which means $I_\ell \ge D_\ell$), we have $a_\ell^* = w(\{\ell\};1) + \frac{\Delta(F^*)}{|F_+^*|} > v_\ell^{\rm sq} + D_\ell$.

We first verify that $(\bar{\pi}, 0, a^*)$ *is a feasible outcome that compensates.* We have:

$$a^*(F^*) = \sum_{i \in F^*} a_i^* = w(F_-^*;1) + w(F_+^*;1) + \Delta(F^*) = w(F^*;1) + \Delta(F^*) = w(F^*;0), \tag{EC.51}$$

and therefore $a^*(F) = a(F) = w(F;0), \forall F \in \bar{\pi}$, so $(\bar{\pi}, 0, a^*)$ is a feasible outcome with $d^\star{=}0$. It compensates because $\Delta(F^*) > 0$ and $a_i \ge w(\{i\};1), \forall i \in \mathcal{L}$ because the outcome $(\bar{\pi}, 0, a)$ itself compensates. A consequence that we use below is that:

$$a^*(S) \;\ge\; w(S;1), \forall S \subseteq F \in \bar{\pi}. \tag{EC.52}$$

Core membership. To prove that this is a core outcome, let (H, π_H) be a deviation with $H \neq \emptyset$.

Suppose $H \subseteq \mathcal{L} \setminus F^*$. Then $a^*(S) = a(S), \forall S \in \pi_H$. Because the set $A(\mathcal{L} \setminus H; \pi_H)$ only depends on deforestation indicators but not on the *allocation*, if (H, π_H) dominates $(\bar{\pi}, 0, a^*)$ it would also dominate $(\bar{\pi}, 0, a)$, which would contradict the latter being a core outcome.

Suppose $H \subsetneq F^*$. By (b)(ii), we have $1 \in A(\mathcal{L} \setminus H; \pi_H)$. For any $S \in \pi_H$, we have $S \subseteq F^*$, and (EC.52) implies that $a^*(S) \geq w(S; 1)$, implying (H, π_H) cannot profitably deviate under pessimism.

Suppose $H = F^*$. If (F^*, π') were a valid deviation (for some $\pi' \in \Pi_{F^*}^{\prec \bar{\pi}}$), then we would have $w(S; d^\star) > a^*(S)$ $\forall S \in \pi'$ by (EC.51), implying that $w(F^*; d^\star) > a^*(F^*) = w(F^*; 0)$. That would yield a contradiction because $w(F^*; d^\star) \leq w(F^*; 0)$ holds in any outcome because $\Delta(F^*) > 0$.

In every case, $(\bar{\pi}, 0, a^*)$ is not dominated, so $(\bar{\pi}, 0, a^*) \in C(\mathcal{L}; \emptyset)$. This proves (c).

Proof of (d). Recall the allocation a^* defined in (EC.50). With F^* denoting the blocker from (a)-(c), fix any $\ell \in F^*_-$, which exists because $F^*_- \neq \emptyset$ by (a). For $\lambda \geq 0$, define the allocation $a^\lambda \in \mathbb{R}^{\mathcal{L}}$ as:

$$a_i^\lambda := \begin{cases} a_\ell^* - \lambda, & i = \ell, \\ a_i^* + \lambda, & i \in F^*_- \setminus \{\ell\}, \\ a_i^* - \dfrac{(|F^*_-| - 2)\,\lambda}{|F^*_+|}, & i \in F^*_+, \\ a_i^*, & i \notin F^*. \end{cases} \tag{EC.53}$$

The transfers inside F^* sum to: $-\lambda + (|F^*_-| - 1)\lambda - (|F^*_-| - 2)\lambda = 0$, so we readily have for any $F \in \bar{\pi}$ that $a^\lambda(F) = a^*(F) = w(F; 0)$, proving that $(\bar{\pi}, 0, a^\lambda)$ is a valid outcome. Moreover, by (c)(i), $a_\ell^\lambda = v_\ell^{\text{sq}} + D_\ell - \lambda < v_\ell^{\text{sq}} + D_\ell$ for $\lambda > 0$, proving that $(\bar{\pi}, 0, a^\lambda)$ does *not* compensate all locals. It remains to show that $(\bar{\pi}, 0, a^\lambda) \in C(\mathcal{L}; \emptyset)$. *Useful inequality concerning sub-coalitions of F^*.* Fix $\emptyset \neq S \subseteq F^*$ and define $p := |S \cap F^*_+|$ and $q := |S \cap (F^*_- \setminus \{\ell\})|$. By (EC.50), $a^*(S) = w(S; 1) + \frac{p\,\Delta(F^*)}{|F^*_+|}$. Adding the values of a_i^λ for $i \in S$, we have:

$$a^\lambda(S) \;=\; \begin{cases} w(S;1) + p \cdot \frac{\Delta(F^*) - (|F^*_-| - 2)\lambda}{|F^*_+|} \;+\; \lambda\, q, & \text{if } \ell \notin S \\ w(S;1) + p \cdot \frac{\Delta(F^*) - (|F^*_-| - 2)\lambda}{|F^*_+|} \;+\; \lambda\,(q - 1), & \text{if } \ell \in S. \end{cases} \tag{EC.54}$$

Set $\bar{\lambda} := \min\!\left(|\Delta_\ell|, \frac{\Delta(F^*)}{\max(1, |F^*| - 2)}\right)$. Because $\Delta_\ell < 0$ and $\Delta(F^*) > 0$, we have $\bar{\lambda} > 0$. Choose $\lambda \in (0, \bar{\lambda}]$. We claim that:

$$a^\lambda(S) \geq w(S; 1), \quad \forall S \subseteq F^* \;\; \text{with} \;\; S \neq \{\ell\}. \tag{EC.55}$$

Define $c(\lambda) := \frac{\Delta(F^*) - (|F^*_-| - 2)\lambda}{|F^*_+|}$. By construction, $c(\lambda) \geq 0$ for every $\lambda \in [0, \bar{\lambda}]$. Indeed, if $|F^*_-| < 2$, then

$$\Delta(F^*) - (|F^*_-| - 2)\lambda \geq \Delta(F^*) > 0.$$

If $|F^*_-| \geq 2$, then $|F^*_-| - 2 \leq \max\{1, |F^*| - 2\}$ and $\lambda \leq \bar{\lambda}$, so

$$(|F^*_-| - 2)\lambda \leq \max\{1, |F^*| - 2\}\bar{\lambda} \leq \Delta(F^*).$$

Thus, the numerator of $c(\lambda)$ is nonnegative in both cases. Consider $S \neq \{\ell\}$. If either $\ell \notin S$ or $\ell \in S$ and $q \geq 1$, then $a^\lambda(S) \geq w(S;1)$ follows from (EC.54) because $\lambda, p, c(\lambda) \geq 0$. If $\ell \in S, q = 0$, then $p \geq 1$ (because $\emptyset \neq S \subseteq F^*$ and $S \cap (F^*_- \setminus \{\ell\}) = \emptyset$), in which case:

$$a^\lambda(S) - w(S;1) \geq c(\lambda) - \lambda = \frac{\Delta(F^*) - (|F^*| - 2)\lambda}{|F^*_+|} \geq 0.$$

In all cases, we conclude that (EC.55) holds.

Core membership. To prove that $(\bar{\pi}, 0, a^\lambda) \in C(\mathcal{L}; \emptyset)$, assume (H, π_H) is a deviation; thus, $H \neq \emptyset$.

Suppose $H \subseteq \mathcal{L} \setminus F^*$. Then, by (EC.53), we have $a_i^\lambda = a_i$ for every $i \in H$. We use the same argument as in the proof of (c): because $A(\mathcal{L} \setminus H; \pi_H)$ does not depend on allocations, a deviation that dominates $(\bar{\pi}, 0, a^\lambda)$ would also dominate $(\bar{\pi}, 0, a)$, contradicting that $(\bar{\pi}, 0, a) \in C(\mathcal{L}; \emptyset)$.

Suppose $H \subsetneq F^*$ and $H \neq \{\ell\}$. By (b)(ii), $1 \in A(\mathcal{L} \setminus H; \pi_H)$. Because $H \neq \{\ell\}$, there exists $S \in \pi_H$ with $S \neq \{\ell\}$, so (EC.55) implies that (H, π_H) does not deviate under pessimism.

Suppose $H = \{\ell\}$. Then, $\pi_H = \{\{\ell\}\}$ and by (b)(iii), we have $0 \in A(\mathcal{L} \setminus \{\ell\}; \{\{\ell\}\})$. This implies that for H to deviate, it must be that

$$a_\ell^\lambda < w(\{\ell\};0) \quad \Leftrightarrow \quad w(\{\ell\};1) - \lambda < w(\{\ell\};1) - |\Delta_\ell| \quad \Leftrightarrow \quad \lambda > |\Delta_\ell|,$$

which does not hold because $\lambda \leq \bar{\lambda} \leq |\Delta_\ell|$. So $H = \{\ell\}$ would not deviate.

Suppose $H = F^*$. Because $a^\lambda(F^*) = w(F^*;0)$, the argument mirrors the one in the proof of (c). For any $\pi' \in \Pi_{F^*}^{\prec \bar{\pi}}$ and any $d^\star \in A(\mathcal{L} \setminus F^*; \pi') \subseteq \{0,1\}$, if (F^*, π') dominated the outcome with a^λ, then $w(S; d^\star) > a^\lambda(S), \forall S \in \pi'$; summing over $S' \in \pi'$ would yield $w(F^*; d^\star) > a^\lambda(F^*) = w(F^*;0)$, which is impossible because $w(F^*; d^\star) \leq w(F^*;0)$ because $\Delta(F^*) > 0$. Hence (F^*, π') does not dominate.

To conclude the proof, note that the outcome $(\bar{\pi}, 0, a^\lambda)$ belongs to the core $C(\mathcal{L}; \emptyset)$ and it does *not* compensate all locals because $a_\ell^\lambda < v_\ell^{\text{sq}} + D_\ell$ from (EC.53). □

Limited cooperation with no family that prefers the incentive. Suppose $\bar{\pi} \neq \{\mathcal{L}\}$ and $I(F) \leq D(F)$ for every $F \in \bar{\pi}$. The following lemma characterizes the core (Part A-iii); its part (iv) also supplies one of the empty-core instances of Part A.

LEMMA EC.13. *Consider the area no-use condition* $\bar{\mathbb{U}}$ *in the cooperative game with transferable utility defined in §3.3, and suppose* $I(F) \leq D(F)$ *for every* $F \in \bar{\pi}$. *Then:*

(i) If the core is nonempty, it contains an outcome with $d^\star = 1$.

(ii) If $\eta > \overline{\eta}$, *the core contains only outcomes with* $d^\star = 1$.

(iii) If $\eta \leq \overline{\eta}$, *the core may contain an outcome with* $d^\star = 0$.

(iv) The core may be empty.

(v) The core may be non-empty and contain only *outcomes with* $d^\star = 1$, *at every blocking cost* $\eta \leq \overline{\eta}$.

Proof of Lemma EC.13. Write $\Delta_\ell := I_\ell - D_\ell$, and recall $w(S;0) = v_S^{\rm sq} + I(S)$, $w(S;1) = v_S^{\rm sq} + D(S)$. By Lemma 2, in the form of (EC.13) and (EC.14), $1 \in T(\pi)$ iff $\Delta(S) \le \eta|\mathcal{L} \setminus S|$ for every $S \in \pi$, and $0 \in T(\pi)$ iff $\sum_{S\in\pi:\Delta(S)>0} \lfloor \Delta(S)/\eta \rfloor \ge \max_{H\in\pi:\Delta(H)\le 0} |H|$.

Part (i). Suppose the core is nonempty. Assume for contradiction that every core outcome is an outcome with $d^\star\!=\!0$, and let $(\pi', 0, a')$ be one such outcome, so $a'(F) = \sum_{S\in\pi', S\subseteq F} w(S;0) = v_F^{\rm sq} + I(F)$ for each $F \in \bar\pi$. Define

$$a_\ell := a'_\ell + \frac{D(F) - I(F)}{|F|}, \qquad \ell \in F \in \bar\pi.$$

Then $a_\ell \ge a'_\ell$ for every ℓ (since $D(F) \ge I(F)$) and $a(F) = w(F;1)$.

We will show that $(\bar\pi, 1, a)$ is in the core. Because $\Delta(F) \le 0$ for every F, we have $\eta_2(\bar\pi) = -\infty$, and by Lemma 2 $1 \in T(\bar\pi)$, so $(\bar\pi, 1, a)$ is a valid outcome. We will show it is undominated. Assume a coalition H deviates and forms $\hat\pi_H$. For the deviation to be profitable, we need $w(H';d) > a(H')$ for all $H' \in \hat\pi_H$ and all $d \in A(\mathcal{L} \setminus H; \hat\pi_H)$, but $a(H') \ge a'(H')$, and so H with partition $\hat\pi_H$ would also be a deviation for the outcome $(\pi', 0, a')$. So H does not dominate $(\bar\pi, 1, a)$. Hence $(\bar\pi, 1, a)$ is in the core, contradicting the fact that the core contains only outcomes with $d^\star\!=\!0$.

Part (ii). Let $\eta > \overline{\eta}$ and take any $\pi \in \Pi_{\mathcal{L}}^{\preceq\bar\pi}$. By (6), every $S \in \pi$ with $\Delta(S) > 0$ satisfies $\Delta(S) \le \overline{\eta} < \eta$, so $\lfloor \Delta(S)/\eta \rfloor = 0$. Moreover $\sum_{S\in\pi} \Delta(S) = \Delta(\mathcal{L}) = \sum_{F\in\bar\pi} \Delta(F) \le 0$, so some $S \in \pi$ has $\Delta(S) \le 0$ and $\max_{H\in\pi:\Delta(H)\le 0} |H| \ge 1$. Therefore $\sum_{S\in\pi:\Delta(S)>0} \lfloor \Delta(S)/\eta \rfloor = 0 < 1 \le \max_{H\in\pi:\Delta(H)\le 0} |H|$, so $0 \notin T(\pi)$. As π was arbitrary, no partition admits an equilibrium with $d^\star = 0$, so no outcome with $d^\star\!=\!0$ is feasible, and the core contains only outcomes with $d^\star\!=\!1$.

Part (iii). Consider the instance

$$\mathcal{L} = \{g_1, b_1, g_2, b_2\}, \qquad \bar\pi = \{F_1, F_2\}, \qquad F_k = \{g_k, b_k\}, \qquad \Delta_{g_k} = \Delta > 0, \;\; \Delta_{b_k} = -\Delta.$$

Then $I(F_k) = D(F_k)$ for each k, and $\overline{\eta} = \Delta$. Fix any $0 < \eta \le \overline{\eta} = \Delta$.

We prove by induction that the core contains an outcome with $d^\star\!=\!0$. More generally, we will show that for any residual set $R \subseteq \mathcal{L}$, and partition $\pi_{\mathcal{L}\setminus R}$ of the locals in $\mathcal{L} \setminus R$ comprised only of singletons, the residual core contains an outcome with $d^\star\!=\!0$.

Claim EC.4. *For every $R \subseteq \mathcal{L}$ and every partition $\pi_{\mathcal{L}\setminus R}$ of $\mathcal{L} \setminus R$ comprised solely of singletons (i.e., $\pi_{\mathcal{L}\setminus R} = \{\{i\} : i \in \mathcal{L} \setminus R\}$), we have*

$$0 \in A(R; \pi_{\mathcal{L}\setminus R}).$$

Moreover, if $R \neq \emptyset$, then the residual outcome

$$(\sigma_R, 0, a^R), \qquad \sigma_R = \{\{\ell\} : \ell \in R\}, \qquad a_\ell^R = v_\ell^{\rm sq} + I_\ell \quad (\ell \in R),$$

lies in $C(R; \pi_{\mathcal{L}\setminus R})$.

Proof of the claim. We prove the claim by induction on $|R|$.

First, the all-singleton partition σ_R defined above always admits the no-deforestation equilibrium indicator. Indeed, in $\sigma_R \cup \pi_{\mathcal{L}\setminus R}$ every coalition is a singleton. In particular, $\{g_1\}$ is one of these coalitions. Since $\Delta_{g_1} = \Delta \geq \eta$, $\sum_{S\in\sigma_R\cup\pi_{\mathcal{L}\setminus R}:\Delta(S)>0} \left\lfloor \frac{\Delta(S)}{\eta} \right\rfloor \geq \left\lfloor \frac{\Delta_{g_1}}{\eta} \right\rfloor \geq 1$. Also, $\max_{H\in\sigma_R\cup\pi_{\mathcal{L}\setminus R}:\Delta(H)\leq 0} |H| = 1$, because every coalition in $\sigma_R \cup \pi_{\mathcal{L}\setminus R}$ is a singleton. Therefore $\eta \leq \eta_2(\sigma_R \cup \pi_{\mathcal{L}\setminus R})$ and by Lemma 2 $0 \in T(\sigma_R \cup \pi_{\mathcal{L}\setminus R})$.

If $R = \emptyset$, then $\sigma_R \cup \pi_{\mathcal{L}\setminus R} = \pi_{\mathcal{L}\setminus R}$, so $0 \in T(\pi_{\mathcal{L}\setminus R}) = A(\emptyset; \pi_{\mathcal{L}\setminus R})$. This proves the base case for the empty residual set.

If $|R| = 1$, the unique partition of R is σ_R, thus the argument above gives $0 \in T(\sigma_R \cup \pi_{\mathcal{L}\setminus R})$. By the definition of the residual core for a singleton residual set,$(\sigma_R, 0, a^R) \in C(R; \pi_{\mathcal{L}\setminus R})$. Hence $0 \in A(R; \pi_{\mathcal{L}\setminus R})$. This proves the claim for singleton residual sets.

Now suppose $|R| \geq 2$ and assume the claim holds for all smaller residual sets. We showed above that $(\sigma_R, 0, a^R)$ is a well-defined residual outcome $(0 \in T(\sigma_R \cup \pi_{\mathcal{L}\setminus R}))$. We show that no candidate deviation dominates it.

Consider any nonempty deviating group $H' \subseteq R$ and any partition $\pi_{H'}$ that it forms. Since each family is a pair, there exists $m \in \{1, 2\}$ such that $H' \subseteq F_m \cap R$. We show that every possible form of such a deviation fails.

First, any deviation whose partition isolates the positive local g_m cannot dominate. If $\{g_m\} \in \pi_{H'}$, then for every equilibrium indicator $d' \in A(R \setminus H'; \pi_{H'} \cup \pi_{\mathcal{L}\setminus R})$, we have

$$w(\{g_m\}; d') \leq w(\{g_m\}; 0) = v^{\mathrm{sq}}_{g_m} + I_{g_m} = a^R(\{g_m\}).$$

Thus, the coalition $\{g_m\}$ in the deviating partition is not strictly better off for every equilibrium indicator in $A(R \setminus H'; \pi_{H'} \cup \pi_{\mathcal{L}\setminus R})$. Therefore, this deviation does not dominate the outcome.

Second, a deviation by an entire family as one coalition cannot dominate because the family is exactly indifferent between the incentive and deforestation. If $\pi_{H'} = \{F_m\}$, then $F_m \subseteq R$ and $\Delta(F_m) = 0$, hence

$$a^R(F_m) = v^{\mathrm{sq}}_{F_m} + I(F_m) = w(F_m; 0) = w(F_m; 1).$$

Therefore, for every equilibrium indicator $d' \in A(R \setminus F_m; \{F_m\} \cup \pi_{\mathcal{L}\setminus R})$, we have $a^R(F_m) = w(F_m; d')$. So this deviation also does not dominate the outcome.

Third, the only remaining possibility is a deviation by the negative singleton $H' = \{b_m\}$. In that case, the smaller residual game has residual set $R \setminus \{b_m\}$ and the partition of the other locals is $\{\{b_m\}\} \cup \pi_{\mathcal{L}\setminus R}$, which is again comprised solely of singletons. By the induction hypothesis, $0 \in A(R \setminus \{b_m\}; \{\{b_m\}\} \cup \pi_{\mathcal{L}\setminus R})$. But then $w(\{b_m\}; 0) = v^{\mathrm{sq}}_{b_m} + I_{b_m} = a^R(\{b_m\})$. Hence $\{b_m\}$ is not strictly better off for every equilibrium indicator in $A(R \setminus \{b_m\}; \{\{b_m\}\} \cup \pi_{\mathcal{L}\setminus R})$, so the negative singleton cannot dominate the outcome.

These three cases exhaust all feasible deviations. Therefore, no candidate deviation dominates $(\sigma_R, 0, a^R)$, so $(\sigma_R, 0, a^R) \in C(R; \pi_{\mathcal{L}\setminus R})$, and $0 \in A(R; \pi_{\mathcal{L}\setminus R})$. This completes the induction and proves the claim.

Applying Claim EC.4 with $R = \mathcal{L}$ and $\pi_{\mathcal{L}\setminus R} = \emptyset$ gives

$$(\sigma_{\mathcal{L}}, 0, a), \qquad a_\ell = v_\ell^{\text{sq}} + I_\ell \quad (\ell \in \mathcal{L}),$$

with $(\sigma_{\mathcal{L}}, 0, a) \in C(\mathcal{L}; \emptyset)$. Thus, the core may contain an outcome with $d^\star\!=\!0$ when $I(F) \le D(F)$ for every $F \in \bar{\pi}$ and $\eta \le \overline{\eta}$.

Part (iv). Consider the instance

$$\mathcal{L} = \{\ell, g, h, e\}, \qquad \bar{\pi} = \{F, \{e\}\}, \qquad F = \{\ell, g, h\},$$

with

$$\Delta_g > 0, \quad \Delta_\ell, \quad \Delta_h, \quad \Delta_e < 0, \quad \Delta_g + \Delta_\ell < 0, \quad \Delta_g + \Delta_h < 0, \quad \text{and} \quad \eta < \Delta_g/3.$$

Then $\Delta(F) = (\Delta_g + \Delta_\ell) + \Delta_h < 0$ and $\Delta(\{e\}) < 0$, so $I(F') < D(F')$ for both families. Note that g is the only local with $\Delta_g > 0$, and every coalition S with $g \in S \neq \{g\}$ prefers deforestation (since $\Delta_g + \Delta_\ell < 0$ and $\Delta_g + \Delta_h < 0$).

We first determine $T(\pi)$ for every partition π. If $\{g\} \notin \pi$, then no coalition of π has $\Delta(S) > 0$, so $\sum_{S\in\pi:\Delta(S)>0} \lfloor \Delta(S)/\eta \rfloor = 0$ while $\max_{H\in\pi:\Delta(H)\le 0} |H| \ge 1$, so $\eta_2(\pi) = -\infty$, and by Lemma 2 $T(\pi) = \{1\}$. If $\{g\} \in \pi$, then $\Delta(\{g\}) = \Delta_g > 3\eta = \eta|\mathcal{L} \setminus \{g\}|$, and all other coalitions $S' \neq \{g\}$ satisfy $\Delta(S') \le 0$, so $\eta < \eta_1(\pi) = \frac{\Delta_g}{3}$. So, by Lemma 2, $\{0\} = T(\pi)$. Thus, an outcome with $d^\star\!=\!0$ requires a partition containing $\{g\}$, and an outcome with $d^\star\!=\!1$ a partition not containing it. We show every outcome is dominated, so $C(\mathcal{L}; \emptyset) = \emptyset$.

outcomes with $d^\star\!=\!0$. Let $(\pi, 0, a)$ be an outcome with $d^\star\!=\!0$; its partition contains $\{g\}$. We will show that because $a(F) = v_F^{\text{sq}} + I(F)$, then F can deviate. Letting F deviate as the single coalition $\{F\}$ leaves as the residual $\{e\}$. The residual core then satisfies $A(\{e\}; \{\{F\}\}) = T(\{F, \{e\}\})$. But because $\{g\} \notin \{F, \{e\}\}$, then $T(\{F, \{e\}\}) = \{1\}$. So F secures $w(F; 1) = v_F^{\text{sq}} + D(F) > v_F^{\text{sq}} + I(F) = a(F)$ because $\Delta(F) < 0$; proving the outcome is dominated.

outcomes with $d^\star\!=\!1$. Let $(\pi, 1, a)$ be an outcome with $d^\star\!=\!1$; its partition does not contain $\{g\}$. We will show that either g would deviate and form partition $\{g\}$ or there is another local $r \in F \setminus \{g\}$ that would deviate and form partition $\{r\}$.

For g not to deviate, the allocation has to satisfy $a_g \ge v_g^{\text{sq}} + I_g > v_g^{\text{sq}} + D_g$. Because $\eta < \Delta_g/3 = \eta_1(\pi'_{\mathcal{L}\setminus\{g\}})$ for any partition $\pi_{\mathcal{L}\setminus\{g\}}$, then by Lemma 2, $T(\{g\} \cup \pi_{\mathcal{L}\setminus\{g\}}) = \{0\}$, which implies that $A(\mathcal{L} \setminus \{g\}; \{\{g\}\}) = \{0\}$ and g can secure by deviating $w(\{g\}; 0) = v_g^{\text{sq}} + I_g$; hence $a_g \ge v_g^{\text{sq}} + I_g > v_g^{\text{sq}} + D_g$.

We now show that the lower bound on a_g leaves another local in F who can profitably deviate. Since $a(F) = v_F^{\text{sq}} + D(F)$ and $a_g \geq v_g^{\text{sq}} + I_g$,

$$a_\ell + a_h = a(F) - a_g \leq v_\ell^{\text{sq}} + v_h^{\text{sq}} + D_\ell + D_h - \Delta_g < v_\ell^{\text{sq}} + v_h^{\text{sq}} + D_\ell + D_h,$$

the last inequality because $\Delta_g > 0$. Hence at least one of ℓ, h — call it r, and write r' for the other — receives

$$a_r < v_r^{\text{sq}} + D_r = w(\{r\}; 1).$$

Consider r deviating as the singleton $\{r\}$ and deforesting. Under pessimism, r obtains $\min_{d' \in A(\mathcal{L}\setminus\{r\};\{\{r\}\})} w(\{r\}; d')$, so it is enough to show that the residual admits only deforestation,

$$A\big(\mathcal{L} \setminus \{r\}; \{\{r\}\}\big) = \{1\}, \qquad \text{where } \mathcal{L} \setminus \{r\} = \{g, r', e\};$$

Then r secures $w(\{r\}; 1) = v_r^{\text{sq}} + D_r > a_r$, so the deviation dominates $(\pi, 1, a)$.

To establish $A(\{g, r', e\}; \{\{r\}\}) = \{1\}$, observe first that, exactly as for $\mathcal{L}$ above, a partition of $\{g, r', e\}$ admits $d^\star = 0$ iff it isolates $\{g\}$ (which blocks the three remaining locals r', e, r because $\eta < \Delta_g/3$), and admits $d^\star = 1$ otherwise. We treat the two outcome types in turn.

No residual outcome with $d^\star = 0$ survives. Any no-deforestation residual outcome uses the partition $\{\{g\}, \{r'\}, \{e\}\}$ and assigns the pair $\{g, r'\}$ the total $v_{\{g,r'\}}^{\text{sq}} + I(\{g, r'\})$. Letting $\{g, r'\}$ deviate as a single coalition leaves $A(\{e\}; \{\{g, r'\}, \{r\}\}) = T(\{\{e\}, \{g, r'\}, \{r\}\}) = \{1\}$ (by Lemma 2 and $\eta_2(\{\{e\}, \{g, r'\}, \{r\}\}) = -\infty$), so $\{g, r'\}$ secures $w(\{g, r'\}; 1) = v_{\{g,r'\}}^{\text{sq}} + D(\{g, r'\})$, which exceeds its no-deforestation total because $\Delta_g + \Delta_{r'} < 0$. Hence, the residual core $C(\mathcal{L} \setminus \{r\}; \{\{r\}\})$ does not contain any outcome with $d^\star = 0$.

A residual outcome with $d^\star = 1$ lies in the residual core $C(\mathcal{L} \setminus \{r\}; \{\{r\}\})$. Take the partition $\{\{g, r'\}, \{e\}\}$ with $d^\star = 1$ and the allocation

$$\hat{a}_g = v_g^{\text{sq}} + I_g, \qquad \hat{a}_{r'} = v_{r'}^{\text{sq}} + D_{r'} - \Delta_g, \qquad \hat{a}_e = v_e^{\text{sq}} + D_e,$$

which is feasible because $\hat{a}_g + \hat{a}_{r'} = v_{\{g,r'\}}^{\text{sq}} + D(\{g, r'\}) = w(\{g, r'\}; 1)$ and $\hat{a}_e = w(\{e\}; 1)$. No coalition improves on it:

- g leaving as a singleton secures at most $w(\{g\}; 0) = v_g^{\text{sq}} + I_g = \hat{a}_g$;
- r' would not deviate because $\{0\} = A(\{g, e\}; \{\{r\}, \{r'\}\})$ (the only partition of $\{g, e\}$ is $\{\{g\}, \{e\}\}$, and $A(\{g, e\}; \{\{r\}, \{r'\}\}) = T(\{\{g\}, \{e\}, \{r\}, \{r'\}\}) = \{0\}$ as shown above), and $w(\{r'\}; 0) = v_{r'}^{\text{sq}} + I_{r'} \leq \hat{a}_{r'}$. Where the last inequality holds because $D_{r'} - \Delta_g \geq I_{r'}$, i.e. $\Delta_g + \Delta_{r'} \leq 0$;
- e can secure at most $w(\{e\}; 1) = \hat{a}_e \geq w(\{e\}; 0)$ ($\Delta_e \leq 0$), and the pair $\{g, r'\}$ similarly can secure at most $w(\{g, r'\}; 1) = \hat{a}_g + \hat{a}_{r'} \geq w(\{g, r'\}; 0)$.

Thus, this outcome with $d^\star\!=\!1$ is undominated, so the residual core is nonempty.

The two cases give $A(\{g,r',e\};\{\{r\}\})=\{1\}$. Therefore r secures $w(\{r\};1)=v_r^{\mathrm{sq}}+D_r>a_r$, and $(\pi,1,a)$ is dominated.

As every outcome is dominated, $C(\mathcal{L};\emptyset)=\emptyset$.

Finally, notice that this exact same instance, but with $\mathcal{L}=F$ would also have an empty core (as the empty core proof did not rely on the existence of the family $\{e\}$). This shows that such an instance can arise with both full and limited cooperation.

Part (v). Consider the instance

$$\mathcal{L}=\{e,g,h\},\qquad \bar\pi=\{F_1,F_2\},\qquad F_1=\{e,g\},\quad F_2=\{h\},\qquad \Delta_e>0,\quad \Delta_e+\Delta_g<0,\quad \Delta_h<0$$

(for example, $\Delta_e=2$, $\Delta_g=-5$, $\Delta_h=-1$). Then $\Delta(F_1)=\Delta_e+\Delta_g<0$ and $\Delta(F_2)=\Delta_h<0$, so $I(F')<D(F')$ for both families, and $\overline{\eta}=\max\{\Delta(S):\Delta(S)>0\}=\Delta_e$, because $\{e\}$ is the only coalition within a family with $\Delta(S)>0$. Fix any $0<\eta\le\overline{\eta}$. We show that $C(\mathcal{L};\emptyset)$ is non-empty and contains *only* outcomes with $d^\star\!=\!1$.

The only two partitions in $\Pi_{\mathcal{L}}^{\prec\bar\pi}$ are $\pi_1=\{\{e,g\},\{h\}\}$ and $\pi_2=\{\{e\},\{g\},\{h\}\}$. By Lemma 2, $T(\pi_1)=\{1\}$ for every $\eta>0$: no coalition of π_1 has $\Delta>0$, so $0\notin T(\pi_1)$, while $\Delta(S)\le 0\le\eta|\mathcal{L}\setminus S|$ gives $1\in T(\pi_1)$; and $0\in T(\pi_2)$, because $\lfloor\Delta_e/\eta\rfloor\ge 1=\max_{H\in\pi_2:\Delta(H)\le 0}|H|$ holds if and only if $\eta\le\Delta_e=\overline{\eta}$. In particular, both T-sets are non-empty and, exactly as in the proof of Lemma EC.7,

$$A\big(R;\pi_{\mathcal{L}\setminus R}\big)=T\big(\{\{i\}:i\in R\}\cup\pi_{\mathcal{L}\setminus R}\big)\qquad\text{for every } R\subsetneq\mathcal{L}\text{ with } R\neq F_1\text{ and every }\pi_{\mathcal{L}\setminus R}\in\Pi_{\mathcal{L}\setminus R}^{\prec\bar\pi}: \tag{EC.56}$$

for $R=\emptyset$ and $|R|=1$ this holds by (5) and Definition 2, and for $R\in\{\{e,h\},\{g,h\}\}$ – whose locals belong to different families, so that the only residual partition is $\{\{i\}:i\in R\}$ and the resulting partition of $\mathcal{L}$ is π_2 – every residual outcome $(\{\{i\}:i\in R\},d^\star,a)$ is not dominated, because a deviating singleton $\{i\}\subseteq R$ faces $A(R\setminus\{i\};\{\{i\}\}\cup\pi_{\mathcal{L}\setminus R})=T(\pi_2)$ and $\min_{d'\in T(\pi_2)}w(\{i\};d')\le w(\{i\};d^\star)=a_i$.

No outcome with $d^\star\!=\!0$ is in the core. Because $T(\pi_1)=\{1\}$, any outcome with $d^\star\!=\!0$ is $(\pi_2,0,a)$ with $a_\ell=w(\{\ell\};0)$ for each local. Let F_1 deviate forming $\{F_1\}$: by (EC.56), $A(\{h\};\{F_1\})=T(\pi_1)=\{1\}$, so the deviation secures $w(F_1;1)>w(F_1;0)=a(F_1)$, where the inequality is $\Delta(F_1)<0$. Every outcome with $d^\star\!=\!0$ is therefore dominated (at every $\eta>0$, since only $T(\pi_1)=\{1\}$ and $\Delta(F_1)<0$ are used).

An outcome with $d^\star\!=\!1$ is in the core. Take $(\pi_1,1,a)$ with

$$a_e=v_e^{\mathrm{sq}}+I_e,\qquad a_g=v_g^{\mathrm{sq}}+D_g-\Delta_e,\qquad a_h=v_h^{\mathrm{sq}}+D_h,$$

feasible because $a_e+a_g=v_{F_1}^{\mathrm{sq}}+D(F_1)=w(F_1;1)$, $a_h=w(\{h\};1)$, and $1\in T(\pi_1)$. By Definition 2, a deviating group lies within one family, so $H\in\big\{\{e\},\{g\},F_1,\{h\}\big\}$; none is profitable:

- $H=\{h\}$: both $w(\{h\};1)=a_h$ and $w(\{h\};0)=a_h+\Delta_h<a_h$, so $\{h\}$ never strictly improves, under any indicator.

- $H = \{e\}$: by (EC.56), $A(\{g,h\};\{\{e\}\}) = T(\pi_2)$, which contains 0, and $w(\{e\};0) = v_e^{\mathrm{sq}} + I_e = a_e$ is not a strict improvement.
- $H = \{g\}$: by (EC.56), $A(\{e,h\};\{\{g\}\}) = T(\pi_2)$, which contains 0, and $w(\{g\};0) - a_g = \Delta_g + \Delta_e = \Delta(F_1) < 0$.
- $H = F_1$: with $\pi_H = \{F_1\}$, $A(\{h\};\{F_1\}) = T(\pi_1) = \{1\}$ and $w(F_1;1) = a(F_1)$ is not a strict improvement; with $\pi_H = \{\{e\},\{g\}\}$, $A(\{h\};\{\{e\},\{g\}\}) = T(\pi_2)$ contains 0, under which $\{e\}$ secures $w(\{e\};0) = a_e$ – again no strict improvement.

Hence $(\pi_1, 1, a) \in C(\mathcal{L};\emptyset)$: for every $0 < \eta \le \overline{\eta}$ the core is non-empty and every core outcome is an outcome with $d^\star = 1$, even though both families strictly prefer deforestation. Together with part (ii) (the case $\eta > \overline{\eta}$), such instances have a non-empty all-deforestation core at *every* blocking cost $\eta > 0$. □

Finally, the next lemma shows that if some family strictly prefers deforestation, the core contains no outcome with $d^\star = 0$ that compensates, whatever the blocking cost; it establishes the corresponding claim of Part A-ii.

LEMMA EC.14. *Consider the area no-use condition $\bar{\mathbb{U}}$ in the cooperative game with transferable utility defined in §3.3. If $I(F) < D(F)$ for some $F \in \bar{\pi}$, the core does not contain outcomes with $d^\star = 0$ that compensate.*

Proof of Lemma EC.14. Consider a family F with $I(F) < D(F)$. The allocation in any outcome with $d^\star = 0$ satisfies

$$\sum_{\ell \in F} a_\ell = \sum_{\ell \in F} (v_\ell^{\mathrm{sq}} + I_\ell) < \sum_{\ell \in F} (v_\ell^{\mathrm{sq}} + D_\ell).$$

Therefore, at least one local in F receives less than $v_\ell^{\mathrm{sq}} + D_\ell$, so the outcome does not compensate all locals. □

EC.6. Modeling Extensions

This section examines several important extensions of our base model.

EC.6.1. Minimum Targeted Cash Payments

Suppose that the incentive is a cash payment that the buyer can target to each local in the area, and the buyer seeks to meet the performance requirement while minimizing his costs. We discuss guidelines for choosing a condition $\mathbb{C}$ and targeting the payments to achieve this.

We first make a few remarks that allow formalizing the problem and avoiding complex terminology. For any local ℓ, the income gain with the incentive I_ℓ equals the monetary payment plus a constant offset. Minimizing the total payment is then equivalent to minimizing $I(\mathcal{L})$, and we refer to I as *payments* to locals. An optimal choice $(\mathbb{C}, I)$ can be found by restricting attention to the condition(s)

$\mathbb{C}$ from the robust recommendations in §3.4 and minimizing $I(\mathcal{L})$ over the corresponding set of constraints in Figure 3.2. We follow this approach, but we also discuss some alternative choices that are also optimal. We consider the scenarios of Figure 3.2, in which the buyer does not induce a preferred type of core outcome; the corresponding analysis for §4.1 is analogous and we omit it. Lastly, because the constraints on I from Figure 3.2 lead to open feasible sets, we derive asymptotically optimal solutions that infimize $I(\mathcal{L})$. For simplicity, we subsequently discuss these asymptotic solutions (and costs), but the implemented payments I_ℓ (and costs) should be interpreted as just above the recommended values.

Brief summary of findings. Note that the individual condition $\mathbb{I}$ and targeted payments D_ℓ to each local $\ell \in \mathcal{L}$ would prevent deforestation and compensate at a total cost approaching $D(\mathcal{L})$, regardless of the scenario. This requires a perfect incentive and highly precise targeting, but remains a useful baseline for the area conditions.

Whenever the buyer seeks **to also compensates** locals, and in scenarios with **full cooperation**, the baseline cost $D(\mathcal{L})$ is optimal, because transfers among locals redistribute value but cannot create it. Under full cooperation the area no-deforestation condition $\bar{\mathbb{D}}$ achieves it and remains the more flexible implementation: it is sufficient to provide an aggregate payment $D(\mathcal{L})$ to the entire community with arbitrary individual payments, because the grand coalition $\{\mathcal{L}\}$ can redistribute them. Under $\bar{\mathbb{D}}$ the buyer also gains nothing by seeking less, since every core outcome that prevents deforestation already compensates every local. In contrast, with limited cooperation and when the buyer cannot induce a preferred outcome, the area conditions cannot compensate while leaving a family unpaid, so targeted individual payments D_ℓ with the individual condition $\mathbb{I}$ are *the only* optimal choice.

To **prevent deforestation with limited cooperation**, cost savings relative to the baseline $D(\mathcal{L})$ are possible and must come from applying the area no-use condition $\bar{\mathbb{U}}$. Such savings arise only if the blocking cost η is sufficiently small – below a threshold that compares η with the average value from deforestation of the locals who would be left unpaid – and we subsequently focus on such cases.

Because every core outcome must have $d^\star = 0$, Figure 3.2 implies that payments under $\bar{\mathbb{U}}$ must satisfy both $\eta_2(I) \geq \eta$, which keeps the core non-empty, and $\underline{\eta}_1(I) > \eta$, which removes every core outcome with $d^\star = 1$. The latter requires paying one family F at least $D(F)$, so that it prefers the incentive, and paying some coalition $S \subseteq F$ at least $D(S) + \eta(|\mathcal{L}| - |S|)$, so that it can credibly block all locals outside S. Adding a local to S replaces the cost η of blocking him with his value from deforestation D_ℓ, so the buyer pays exactly the members of F with $D_\ell < \eta$ and blocks the others; if no member of F is less costly to pay than to block, he must still pay one, and pays the least expensive.

The former requirement determines which *families* are paid. Every family that is not paid receives nothing at all, rather than a reduced payment, and every member of a paid family other than F

receives exactly D_ℓ, except that a single family may receive the additional value that finances the joint blocking. A family other than F is worth paying only if leaving it unpaid would raise the blocking burden, which requires it to be larger than every family left unpaid, and only if buying it off costs less than financing that burden, which requires its average value from deforestation to be below the blocking cost, $D(F')/|F'| < \eta$. Small families, and families whose members value deforestation highly, are therefore the ones left out. Paying a second family can be strictly cheaper than paying one family alone, and when all families have the same size, or when $D_\ell \geq \eta$ for every local, paying a single local – the member with the smallest D_ℓ in one family – is optimal.

These results highlight that any cost savings from preventing deforestation with $\bar{\mathbb{U}}$ may inherently conflict with distributive justice. A least-cost implementation with $\bar{\mathbb{U}}$ would require the buyer to pay locals and families that are inexpensive or strategically useful for creating blocking threats, while paying nothing to all other locals. The design may therefore concentrate payments on locals or families with the lowest opportunity costs/values from deforestation, or pay a large family because it would be costly to block, rather than directing payments toward those who lose the most from foregoing deforestation (who are often also the poorest locals). Requiring compensation eliminates this selectivity and restores the baseline cost $D(\mathcal{L})$.

Formal Analysis. We next formalize the results that support the recommendations above. We maintain the normalization that one unit of value delivered to a local costs the buyer one unit, so that the buyer's total payment is $I(\mathcal{L})$, and the base-model assumptions $D_\ell > 0$ and $I_\ell \neq D_\ell$ for every $\ell \in \mathcal{L}$. We write

$$\mathcal{I} := \left\{ I \in \mathbb{R}_+^{\mathcal{L}} : I_\ell \neq D_\ell \text{ for every } \ell \in \mathcal{L} \right\},$$

and, as in §3.4, we deem a conditional incentive admissible only when the relevant theorem states that the core is non-empty and *all* core outcomes meet the performance requirement. Every value below is an infimum: the requirements in Figure 3.2 are strict inequalities, so the payments that implement them should be read as slightly above the stated amounts.

Our first result shows that the compensation requirement fixes the buyer's cost at $D(\mathcal{L})$, whatever the condition and whatever the scenario, and that under $\bar{\mathbb{D}}$ the same cost is already incurred to prevent deforestation alone.

Proposition 1 (Preventing deforestation with compensation). Consider any family partition $\bar{\pi}$ and any condition $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$.

(i) If $(I, \mathbb{C})$ prevents deforestation and compensates, then $I(\mathcal{L}) \geq D(\mathcal{L})$, and this bound is approached under $\mathbb{I}$ by $I_\ell = D_\ell + \varepsilon/|\mathcal{L}|$ for every $\ell \in \mathcal{L}$.

(ii) Under $\bar{\mathbb{D}}$, every core outcome with $d^\star = 0$ compensates. Wherever $\bar{\mathbb{D}}$ is admissible, preventing deforestation therefore costs $D(\mathcal{L})$ as well; under full cooperation this cost is approached by paying a single local $D(\mathcal{L}) + \varepsilon$.

(iii) Under $\bar{\mathbb{U}}$ and limited cooperation, compensation fails outright whenever one family prefers the incentive and another satisfies $I(F) < D(F)$: no core outcome with $d^\star = 0$ then compensates. The buyer cannot compensate while leaving a family unpaid.

Proof. For part (i), let $(\pi, 0, a)$ be a core outcome that compensates every local. Because no local deforests, every local receives the incentive under each of $\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}$, so feasibility gives $\sum_{\ell\in\mathcal{L}} a_\ell = v^{\mathrm{sq}}(\mathcal{L}) + I(\mathcal{L})$, while compensation gives $\sum_{\ell\in\mathcal{L}} a_\ell \geq v^{\mathrm{sq}}(\mathcal{L}) + D(\mathcal{L})$. The stated perfect incentive costs $D(\mathcal{L}) + \varepsilon$ and meets both requirements under $\mathbb{I}$.

Part (ii) is Theorem 1-A: under $\bar{\mathbb{D}}$, an uncompensated local would deviate, deforest, and earn strictly more, so no core outcome with $d^\star = 0$ can leave a local uncompensated. Preventing deforestation therefore implies compensating, and part (i) applies. For the upper bound, fix $\ell_0 \in \mathcal{L}$, set $I_{\ell_0} = D(\mathcal{L}) + \varepsilon$ and $I_\ell = 0$ for $\ell \neq \ell_0$, and apply Theorem 1-(A,B): all core outcomes then have $d^\star = 0$ and compensate every local.

Part (iii) is Theorem 2-ii. □

The compensation bound is independent of the family structure and of the blocking cost: transfers among locals redistribute value but cannot create it. Under $\bar{\mathbb{D}}$ the buyer gains nothing by seeking less, since preventing deforestation already compensates every local; and under $\bar{\mathbb{U}}$ he cannot compensate while leaving a family unpaid. Only when he merely prevents deforestation, under limited cooperation and without inducing a preferred type of core outcome, can he pay less than $D(\mathcal{L})$ – and only under $\bar{\mathbb{U}}$, by substituting a credible threat of blocking for compensation. We study that case for the rest of this section, and we write

$$\Delta_\ell := I_\ell - D_\ell, \qquad \Delta(S) := \sum_{\ell\in S} \Delta_\ell,$$

so that $I(\mathcal{L}) = D(\mathcal{L}) + \Delta(\mathcal{L})$ and $I_\ell \geq 0$ reads $\Delta_\ell \geq -D_\ell$. For $I \in \mathcal{I}$, let

$$\mathcal{F}(I) := \{F \in \bar{\pi} : \Delta(F) > 0\}, \qquad m(\mathcal{F}) := \max_{H\in\bar{\pi}\setminus\mathcal{F}} |H|,$$

with $m(\bar{\pi}) = 0$; thus $\mathcal{F}(I)$ collects the families that prefer the incentive in aggregate, and $m(\mathcal{F})$ is the size of the largest family that does not. By Figure 3.2, the buyer must choose I so that all core outcomes under $\bar{\mathbb{U}}$ have $d^\star = 0$, which by Theorem 2 requires both $\eta \leq \eta_2(I)$, which keeps the core non-empty, and $\eta < \underline{\eta}_1(I)$, which removes every core outcome with deforestation. In the Δ notation, $\eta \leq \eta_2(I)$ states that for every $F' \in \bar{\pi}$ and every $H \subseteq F'$ with $\Delta(H) \leq 0$,

$$\sum_{G\in\bar{\pi}:\, \Delta(G\setminus H)>0} \left\lfloor \frac{\Delta(G\setminus H)}{\eta} \right\rfloor \geq |H|, \tag{EC.57}$$

and $\eta < \underline{\eta}_1(I)$ states that some $F \in \mathcal{F}(I)$ and some $\emptyset \neq S \subseteq F$ satisfy $\Delta(S) > \eta(|\mathcal{L}| - |S|)$. Applying (EC.57) to $H = X$ for a family $X \in \bar{\pi} \setminus \mathcal{F}(I)$, and noting that $X \setminus X = \emptyset$ so that X contributes nothing to its own blocking, gives

$$\sum_{G\in\mathcal{F}(I)} \left\lfloor \frac{\Delta(G)}{\eta} \right\rfloor \geq |X|. \tag{EC.58}$$

The first requirement identifies which locals of a family the buyer pays.

Proposition 2 (The blocking coalition). Suppose $\bar{\pi} \neq \{\mathcal{L}\}$, the buyer does not induce a preferred type of core outcome, and $I \in \mathcal{I}$ meets both requirements under $\bar{\mathbb{U}}$. Then some family F and some $\emptyset \neq S \subseteq F$ satisfy

$$I(F) > D(F) \qquad \text{and} \qquad I(S) > D(S) + \eta\big(|\mathcal{L}| - |S|\big).$$

Moreover

$$D(S) + \eta\big(|\mathcal{L}| - |S|\big) = \eta|\mathcal{L}| + \sum_{\ell \in S}\big(D_\ell - \eta\big), \tag{EC.59}$$

so among the coalitions of a given family F this requirement is weakest at

$$S^*(F) = \{\ell \in F : D_\ell < \eta\},$$

to which any subset of the locals with $D_\ell = \eta$ may be added; if no local of F has $D_\ell < \eta$, it is weakest at a singleton containing a local with the smallest D_ℓ in F.

Proof. The displayed inequalities restate $\eta < \underline{\eta}_1(I)$ and the side condition $\Delta(F) > 0$ in (9). Identity (EC.59) is immediate. Each local ℓ contributes $D_\ell - \eta$ to its right-hand side, so the sum is smallest when S contains exactly the locals with $D_\ell < \eta$, locals with $D_\ell = \eta$ being immaterial. Since $\underline{\eta}_1$ requires $S \neq \emptyset$, when every local of F has $D_\ell > \eta$ the smallest sum over non-empty coalitions is attained by a singleton containing a local with the smallest D_ℓ. $\square$

Adding a local to S replaces the cost η of blocking him with his value from deforestation D_ℓ, so the buyer pays exactly those members of F who are less costly to pay than to block – unless no member is, in which case he must still pay one, and pays the least expensive. The second requirement determines which *families* are paid.

Proposition 3 (Structure of least-cost designs). Suppose $\bar{\pi} \neq \{\mathcal{L}\}$, the buyer does not induce a preferred type of core outcome, and $I \in \mathcal{I}$ meets both requirements under $\bar{\mathbb{U}}$. Let $\mathcal{F} := \mathcal{F}(I)$ and let $F \in \mathcal{F}$ and $\emptyset \neq S \subseteq F$ be as in Proposition 2. Then, for every $\varepsilon > 0$, there is an incentive $I' \in \mathcal{I}$ meeting both requirements with the same $\mathcal{F}$, F and S, and with $I(\mathcal{L})' \leq I(\mathcal{L}) + \varepsilon$, such that:

(i) every local of every family $H \in \bar{\pi} \setminus \mathcal{F}$ receives nothing, $I'_\ell = 0$;

(ii) every local of every family in $\mathcal{F} \setminus \{F\}$ receives more than his value from deforestation, so only F pays a member less than D_ℓ;

(iii) at most one family in $\mathcal{F} \setminus \{F\}$ has $\Delta'(F') \geq \eta$; every other family in $\mathcal{F} \setminus \{F\}$ contributes nothing to the left-hand side of (EC.57).

Moreover, if $F' \in \mathcal{F} \setminus \{F\}$ contributes nothing to (EC.57) and satisfies

$$D(F') > \eta\big(|F'| - m(\mathcal{F})\big), \tag{EC.60}$$

then withdrawing its payments, and adding $\eta\max\{|F'|-m(\mathcal{F}),0\}$ to the payment of any one local of F, meets both requirements at strictly lower cost. Consequently, at a least-cost design every such family satisfies

$$D(F')\le\eta\big(|F'|-m(\mathcal{F})\big), \tag{EC.61}$$

so it is strictly larger than every family that receives nothing and, whenever some family does, $D(F')/|F'|<\eta$. At least one family receives nothing whenever the cost falls below $D(\mathcal{L})$, because $\mathcal{F}=\bar\pi$ would give $I(\mathcal{L})\ge\sum_{G\in\bar\pi}I(G)>D(\mathcal{L})$.

Proof. For part (i), fix $X\in\bar\pi\setminus\mathcal{F}$ and set $I'_\ell=0$ for $\ell\in X$, which lowers the cost by $I(X)\ge 0$ and leaves $\mathcal{F}$, F and S unchanged. Consider any H admissible in (EC.57). If $H\cap X=\emptyset$, every term with $G\ne X$ is unchanged, while the term for $G=X$ is absent both before and after because $\Delta(X)\le 0$ and $\Delta'(X)\le 0$. If $H\subseteq X$, then $|H|\le|X|$ and the families of $\mathcal{F}$, all disjoint from H, already supply $|X|$ units by (EC.58). Iterating over $\bar\pi\setminus\mathcal{F}$ gives the claim.

For part (ii), fix $F'\in\mathcal{F}\setminus\{F\}$, let $\sigma:=\Delta(F')>0$, and replace the payments in F' by $I'_\ell=D_\ell+\sigma/|F'|$. The aggregate $I(F')$, and hence the cost, is unchanged; no coalition inside F' is admissible in (EC.57) any longer; and for every admissible H, now disjoint from F', the term for $G=F'$ equals $\lfloor\sigma/\eta\rfloor$ as before. The pair (F,S) lies in a different family and is unaffected.

For part (iii), let $F'_0,\dots,F'_k$ be the families of $\mathcal{F}\setminus\{F\}$, with surpluses $\sigma_0,\dots,\sigma_k>0$, each paid as in part (ii). Give F'_0 the aggregate surplus $\sigma_0+\cdots+\sigma_k$ and give each F'_j, $j\ge 1$, the surplus $\min\{\eta,\varepsilon\}/2^j$, in each case split equally among its members on top of their values from deforestation. The cost rises by less than ε; every member of every such family still receives more than his value from deforestation; $\lfloor\Delta'(F'_j)/\eta\rfloor=0$ for $j\ge 1$; and since $\lfloor(\sigma_0+\cdots+\sigma_k)/\eta\rfloor\ge\sum_j\lfloor\sigma_j/\eta\rfloor$, the left-hand side of (EC.57) does not fall for any admissible H, all of which are disjoint from these families.

For the final claim, let F' be as stated, write $k:=\max\{|F'|-m(\mathcal{F}),0\}$, withdraw the payments to F', and add ηk to the payment of a local $\ell^*\in F$. Deleting F' removes nothing from the left-hand side of (EC.57), because F' contributed nothing. Adding ηk to ℓ^* can only help: if $\ell^*\notin H$ it raises $\Delta(F\setminus H)$ by ηk, and hence the term for $G=F$ by at least k; if $\ell^*\in H$, then $H\subseteq F$, so $\Delta(F\setminus H)$ is unchanged while $\Delta(H)$ rises, and H either remains admissible with its requirement still met or ceases to be admissible. Every $H\subseteq F'$ is newly admissible, but $|H|\le|F'|\le m(\mathcal{F})+k$, and the families of $\mathcal{F}\setminus\{F'\}$, all disjoint from H, supply at least $m(\mathcal{F})+k$ units by (EC.58) together with the k added ones. Neither $\Delta(S)$ nor $\Delta(F)$ falls, so (F,S) still witnesses the first requirement. The cost changes by $\eta k-I(F')<\eta k-D(F')<0$ by (EC.60). Finally, (EC.61) and $D(F')>0$ give $|F'|>m(\mathcal{F})$, and dividing by $|F'|$ gives $D(F')/|F'|\le\eta(1-m(\mathcal{F})/|F'|)$, which is below η when $m(\mathcal{F})\ge 1$. □

A family other than F is therefore worth paying only if leaving it unpaid would raise the blocking burden – which requires it to be larger than every family left unpaid – and only if buying it off costs less than financing that burden. Condition (EC.61) is the family-level counterpart of the rule governing S in Proposition 2: a local is paid when $D_\ell < \eta$, and a family is paid only when $D(F')/|F'| < \eta$. In both cases the buyer pays whoever is less costly to pay than to block. The next instance shows that paying such a second family can be strictly cheaper than paying one family alone.

Instance 1. Let $\eta = 1$, $|\mathcal{L}| = 9$, and $\bar{\pi} = \{F, G, \{c\}\}$, where $F = \{\ell_1, \dots, \ell_5\}$, $G = \{g_1, g_2, g_3\}$,

$$D_{\ell_j} = \tfrac{1}{20}\ (j \le 4), \qquad D_{\ell_5} = \tfrac{7}{2}, \qquad D_{g_i} = \tfrac{1}{10}\ (i \le 3), \qquad D_c = 40.$$

Some incentive that pays members of both F and G meets both requirements at a cost approaching $\frac{11}{2}$, whereas every incentive that meets both requirements and pays the members of a single family costs more than $\frac{63}{10}$.

Proof. For the first claim, fix small $\delta, \varepsilon > 0$ and set $\Delta_{\ell_1} = 4 + \varepsilon$, $\Delta_{\ell_2} = \Delta_{\ell_3} = \delta$, $\Delta_{\ell_4} = 1$, $\Delta_{\ell_5} = -\frac{7}{2}$, $\Delta_{g_i} = \delta$ for every i, and $\Delta_c = -40$; thus every member of G is paid just above his value from deforestation and c is paid nothing. The cost is $\frac{11}{2} + \varepsilon + 5\delta$. The coalition $S = \{\ell_1, \dots, \ell_4\}$ has $\Delta(S) = 5 + \varepsilon + 2\delta > 5 = \eta(|\mathcal{L}| - |S|)$ and $\Delta(F) = \frac{3}{2} + \varepsilon + 2\delta > 0$, so the first requirement holds. For (EC.57), the admissible coalitions are $\{c\}$ and the subsets of F that contain ℓ_5 but not ℓ_1, since any $H \ni \ell_1$ has $\Delta(H) \ge 4 + \varepsilon - \frac{7}{2} > 0$. Against $\{c\}$, the family F supplies $\lfloor \Delta(F) \rfloor = 1$ unit; against $H \subseteq \{\ell_2, \dots, \ell_5\}$ with $\ell_5 \in H$, it supplies $\lfloor \Delta(F \setminus H) \rfloor \ge \lfloor \Delta_{\ell_1} \rfloor = 4 \ge |H|$ units. Letting $\delta, \varepsilon \downarrow 0$ gives the stated cost.

For the second claim, consider an incentive meeting both requirements whose payments lie in a single family $\hat{F}$, so $\Delta_\ell = -D_\ell$ outside $\hat{F}$. If $\hat{F} = \{c\}$, the only candidate coalition in Proposition 2 is $S = \{c\}$, so $\Delta_c > 8\eta$ and the cost exceeds 48. If $\hat{F} = G$, some $\emptyset \ne S' \subseteq G$ has $\Delta(S') > \eta(9 - |S'|)$, so by (EC.59) the cost is at least $I(S') > D(S') + \eta(9 - |S'|) \ge \frac{63}{10}$. If $\hat{F} = F$, then $H = G$ is admissible in (EC.57) and only F can supply blocking capacity, so $\lfloor \Delta(F)/\eta \rfloor \ge 3$ and the cost is at least $D(F) + 3\eta = \frac{67}{10}$. $\square$

In this instance G is paid because, at size three, it is too large for F to block, whereas the singleton $\{c\}$ is blocked rather than paid even though its member has by far the largest value from deforestation. In two cases the trade-off admits an exact solution, and the buyer optimally concentrates the entire payment on a single local. Write $(x)^+ := \max\{x, 0\}$.

Proposition 4 (Two exactly solvable cases). Suppose $\bar{\pi} \ne \{\mathcal{L}\}$ and the buyer does not induce a preferred type of core outcome. Assume that either

(a) every family has the same size, or

(b) $D_\ell \ge \eta$ for every $\ell \in \mathcal{L}$.

For $F \in \bar{\pi}$, let $\ell_F \in F$ attain $d_F := \min_{\ell \in F} D_\ell$, and define

$$Q_F := \sum_{\ell \in F} \big(\eta - D_\ell\big)^+, \qquad m_F := \max_{F' \in \bar{\pi} \setminus \{F\}} |F'|,$$

$$c(F) := \max\Big\{\max\{d_F,\eta\} + \eta\big(|\mathcal{L}|-1\big) - Q_F,\ D(F) + \eta\, m_F\Big\}. \tag{EC.62}$$

Then the infimal cost of preventing deforestation is $\min\big\{D(\mathcal{L}),\ \min_{F\in\bar\pi} c(F)\big\}$. For a minimizing family F, the second term is approached under $\bar{\mathbb{U}}$ by paying the single local ℓ_F an amount just above $c(F)$ and paying every other local nothing.

Proof. *Upper bound.* Fix F, write $n := |\mathcal{L}|$, $R := F\setminus\{\ell_F\}$ and $P := \sum_{\ell\in R}(\eta - D_\ell)^+ = Q_F - (\eta - d_F)^+$, and for $t>0$ let $I_{\ell_F} = D_{\ell_F} + t$ and $I_\ell = 0$ otherwise, at cost $D_{\ell_F} + t$. By Proposition 2, a candidate coalition must contain ℓ_F, the only local with $\Delta_\ell > 0$; writing it as $\{\ell_F\}\cup T$ with $T\subseteq R$, the first requirement reads $t > \eta(n-1) - \sum_{\ell\in T}(\eta - D_\ell)$, weakest at $T = \{\ell\in R : D_\ell < \eta\}$, giving $t > \eta(n-1) - P$; and $\Delta(F) > 0$ reads $t > D(R)$. In (EC.57), no admissible H contains ℓ_F once $t > D(R)$, since then $\Delta(H) \ge t - D(R) > 0$. If $H\cap F = \emptyset$, only F supplies capacity and $F\setminus H = F$, so the requirement is $t \ge D(R) + \eta|H|$, most demanding at a largest family other than F: $t \ge D(R) + \eta m_F$. If $\emptyset \ne H \subseteq R$, it is $t \ge D(R) + \sum_{\ell\in H}(\eta - D_\ell)$, most demanding at $t \ge D(R) + P$. Under (a), $P < \eta(|F|-1) < \eta m_F$ because every term of P is below η and $m_F = |F|$; under (b), $P = 0 \le \eta m_F$. So both requirements hold for every $t > \max\{\eta(n-1) - P,\ D(R) + \eta m_F\}$, and the cost decreases to $c(F)$, using $D_{\ell_F} + (\eta - d_F)^+ = \max\{d_F,\eta\}$ for the first term and $D_{\ell_F} + D(R) = D(F)$ for the second. Together with $\mathbb{I}$, which approaches $D(\mathcal{L})$ by Proposition 1-(i), this gives the upper bound.

Lower bound. Let I meet both requirements under $\bar{\mathbb{U}}$, with $\mathcal{F} = \mathcal{F}(I)$, F and S as in Proposition 2. By (EC.59),

$$D(S) + \eta(n - |S|) = \eta n + \sum_{\ell\in S}\big(D_\ell - \eta\big) \ge \eta n + \sum_{\ell\in S}\big(D_\ell - \eta\big)^+ - Q_F.$$

If $d_F \le \eta$ the right-hand side is at least $\eta n - Q_F = \max\{d_F,\eta\} + \eta(n-1) - Q_F$; if $d_F > \eta$ then $Q_F = 0$ and, as $S \ne \emptyset$, it is at least $d_F + \eta(n-1)$. Hence $I(\mathcal{L}) \ge I(S) > \max\{d_F,\eta\} + \eta(n-1) - Q_F$. Next, if $\mathcal{F} = \bar\pi$ then $I(\mathcal{L}) > D(\mathcal{L})$; otherwise (EC.58), applied to a largest family of $\bar\pi\setminus\mathcal{F}$, gives $\sum_{G\in\mathcal{F}}\Delta(G) \ge \eta\, m(\mathcal{F})$ and hence $I(\mathcal{L}) \ge \sum_{G\in\mathcal{F}} I(G) \ge D(F) + \eta\, m(\mathcal{F})$. Under (a), $m(\mathcal{F}) = m_F$ whenever $\mathcal{F} \ne \bar\pi$. Under (b), let $\hat F \ne F$ attain m_F: if $\hat F \notin \mathcal{F}$ then $m(\mathcal{F}) \ge m_F$; and if $\hat F \in \mathcal{F}$ then $I(\mathcal{L}) \ge I(F) + I(\hat F) > D(F) + D(\hat F) \ge D(F) + \eta m_F$, using $D_\ell \ge \eta$ on $\hat F$. In every case $I(\mathcal{L}) \ge \min\{D(\mathcal{L}), c(F)\}$, and $\mathbb{I}$ costs at least $D(\mathcal{L})$. □

The first term of (EC.62) is the cost of enabling one local to block every other local, and it selects a local with the smallest value from deforestation; the second is the cost of making his family prefer the incentive and of financing a blocking threat against the largest other family. Neither hypothesis can be dropped: in Instance 1, which violates both, the buyer strictly prefers to pay a second family.

Finally, the buyer turns to $\bar{\mathbb{U}}$ only when blocking is inexpensive enough to substitute for compensation. Let $K_{\bar{\mathbb{U}}}(\eta)$ denote the infimal cost of preventing deforestation under $\bar{\mathbb{U}}$ at blocking cost η, and recall from Proposition 1-(i) that $\mathbb{I}$ approaches $D(\mathcal{L})$.

Proposition 5 (Choosing between $\mathbb{I}$ and $\bar{\mathbb{U}}$). Suppose $\bar{\pi} \neq \{\mathcal{L}\}$ and the buyer does not induce a preferred type of core outcome.

(i) $K_{\bar{\mathbb{U}}}(\eta)$ is nondecreasing in η. Hence, writing

$$\eta^{\mathrm{sw}} := \sup\{\eta > 0 : K_{\bar{\mathbb{U}}}(\eta) < D(\mathcal{L})\},$$

the area no-use condition is strictly cheaper than the individual condition for every $\eta < \eta^{\mathrm{sw}}$, and the individual condition is weakly cheaper for every $\eta > \eta^{\mathrm{sw}}$.

(ii) The threshold satisfies $\underline{\eta}^{\mathrm{sw}} \leq \eta^{\mathrm{sw}} \leq \overline{\eta}^{\mathrm{sw}}$, where

$$\underline{\eta}^{\mathrm{sw}} := \max_{F \in \bar{\pi}} \frac{D(\mathcal{L} \setminus F)}{|\mathcal{L} \setminus F|}, \qquad \overline{\eta}^{\mathrm{sw}} := \max_{F \in \bar{\pi},\, \emptyset \neq S \subseteq F} \frac{D(\mathcal{L}) - D(S)}{|\mathcal{L}| - |S|}.$$

Proof. For part (i), both requirements are of the form "η small enough": if I meets $\eta \leq \eta_2(I)$ and $\eta < \underline{\eta}_1(I)$, it meets them at every smaller blocking cost. Hence $K_{\bar{\mathbb{U}}}$ is nondecreasing and the comparison with the constant $D(\mathcal{L})$ switches once.

For the lower bound in part (ii), fix $F \in \bar{\pi}$, put $k := |\mathcal{L} \setminus F|$, choose $\ell_0 \in F$, and pay $I_{\ell_0} = D_{\ell_0} + \eta k + \varepsilon$, $I_\ell = D_\ell + \varepsilon$ for $\ell \in F \setminus \{\ell_0\}$, and $I_\ell = 0$ outside F. Then $\Delta(F) = \eta k + |F|\varepsilon$, so $S = F$ satisfies $\Delta(F) > \eta(|\mathcal{L}| - |F|)$ and the first requirement holds. Every member of F has $\Delta_\ell > 0$, so every coalition admissible in (EC.57) lies outside F and has at most k members, while F supplies $\lfloor \Delta(F)/\eta \rfloor \geq k$ units; the second requirement holds too. The cost decreases to $D(F) + \eta k$, which is below $D(\mathcal{L})$ exactly when $\eta < D(\mathcal{L} \setminus F)/k$.

For the upper bound, let I meet both requirements under $\bar{\mathbb{U}}$ and let F and S be as in Proposition 2. Then $I(\mathcal{L}) \geq I(S) > D(S) + \eta(|\mathcal{L}| - |S|)$, so costing less than $D(\mathcal{L})$ requires $\eta < (D(\mathcal{L}) - D(S))/(|\mathcal{L}| - |S|)$. □

Savings under $\bar{\mathbb{U}}$ are therefore available only when the blocking cost is small relative to the value the excluded locals forgo: $\underline{\eta}^{\mathrm{sw}}$ compares η with the average value from deforestation of the locals left outside a single family, and $\overline{\eta}^{\mathrm{sw}}$ with the average outside the coalition that carries the blocking threat.

EC.6.2. Deforestation By Logging and Blocking Deforestation Attempts.

Whereas our base model assumes that locals cannot prevent other locals from deforesting land, blocking deforestation attempts may be plausible when forest is cleared through logging rather than fire. Logging generally requires observable preparation and the movement of heavy equipment (e.g., feller-bunchers, harvesters, and trucks) before clearing occurs. Locals may therefore be able to prevent clearing by blocking access roads, disabling equipment, reporting illegal activity, or physically protecting trees (Donald 2021, Evans 2013, Burton 2004). We extend the model to capture this distinction by allowing locals to block deforestation attempts and – if forest clearing is successful – to subsequently block the associated economic use. We next formalize the main modeling changes.

Deforestation Attempts. The locals within each coalition $S \in \pi$ jointly decide whether each local $\ell \in S$ should *attempt to* deforest land to potentially generate income ($d_\ell = 1$, with cost c_ℓ) or leave the forest intact ($d_\ell = 0$). The decision $d_\ell = 1$ can be thought of as bringing heavy equipment to the area, and c_ℓ captures the cost of making that attempt. The notation d_S and d remains the same as in the base model, with the understanding that it refers to attempts to deforest.

Blocking Deforestation Attempts. Having observed all deforestation attempts d, the locals in each coalition jointly decide whether to block the clearing of forest. Blocking one local ℓ from clearing forest incurs a cost $\eta_{\bar{\mathbb{D}}} \geq 0$, and blocking is successful if done. Mirroring the notation in the base model, we represent these *deforestation-blocking* decisions for all locals in coalition S with the matrix $B_S^{\bar{\mathbb{D}}}(d)$ and for all locals in $\mathcal{L}$ with the matrix $B^{\bar{\mathbb{D}}}(d)$. In both matrices, $B_{\ell i}^{\bar{\mathbb{D}}} = 1$ if and only if ℓ blocks i, and $\max_{\ell \in \mathcal{L}} B_{\ell i}^{\bar{\mathbb{D}}} = 1$ indicates that i is blocked from deforesting land. Coalition $S \in \pi$ thus incurs a total blocking cost of $\eta_{\bar{\mathbb{D}}} \cdot \sum_{\ell \in S, i \in \mathcal{L}} B_{\ell i}^{\bar{\mathbb{D}}}$, shared among the locals in the coalition.

Blocking Use. Having observed all deforestation attempts d and all deforestation-blocking decisions $B^{\bar{\mathbb{D}}}$, the locals in each coalition then jointly decide whether to block each local $\ell \in \mathcal{L}$ from using deforested land to generate income, at a cost $\eta_{\bar{\mathbb{U}}}$ per local blocked. Because an action that prevents deforesting land also prevents the associated use, whereas use may be blocked through less costly actions (like those discussed in the paper's introduction), we assume $0 < \eta_{\bar{\mathbb{U}}} < \eta_{\bar{\mathbb{D}}}$. Blocking use is also successful if done. We use the matrices $B_S^{\bar{\mathbb{U}}}(d, B^{\bar{\mathbb{D}}})$ and $B^{\bar{\mathbb{U}}}(d, B^{\bar{\mathbb{D}}})$ to denote the *use-blocking* decisions of coalition S and of all locals, respectively. Use-blocking decisions depend on d and $B^{\bar{\mathbb{D}}}$ because they occur after observing deforestation attempts and deforestation-blocking decisions. As in the base model, both kinds of blocking are directed at locals in other coalitions: $B_{i\ell}^{\bar{\mathbb{D}}} = B_{i\ell}^{\bar{\mathbb{U}}} = 0$ whenever i and ℓ belong to the same coalition.

Conditions. The individual condition $\mathbb{I}$ is unchanged; the two area conditions can be written as:

1. *Area No-Deforestation Condition,* $\bar{\mathbb{D}}$. Each local $\ell \in \mathcal{L}$ receives the incentive if and only if no local deforests land:

$$\bar{\mathbb{D}}_\ell(d, B^{\bar{\mathbb{D}}}, B^{\bar{\mathbb{U}}}) = \text{yes} \;\Leftrightarrow\; d_i \cdot \big(1 - \max_{g \in \mathcal{L}} B_{gi}^{\bar{\mathbb{D}}}\big) = 0, \quad \forall i \in \mathcal{L}.$$

2. *Area No-Use Condition,* $\bar{\mathbb{U}}$. Each local $\ell \in \mathcal{L}$ receives the incentive if and only if no local uses land that he deforested to generate income:

$$\bar{\mathbb{U}}_\ell(d, B^{\bar{\mathbb{D}}}, B^{\bar{\mathbb{U}}}) = \text{yes} \;\Leftrightarrow\; d_i \cdot \big(1 - \max_{g \in \mathcal{L}} B_{gi}^{\bar{\mathbb{D}}}\big) \cdot \big(1 - \max_{g \in \mathcal{L}} B_{gi}^{\bar{\mathbb{U}}}\big) = 0, \quad \forall i \in \mathcal{L}.$$

The area no-deforestation condition $\bar{\mathbb{D}}$ requires that any deforestation attempt should be blocked. The area no-use condition $\bar{\mathbb{U}}$ is again more flexible, requiring that any local with a deforestation attempt should be blocked either from clearing forest or from generating income from clearing.

Decision Timeline and Overall Game Formulation. Given an incentive I and condition $\mathbb{C}$, the locals first organize themselves into a partition $\pi \prec \bar{\pi}$ of coalitions. Subsequently, the coalitions $S \in \pi$ engage in a three-stage, non-cooperative game wherein each coalition S chooses its deforestation attempts d_S, followed by its deforestation-blocking decisions $B_S^{\bar{\mathbb{D}}}(d)$ and use-blocking decisions $B_S^{\bar{\mathbb{U}}}(d)$, to maximize their aggregate net income $V_S(d, B^{\bar{\mathbb{D}}}, B^{\bar{\mathbb{U}}}) = \sum_{\ell \in S} V_\ell(d, B^{\bar{\mathbb{D}}}, B^{\bar{\mathbb{U}}})$, where each local's net income is defined as:

$$V_\ell(d, B^{\bar{\mathbb{D}}}, B^{\bar{\mathbb{U}}}) := \begin{cases} v_\ell^{\text{sq}} + I_\ell - c_\ell \cdot d_\ell - \eta_{\bar{\mathbb{D}}} \sum_{i \in \mathcal{L}} B_{\ell i}^{\bar{\mathbb{D}}} - \eta_{\bar{\mathbb{U}}} \sum_{i \in \mathcal{L}} B_{\ell i}^{\bar{\mathbb{U}}} & \text{if } \mathbb{C}_\ell(d, B^{\bar{\mathbb{D}}}, B^{\bar{\mathbb{U}}}) = \text{yes} \\ v_\ell^{\text{sq}} + D_\ell - \eta_{\bar{\mathbb{D}}} \sum_{i \in \mathcal{L}} B_{\ell i}^{\bar{\mathbb{D}}} - \eta_{\bar{\mathbb{U}}} \sum_{i \in \mathcal{L}} B_{\ell i}^{\bar{\mathbb{U}}} & \text{if } \mathbb{C}_\ell(d, B^{\bar{\mathbb{D}}}, B^{\bar{\mathbb{U}}}) = \text{no}. \end{cases} \tag{EC.63}$$

Here, D_ℓ is local ℓ's net income gain from timber extraction and production on the cleared land.

All other components of the model remain the same as in §2.

Analysis. As in the base model, the individual condition $\mathbb{I}$ depends only on deforestation decisions and is therefore unaffected by blocking, so we focus on the area conditions.

Fix a partition $\pi \in \Pi_{\mathcal{L}}^{\prec \bar{\pi}}$ and consider the induced non-cooperative game among coalitions $S \in \pi$. A subgame-perfect equilibrium is a set of deforestation attempts $d^\star$, deforestation-blocking policies $B_S^{\bar{\mathbb{D}}^\star}(d)$, and use-blocking policies $B_S^{\bar{\mathbb{U}}^\star}(d, B^{\bar{\mathbb{D}}})$ that satisfy, for every coalition $S \in \pi$, the following:

$$d_S^\star \in \operatorname*{arg\,max}_{d_S \in \{0,1\}^S} V_S\Big([d_S, d_{-S}^\star], B^{\bar{\mathbb{D}}^\star}([d_S, d_{-S}^\star]), B^{\bar{\mathbb{U}}^\star}\big([d_S, d_{-S}^\star], B^{\bar{\mathbb{D}}^\star}([d_S, d_{-S}^\star])\big)\Big)$$

$$B_S^{\bar{\mathbb{D}}^\star}(d) \in \operatorname*{arg\,max}_{B_S^{\bar{\mathbb{D}}} \in \{0,1\}^{S \times \mathcal{L}}} V_S\Big(d, [B_S^{\bar{\mathbb{D}}}, {B_{-S}^{\bar{\mathbb{D}}}}^\star(d)], B^{\bar{\mathbb{U}}^\star}(d, B^{\bar{\mathbb{D}}})\Big),$$

$$B_S^{\bar{\mathbb{U}}^\star}(d, B^{\bar{\mathbb{D}}}) \in \operatorname*{arg\,max}_{B_S^{\bar{\mathbb{U}}} \in \{0,1\}^{S \times \mathcal{L}}} V_S\Big(d, B^{\bar{\mathbb{D}}}, [B_S^{\bar{\mathbb{U}}}, {B_{-S}^{\bar{\mathbb{U}}}}^\star(d, B^{\bar{\mathbb{D}}})]\Big).$$

We refer to this as the *extended* non-cooperative game to draw distinctions from our base model. We denote by $\hat{\mathcal{Q}}(\pi, \mathbb{C})$ the set of all subgame-perfect equilibria of this extended, three-stage game.

Our next result summarizes an intuitive observation: under $\bar{\mathbb{U}}$, coalitions block through the least costly means, so they block use rather than block deforestation; under $\bar{\mathbb{D}}$, blocking use cannot restore compliance once clearing occurs, so only blocking deforestation attempts is relevant. In equilibrium then, no blocking of deforestation attempts occurs under $\bar{\mathbb{U}}$, and no blocking of use occurs under $\bar{\mathbb{D}}$.

LEMMA EC.15. *Consider a partition $\pi \in \Pi_{\mathcal{L}}$. Under $\bar{\mathbb{D}}$, no blocking of use occurs in any equilibrium, i.e., $B_{i\ell}^{\bar{\mathbb{U}}^\star} = 0$ for every $i, \ell \in \mathcal{L}$ and any equilibrium in $\hat{\mathcal{Q}}(\pi, \bar{\mathbb{D}})$. With $\bar{\mathbb{U}}$, no blocking of deforestation occurs in any equilibrium, i.e., $B_{i\ell}^{\bar{\mathbb{D}}^\star} = 0$ for any equilibrium in $\hat{\mathcal{Q}}(\pi, \bar{\mathbb{U}})$.*

Proof. Under $\bar{\mathbb{D}}$, compliance depends only on successful deforestation, so blocking use cannot affect incentives and is strictly costly. Hence no equilibrium in $\hat{\mathcal{Q}}(\pi, \bar{\mathbb{D}})$ involves blocking use. Under $\bar{\mathbb{U}}$, compliance depends on $(1 - \max_{i \in \mathcal{L}} B_{i\ell}^{\bar{\mathbb{D}}^\star})(1 - \max_{i \in \mathcal{L}} B_{i\ell}^{\bar{\mathbb{U}}^\star})$. If $B_{i\ell}^{\bar{\mathbb{D}}^\star} = B_{i\ell}^{\bar{\mathbb{U}}^\star} = 1$, local i can deviate by performing only one blocking action. Under $\bar{\mathbb{U}}$, because $\eta_{\bar{\mathbb{U}}} < \eta_{\bar{\mathbb{D}}}$, the cheapest deviation is to block use only, implying that blocking deforestation does not occur in equilibrium. □

With this observation, we next prove structural results mirroring those in §3.2. For both $\bar{\mathbb{D}}$ and $\bar{\mathbb{U}}$, Lemma EC.16(i) shows that the set of equilibria in the extended non-cooperative game is non-empty, and in every equilibrium, no blocking occurs and either all locals attempt to deforest or no local attempts to deforest (as in Lemma 1). Then, Lemma EC.16(ii) proves the most critical result: that the set of equilibrium indicators $d^\star$ in the extended non-cooperative game under $\bar{\mathbb{D}}$ and $\bar{\mathbb{U}}$ is *exactly equal* to the set of equilibrium indicators in the non-cooperative game under $\bar{\mathbb{U}}$ from §3.2 with blocking costs $\eta = \eta_{\bar{\mathbb{D}}}$ and $\eta = \eta_{\bar{\mathbb{U}}}$, respectively.

LEMMA EC.16. *Consider any partition $\pi \in \Pi_{\mathcal{L}}^{\prec \bar{\pi}}$ and $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$.*

i. The set $\hat{\mathcal{Q}}(\pi, \mathbb{C})$ is non-empty and any equilibrium in $\hat{\mathcal{Q}}(\pi, \mathbb{C})$ satisfies $B^{\bar{\mathbb{D}}^\star}(d^\star) = 0$, $B^{\bar{\mathbb{U}}^\star}(d^\star, B^{\bar{\mathbb{D}}^\star}) = 0$, and either $d^\star = 0$ or $d^\star = 1$.

ii. With $T(\pi, \mathbb{C})$ defined as in §2 and the thresholds $\eta_{1,2}(\pi)$ defined in Lemma 2, we have:

$$T(\pi, \mathbb{C}) = \begin{cases} \{0\} & \text{if } \eta_{\mathbb{C}} < \eta_1(\pi) \\ \{1\} & \text{if } \eta_{\mathbb{C}} > \eta_2(\pi) \\ \{0,1\} & \text{otherwise.} \end{cases}$$

Proof. Lemma EC.15 implies that under $\bar{\mathbb{U}}$, only blocking of use occurs in equilibrium, so the game is identical to the base model with blocking cost $\eta_{\bar{\mathbb{U}}}$. Moreover, Lemma EC.15 also implies that under $\bar{\mathbb{D}}$, blocking use is irrelevant for compliance and therefore never occurs, reducing the game to one with only deforestation attempts and deforestation blocking, which is also equivalent to the base model with blocking cost $\eta_{\bar{\mathbb{D}}}$. The characterization of equilibria and thresholds therefore follows from the arguments used in Lemma 1 and Lemma 2. □

The important consequence of Lemma EC.16 is that the core under $\bar{\mathbb{D}}$ and the core under $\bar{\mathbb{U}}$ in this extension exactly equal the base-model core under $\bar{\mathbb{U}}$ with blocking cost $\eta = \eta_{\bar{\mathbb{D}}}$ and $\eta = \eta_{\bar{\mathbb{U}}}$, respectively. This implies that all the findings concerning $\bar{\mathbb{U}}$ in §3.3 directly apply to both $\bar{\mathbb{D}}$ and $\bar{\mathbb{U}}$ in this extension. Moreover, because the base-model $\bar{\mathbb{D}}$ is equivalent to the base-model $\bar{\mathbb{U}}$ with blocking cost $\eta = +\infty$, any relevant comparisons – such as comparing $\bar{\mathbb{D}}$ in the extension with $\bar{\mathbb{D}}$ in the base model or with $\bar{\mathbb{U}}$ in the extension – are equivalent to comparing two base-model conditions $\bar{\mathbb{U}}$ under different values of the blocking cost η. For the subsequent results, it therefore helps to recall the discussion at the end of §3.3 on the effect of changing η on the no-use condition $\bar{\mathbb{U}}$.

Consider first how the effectiveness of the area no-deforestation condition $\bar{\mathbb{D}}$ is impacted by allowing locals to block deforestation attempts. The equivalence above shows that comparing $\bar{\mathbb{D}}$ in the base-model with $\bar{\mathbb{D}}$ in the extension is the same as comparing (base-model) $\bar{\mathbb{U}}$ under blocking costs of $\eta = +\infty$ and $\eta = \eta_{\bar{\mathbb{D}}}$, respectively. As discussed at the end of §3.3, lowering the blocking cost has ambiguous effects on $\bar{\mathbb{U}}$'s performance: with full cooperation or when compensation is required, lowering η *worsens* performance because the core may become empty or may contain no outcomes with $d^\star = 0$ that compensate; with limited cooperation, lowering η can improve performance by maintaining outcomes with $d^\star = 0$ under a wider set of problem instances, but only provided the core remains non-empty. The area no-deforestation condition recovers its base-model performance only when $\eta_{\bar{\mathbb{D}}} > \overline{\eta}$, so that no coalition can profitably block even one deforestation attempt.

This observation directly affects our recommendations concerning the best conditional incentive, which are summarized in Figure EC.1. Although these recommendations exhibit similar patterns to those in §3.4, some changes are worth highlighting.

Scenario	**Prevent Deforestation**	**... and Compensate**
Full cooperation: $\bar{\pi} = \{\mathcal{L}\}$. If core has an outcome with $d^\star = 0$, that type of outcome occurs.	Area No-Deforestation $\bar{\mathbb{D}}$ (with $\eta = \eta_{\bar{\mathbb{D}}}$) or Area No-Use $\bar{\mathbb{U}}$ (with $\eta = \eta_{\bar{\mathbb{U}}}$) with: $I(\mathcal{L}) > D(\mathcal{L})$, and either $\eta_2(I) \geq \eta$ or $\overline{\eta}_1(I) \leq \eta$	Area No-Deforestation $\bar{\mathbb{D}}$ with: $I(\mathcal{L}) > D(\mathcal{L})$ and $\overline{\eta}_1(I) \leq \eta_{\bar{\mathbb{D}}}$ or Individual $\mathbb{I}$ with: $I_\ell > D_\ell$ for all $\ell \in \mathcal{L}$
Full cooperation: $\bar{\pi} = \{\mathcal{L}\}$. Any type of outcome in the core can occur.	Area No-Deforestation $\bar{\mathbb{D}}$ (with $\eta = \eta_{\bar{\mathbb{D}}}$) or Area No-Use $\bar{\mathbb{U}}$ (with $\eta = \eta_{\bar{\mathbb{U}}}$) with: $I(\mathcal{L}) > D(\mathcal{L})$, and either $\eta_2(I) \geq \eta$ or $\overline{\eta}_1(I) \leq \eta$	Area No-Deforestation $\bar{\mathbb{D}}$ with: $I(\mathcal{L}) > D(\mathcal{L})$ and $\overline{\eta}(I) < \eta_{\bar{\mathbb{D}}}$ or Individual $\mathbb{I}$ with: $I_\ell > D_\ell$ for all $\ell \in \mathcal{L}$
Limited cooperation: $\bar{\pi} \neq \{\mathcal{L}\}$. If core has an outcome with $d^\star = 0$, that type of outcome occurs.	Area No-Use $\bar{\mathbb{U}}$ with: $I(F) > D(F)$ for some $F \in \bar{\pi}$ and $\eta_2(I) \geq \eta_{\bar{\mathbb{U}}}$	Area No-Deforestation $\bar{\mathbb{D}}$ with: $I(F) > D(F)$ for all $F \in \bar{\pi}$, $\overline{\eta}_1(I) \leq \eta_{\bar{\mathbb{D}}}$ or Individual $\mathbb{I}$ with: $I_\ell > D_\ell$ for all $\ell \in \mathcal{L}$
Limited cooperation: $\bar{\pi} \neq \{\mathcal{L}\}$. Any type of outcome in the core can occur.	Area No-Use $\bar{\mathbb{U}}$ with: $I(F) > D(F)$ for some $F \in \bar{\pi}$, and $\underline{\eta}_1(I) > \eta_{\bar{\mathbb{U}}}$ and $\eta_2(I) \geq \eta_{\bar{\mathbb{U}}}$ or Individual $\mathbb{I}$ with: $I_\ell > D_\ell$ for all $\ell \in \mathcal{L}$	Individual $\mathbb{I}$ with: $I_\ell > D_\ell$ for all $\ell \in \mathcal{L}$

Figure EC.1 Recommended condition $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$ and incentive I to prevent deforestation, and to also compensate, for the extension that allows for blocking of both deforestation and use, depending on cooperation in the area and on whether all or some outcomes in the core must have $d^\star = 0$. Each inequality has a distinct color to highlight similarities and differences between the conditions on problem parameters.

Under full cooperation, the key change is that $\bar{\mathbb{D}}$ is no longer the uniformly best condition to use (as in the base model). To prevent deforestation, either $\bar{\mathbb{D}}$ or $\bar{\mathbb{U}}$ may now be preferred. Under either condition, if locals collectively prefer the incentive and the core is non-empty, every core outcome has $d^\star\!=\!0$; however, core non-emptiness is evaluated using the core of the base-model $\bar{\mathbb{U}}$ with $\eta_{\bar{\mathbb{D}}}$ under $\bar{\mathbb{D}}$ and $\eta_{\bar{\mathbb{U}}}$ under $\bar{\mathbb{U}}$, respectively, so either blocking cost may lie in the intermediate range in which the core can be empty. When both cores are non-empty, the two conditions are equivalent for preventing deforestation. To also compensate, $\bar{\mathbb{D}}$ remains more effective than $\bar{\mathbb{U}}$, because its higher blocking cost $\eta_{\bar{\mathbb{D}}} > \eta_{\bar{\mathbb{U}}}$ makes it more appealing for compensation. Nevertheless, $\bar{\mathbb{D}}$ is no longer uniformly best: when any type of outcome in the core could occur, an incentive with $\bar{\mathbb{D}}$ now requires the stronger condition that no coalition can profitably block even one deforestation attempt; when it suffices for the core to have an outcome with $d = 0$ for that type of outcome to occur, an incentive with $\bar{\mathbb{D}}$ now also requires that no coalition can profitably block all other locals. In both cases, if the applicable requirement fails, the individual condition $\mathbb{I}$ with a perfect incentive may be preferred.

For the same reason, under limited cooperation, the recommendations may change when the buyer seeks both to prevent deforestation and compensate. When an outcome with $d^\star = 0$ occurs if it is in the core, recall that in §4.1 the recommendation was to use $\bar{\mathbb{D}}$ and an incentive that is preferred by every family, because in that case, all core outcomes with $d^\star\!=\!0$ are maintained through utility transfers and therefore compensate. In the extension, however, a coalition may instead withhold these transfers and rely on blocking deforestation attempts. The recommendation for $\bar{\mathbb{D}}$ therefore requires both that every family prefer the incentive and that $\eta_{\bar{\mathbb{D}}}$ be sufficiently high that no coalition can profitably avoid transfers by blocking all other locals. When this additional requirement fails, $\mathbb{I}$ with a perfect incentive may be preferred. If all outcomes in the core should have $d^\star = 0$ to prevent deforestation, the individual condition was already recommended in the base model, so that recommendation is unchanged when $\bar{\mathbb{D}}$ is weakened.

Lastly, under limited cooperation, the recommendations for preventing deforestation do not change. Allowing deforestation attempts to be blocked makes $\bar{\mathbb{D}}$ more effective for this purpose than it was in the base model, because blocking threats can now maintain outcomes with $d^\star\!=\!0$. However, the same effect is stronger under $\bar{\mathbb{U}}$, since $\eta_{\bar{\mathbb{U}}} < \eta_{\bar{\mathbb{D}}}$: any low-blocking-cost requirement satisfied under $\bar{\mathbb{D}}$ is also satisfied under $\bar{\mathbb{U}}$, whereas the converse need not hold. The set of circumstances in which $\bar{\mathbb{D}}$ can be recommended for preventing deforestation is therefore contained in the corresponding set under $\bar{\mathbb{U}}$. Accordingly, Figure EC.1 continues to recommend $\bar{\mathbb{U}}$, or $\mathbb{I}$ with a perfect incentive, in these cases.

Overall, allowing deforestation attempts to be blocked then makes $\bar{\mathbb{D}}$ (paradoxically) *less appealing* as a recommendation. This change makes $\bar{\mathbb{D}}$ more similar to $\bar{\mathbb{U}}$. This weakens $\bar{\mathbb{D}}$ under full cooperation and when compensation is required, which were exactly the circumstances in which $\bar{\mathbb{D}}$ was preferred in the base model; the change strengthens $\bar{\mathbb{D}}$ in the limited-cooperation cases where inexpensive blocking helps prevent deforestation, but $\bar{\mathbb{U}}$ remains more effective in precisely those cases because blocking use is less costly. Conversely, the revised recommendations add alternatives or restrictions where $\bar{\mathbb{D}}$ was previously unequivocally recommended, without adding $\bar{\mathbb{D}}$ to the limited-cooperation recommendations for preventing deforestation.

EC.6.3. When Blocking Use Is Not Always Successful

We next consider a setting where blocking use is not always successful. Specifically, locals make blocking *attempts*, and *all* blocking attempts succeed with probability p (and with probability $1-p$, all blocking attempts fail). Only the area no-use condition is affected. Under full cooperation, if locals collectively prefer the incentive, every core outcome has $d^\star = 0$ for every p, although the core need not be nonempty (just as in the base model). When blocking is sufficiently likely to succeed ($p > p_2$), the full base-model characterization extends under both full and limited cooperation, with the relevant blocking-cost thresholds multiplied by p. For lower values of p under limited cooperation, more complex equilibria can arise, with deforestation and blocking attempts occurring simultaneously. If $p < p_1$, no core outcome has $d^\star = 0$, whereas if $p_1 < p < p_2$, instances exist in which the core contains an outcome with deforestation for every blocking cost $\eta > 0$.

We now formalize the model changes and the results for this extension.

Blocking attempts that may fail. Having observed all the deforestation decisions d, the locals in each coalition $S \in \pi$ jointly decide whether to *attempt to block* other locals $\ell \in \mathcal{L} \setminus S$, i.e., try to prevent them from using land they deforested to generate income. We represent the blocking attempts for all locals i in coalition S as a matrix $B_S(d) \in \{0,1\}^{S \times \mathcal{L}}$ and the blocking attempts for all locals $\ell \in \mathcal{L}$ as the matrix $B(d) \in \{0,1\}^{\mathcal{L} \times \mathcal{L}}$. For both matrices, $B_{i\ell} = 1$ if and only if i attempts to block ℓ, $B_{i\ell} = 0$ if i and ℓ belong to the same coalition $S \in \pi$. The cost to attempt to block one local is $\eta > 0$, so local $i \in \mathcal{L}$ incurs total blocking cost $\eta \cdot \sum_{\ell \in \mathcal{L}} B_{i\ell}$.

Blocking attempts are not always successful: with probability $p \in (0,1]$, *all* blocking attempts are successful, and with probability $1-p$ *all* blocking attempts fail. We represent blocking success with a Bernoulli random variable θ taking value 1 with probability p. For analytical tractability, we assume that p is independent of the number of locals being blocked or the number of locals attempting blocking; this is reasonable when the success of separate blocking attempts is highly correlated. (For instance, this is the case when blocking involves notifying authorities, which respond with probability p independent of how many notifications they receive and successfully block all production when they respond.) Thus, local i successfully blocks ℓ if and only if $\theta \cdot B_{i\ell} = 1$, and ℓ is successfully blocked from using deforested land if and only if $\theta \cdot \max_{i \in \mathcal{L}} B_{i\ell} = 1$.

Conditions. Only the *area no-use condition* $\bar{\mathbb{U}}$ is affected by this extension: under $\bar{\mathbb{U}}$, each local $\ell \in \mathcal{L}$ receives the incentive if and only if no local uses land that he deforested to generate income, $\bar{\mathbb{U}}_\ell(d, B, \theta) = \text{yes} \;\Leftrightarrow\; d_i \cdot \left(1 - \theta \cdot \max_{g \in \mathcal{L}} B_{gi}\right) = 0, \forall i \in \mathcal{L}$. Different from the base model, compliance with $\bar{\mathbb{U}}$ now depends on whether the blocking attempts are successful, as indicated by θ.

Coalition objectives. The net income gains with the incentive I and with deforestation D are defined as in our base model. Consistent with the base model, we assume that if locals do not comply with condition $\bar{\mathbb{U}}$ (which can also occur here because blocking attempts fail), they have recourse and they engage in deforestation. Every coalition $S \in \pi$ chooses its deforestation decisions and blocking attempts to maximize its *expected* net income given by:

$$V_S(d,B) := \begin{cases} \sum_{\ell \in S} \left(v_\ell^{\mathrm{sq}} + I_\ell - \eta \sum_{i \in \mathcal{L}} B_{\ell i} \right) & \text{if } d = 0 \\ \sum_{\ell \in S} \left(v_\ell^{\mathrm{sq}} + p\big(I_\ell - c_\ell \cdot d_\ell\big) + (1-p)\, D_\ell - \eta \sum_{i \in \mathcal{L}} B_{\ell i} \right) & \text{if } d \neq 0 \text{ and } \bar{\mathbb{U}}_\ell(d,B,1) = \text{yes} \\ \sum_{\ell \in S} \left(v_\ell^{\mathrm{sq}} + D_\ell - \eta \sum_{i \in \mathcal{L}} B_{\ell i} \right) & \text{if } \bar{\mathbb{U}}_\ell(d,B,1) = \text{no} \end{cases} \tag{EC.64}$$

The first case reflects that when no local deforests, every local complies with $\bar{\mathbb{U}}$ regardless of θ; indeed, $\bar{\mathbb{U}}_\ell(d,B,0) = \text{yes}$ if and only if $d = 0$, because no blocking attempt succeeds when $\theta = 0$.

To appreciate the second and third cases, note that when some local deforests land, compliance with $\bar{\mathbb{U}}$ can only hold if blocking attempts cover *every* deforesting local and blocking succeeds ($\theta = 1$). In the second case, blocking attempts indeed cover every deforesting local, so with probability p, blocking attempts succeed, locals comply, and each local earns the incentive (with deforesting locals incurring the cost c_ℓ without deriving income from deforestation), whereas with probability $1-p$, all blocking attempts fail, no local complies, and each local derives income from deforestation. In the third case, some deforesting local is not covered by any blocking attempt, so compliance fails; the net income is then independent of θ, and each local derives income from deforestation.

In analogy to §2, coalitions maximize their *expected* net income, and the allocations a are interpreted as ex-ante allocations of expected net income (implementable, for instance, through state-contingent transfers within each coalition).

For analysis, we define quantities mirroring those in §3. For partition $\pi \in \Pi_{\mathcal{L}}^{\preceq \bar{\pi}}$, let $\mathcal{Q}_p^{\mathrm{NC}}(\pi, \bar{\mathbb{U}})$ denote the set of all pure-strategy, subgame-perfect equilibria in the non-cooperative game between coalitions $S \in \pi$ under $\bar{\mathbb{U}}$, let $w_p\big(S; \pi, (d^\star, B^\star)\big) := V_S(d^\star, B^\star)$ denote the partition function corresponding to the expected net income of coalition $S \in \pi$ in equilibrium $(d^\star, B^\star) \in \mathcal{Q}_p^{\mathrm{NC}}(\pi, \bar{\mathbb{U}})$, and let $W_p(\pi) := \big\{ w_p\big(\cdot\,; \pi, (d^\star, B^\star)\big) : (d^\star, B^\star) \in \mathcal{Q}_p^{\mathrm{NC}}(\pi, \bar{\mathbb{U}}) \big\}$ denote the correspondence containing all partition functions for any feasible partition π. The main modification compared to the base model is that w_p and W_p now depend on the probability p and on the full equilibrium decisions $d^\star \in \{0,1\}^{\mathcal{L}}$ and $B^\star \in \{0,1\}^{\mathcal{L} \times \mathcal{L}}$. The pessimistic recursive core is defined exactly as in §3 using the partition function w_p and the correspondence W_p instead of w and W, respectively.

EC.6.3.1. Main Results. When blocking use can fail, the non-cooperative game of deforestation and blocking can entail more complex equilibria, wherein some locals deforest land even if other locals subsequently attempt to block them. The next example showcases this.

EXAMPLE EC.1. Consider the instance $\mathcal{L} = \{g, \ell\}$, $\bar{\pi} = \{\{g\}, \{\ell\}\}$, $I_g > D_g$, and $I_\ell < D_\ell$. Under the area no-use condition $\bar{\mathbb{U}}$, the set of sub-game perfect equilibria $\mathcal{Q}_p^{\mathrm{NC}}(\pi, \bar{\mathbb{U}})$ contains an equilibrium with $d_g^\star = 0$, $d_\ell^\star = 1$, and $B_{g\ell}^\star = 1$ if the following conditions hold:

$$p(v_g^{\mathrm{sq}} + I_g) + (1-p)(v_g^{\mathrm{sq}} + D_g) - \eta > v_g^{\mathrm{sq}} + D_g \quad \Leftrightarrow \quad p > \eta/(I_g - D_g) \tag{EC.65a}$$

$$p(v_\ell^{\mathrm{sq}} + I_\ell - c_\ell) + (1-p)(v_\ell^{\mathrm{sq}} + D_\ell) > v_\ell^{\mathrm{sq}} + I_\ell \quad \Leftrightarrow \quad p < 1 - c_\ell/(c_\ell + D_\ell - I_\ell). \tag{EC.65b}$$

The complex equilibrium arises because (EC.65a) implies that g prefers to set $d_g = 0$ and attempt to block ℓ, whereas (EC.65b) implies that ℓ prefers to set $d_\ell = 1$ even though g would subsequently attempt to block him. This occurs only if the probability of blocking being successful is neither too large nor too small: p must be large enough so that g benefits enough by attempting to block, but small enough so that ℓ prefers to engage in deforestation even though g would subsequently attempt to block him. Otherwise, if $p < \eta/(I_g - D_g)$ or $p > 1 - c_\ell/(c_\ell + D_\ell - I_\ell)$, then $\mathcal{Q}_p^{\mathrm{NC}}(\pi, \bar{\mathbb{U}})$ contains a unique equilibrium wherein $d^\star \in \{0, 1\}$ and $B^\star = 0$. Notably, our base-case model where $p = 1$ falls exactly in the latter category.

Such complex equilibria highlight one reason why this extended setting is significantly more challenging to analyze than the base model. To that end, our subsequent goal will be to provide a partial characterization that demonstrates that several base-model qualitative results and the recommendations in §3.4 are robust. Specifically, we will prove the following result.

PROPOSITION EC.1. *Consider the following thresholds satisfying $p_1 \le p_2$:*

$$p_1 = \max_{F,S}\left\{\frac{D(F) - I(F)}{c(S) + D(S) - I(S)} : F \in \bar{\pi}, S \subseteq F, I(F) < D(F), I(S) < D(S), I(F \setminus S) \ge D(F \setminus S)\right\}$$
$$p_2 = \max_{F,S}\left\{\frac{D(S) - I(S)}{\min_{i \in S} c_i + D(S) - I(S)} : F \in \bar{\pi},\ S \subseteq F,\ I(S) < D(S)\right\}.$$

For a conditional incentive $(I, \bar{\mathbb{U}})$, we have:

(i) Suppose $p > p_2$. The core equals the core in the base-model under $\bar{\mathbb{U}}$ with blocking cost η/p.

(ii) Suppose $p \le p_2$, $\bar{\pi} = \{\mathcal{L}\}$, and $I(\mathcal{L}) > D(\mathcal{L})$. Every core outcome has $d^\star = 0$ for every $p \in (0, p_2]$.

(iii) Suppose $p \le p_2$, $\bar{\pi} \ne \{\mathcal{L}\}$, and there exists $F \in \bar{\pi}$ with $I(F) > D(F)$. Then, the core under $(I, \bar{\mathbb{U}})$ cannot have outcomes with $d^\star = 0$ when $p < p_1$. There exist instances satisfying $p_1 < p < p_2$ for which, for every $\eta > 0$, the core under $(I, \bar{\mathbb{U}})$ is nonempty and has outcomes with $d^\star \ne 0$.

(iv) Suppose $p \le p_2$ and $I(F) \le D(F)$ for every $F \in \bar{\pi}$. Then for every $\eta > 0$, whenever the core is nonempty, it has an outcome with $d^\star = 1$.

It is illustrative to compare the results in Proposition EC.1 with the base-model results in Theorem 2. Part (i) of Proposition EC.1 shows that all base-model results can be readily extended to this setting if blocking attempts are sufficiently likely to succeed, $p > p_2$, by adjusting the blocking cost as η/p. In this parameter regime then, a probabilistic failure of blocking is equivalent to a larger blocking cost in the base model.

To understand the origins of the critical threshold p_2, consider a coalition S that strictly prefers deforestation and that keeps a nonempty subset $R \subseteq S$ of its members deforesting land while facing the prospect of being tentatively blocked. With probability p, blocking succeeds, the members of S comply, and S earns the incentive net of the wasted deforestation costs $c(R)$, whereas with probability $1-p$, blocking fails and S derives its income with deforestation. Not deforesting at all yields higher expected net income than deploying the "trigger" R if and only if:

$$v^{\mathrm{sq}}(S) + I(S) > p\big(v^{\mathrm{sq}}(S) + I(S) - c(R)\big) + (1-p)(v^{\mathrm{sq}}(S) + D(S)) \;\Leftrightarrow\; p > \frac{D(S) - I(S)}{c(R) + D(S) - I(S)}.$$

The value p_2 corresponds to the largest possible right-hand-side value. That is achieved when $c(R)$ is smallest, which corresponds to taking $R = \{\ell\}$ for a local $\ell \in S$ with the smallest deforestation cost c_ℓ. The condition $p > p_2$ then states that blocking is sufficiently likely to succeed that every coalition S that strictly prefers deforestation would rather not deforest than take the cheapest possible gamble on enforcement failure, which is done by letting its member with the lowest deforestation cost clear forest. Thus, this sufficient condition ensures that no coalition would take such gambles, which (as we will see) will preserve all the qualitative results in our base model concerning the non-cooperative game and the core of the TU cooperative game.

When the probability of successful blocking is not that high, $p \le p_2$, the game is significantly more complex. For this case, our analysis will show that complex equilibria exist in the non-cooperative game between coalitions, wherein locals in certain coalitions deforest land even though they anticipate that other coalitions would subsequently attempt to block them. This makes a comprehensive characterization of the core of the TU game very challenging. We instead focus on mirroring a subset of the results in Theorem 2: we seek to understand the conditions under which the core of this more complex game only contains outcomes with $d^\star = 0$, which would allow a buyer to prevent deforestation even when they are unable to induce their preferred core outcomes. To that end, cases (ii)-(iv) show that several qualitative results from Theorem 2 continue to hold.

With full cooperation and when locals collectively prefer the incentive, every core outcome, if any, has $d^\star = 0$ for every $p \le p_2$. This mirrors the strength of the finding in §3.4 that such scenarios only lead to core outcomes with $d^\star = 0$. However, as in the base model, the core could be empty.

With limited cooperation and when at least one family prefers the incentive, the core may have no outcomes with $d^\star = 0$ for any blocking cost η if blocking is not too likely to succeed, $p \in (p_1, p_2)$, and is guaranteed to have outcomes with $d^\star = 1$ if blocking is very unlikely to succeed, $p < p_1$.

To understand threshold p_1, consider an outcome with $d^\star{=}0$ and a family F that strictly prefers deforestation, containing a sub-coalition S that strictly prefers deforestation while its complement $F \setminus S$ either prefers the incentive or is indifferent. Because the outcome remains in the core only if $F \setminus S$ does not deviate, the locals in $F \setminus S$ must be allocated at least their deforestation net income. Under $d^\star{=}0$, the total net income of family F is $v^{\mathrm{sq}}(F) + I(F)$, so coalition S can receive at most

$$v^{\mathrm{sq}}(S) + I(S) + \big(I(F \setminus S) - D(F \setminus S)\big).$$

Even granted this largest possible share, coalition S would prefer to deviate and deforest land if its expected payoff from doing so exceeds that share even under a continuation in which every deforesting member of S is tentatively blocked. In that case, with probability p the blocking attempts succeed and S complies, earning the incentive net of the deforestation cost, with gains of $I(S) - c(S)$, whereas with probability $1-p$ the attempts fail and S uses deforested land, with gains of $D(S)$. So S's expected net income from the deviation would dominate their initial share if:

$$\begin{aligned} v^{\mathrm{sq}}(S) + p\big(I(S) - c(S)\big) + (1-p)D(S) > v^{\mathrm{sq}}(S) + I(S) + \big(I(F \setminus S) - D(F \setminus S)\big) \\ \Leftrightarrow \quad p < \frac{D(F) - I(F)}{c(S) + D(S) - I(S)}. \end{aligned}$$

The threshold p_1 corresponds to the largest possible left-hand-side in the inequality above. If $p < p_1$, blocking is so unreliable that some sub-coalition S would rather deforest land and risk being tentatively blocked than accept its share of an outcome with $d^\star{=}0$, even when that share absorbs the entire surplus that the rest of its family can offer with the incentive. Under these circumstances, no core outcome can have $d^\star{=}0$.

Lastly, Part (iv) of Proposition EC.1 shows that if no family prefers the incentive, a nonempty core has outcomes with $d^\star{=}1$ for any $p \le p_2$.

To prove Proposition EC.1, we subsequently distinguish cases depending on the cooperation level and on whether $p > p_2$ holds. §EC.6.3 presents results on the non-cooperative game, and §EC.6.3.3 provides sufficient results for the TU cooperative game to allow checking Proposition EC.1.

Consider the non-cooperative game between coalitions of locals for a fixed partition $\pi \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$. The thresholds p_1, p_2 are as defined in Proposition EC.1. We first prove that the non-cooperative game between coalitions admits pure-strategy subgame-perfect equilibria, and we exhibit one such equilibrium that takes a particularly simple form, wherein either (i) all locals deforest land or no local deforest land and there is no blocking, or (ii) exactly one local deforests land and exactly one attempt is made to block him. (The latter equilibrium resembles the example seen in our discussion of the threshold p_2, wherein a single local deforests land to "trigger" blocking attempts.)

Lemma EC.17 **(Existence of a pure-strategy equilibrium)**. *For every $\pi \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$, every $p \in (0,1]$, and every $\eta > 0$, the set of equilibria is nonempty, $\mathcal{Q}_p^{\mathrm{NC}}(\pi, \bar{\mathbb{U}}) \neq \emptyset$. Moreover, $\mathcal{Q}_p^{\mathrm{NC}}(\pi, \bar{\mathbb{U}})$ contains a pure-strategy subgame-perfect equilibrium wherein equilibrium deforestation and blocking decisions take one of the following three forms:*

(i) $d^\star = 1$ *and* $B^\star = 0$;

(ii) $d^\star = 0$ *and* $B^\star = 0$;

(iii) exactly one local h deforests, and a coalition $S^\star \in \pi$ with $h \notin S^\star$ makes exactly one attempt to block h, while no other blocking attempt is made.

Consequently, $W_p(\pi) \neq \emptyset$ for every $\pi \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$.

Proof. For every coalition $S \in \pi$, define

$$\Delta(S) := I(S) - D(S), \qquad \underline{c}(S) := \min_{i \in S} c_i.$$

We normalize coalition S's expected net income by subtracting the constant $v^{\mathrm{sq}}(S) + D(S)$.

We first make a useful observation about the blocking subgame. Fix a nonempty set X of locals who deforest. At $B = 0$, only a coalition S satisfying $S \cap X = \emptyset$ can unilaterally induce compliance, because a coalition cannot block its own members. Doing so requires $|X|$ blocking attempts and yields normalized payoff $p\Delta(S) - \eta|X|$. Hence, if

$$p\Delta(S) \leq \eta|X| \qquad \forall S \in \pi \text{ such that } S \cap X = \emptyset,$$

then $B = 0$ is a pure-strategy equilibrium of the blocking subgame. Otherwise, any coalition S satisfying the reverse strict inequality can block every member of X exactly once, while all other coalitions make no blocking attempts; this is also a pure-strategy equilibrium. If $X = \emptyset$, $B = 0$ is the unique equilibrium. Thus, every blocking subgame admits a pure-strategy equilibrium.

If $\pi = \{\mathcal{L}\}$, no blocking is feasible. The grand coalition therefore chooses $d = 0$ when $\Delta(\mathcal{L}) > 0$ and chooses $d = 1$ otherwise, where the latter follows from the deforestation tie-breaking rule. Hence the result holds under full cooperation.

Suppose henceforth that $\pi \neq \{\mathcal{L}\}$. First, assume that

$$p\Delta(S) \leq \eta|\mathcal{L} \setminus S| \qquad \forall S \in \pi. \tag{EC.66}$$

Then $d = 1$ and $B = 0$ form an equilibrium. Indeed, if a coalition H deviates but leaves at least one of its members deforesting, every coalition still contains a deforesting local, so no coalition can unilaterally induce compliance and $B = 0$ remains an equilibrium. If H instead sets $d_H = 0$, then H is the only coalition that could block all remaining deforesters, but (EC.66) implies that doing so is not profitable. In either case, H receives the same expected net income as under universal deforestation, and the tie-breaking rule selects $d_H = 1$.

Now suppose that (EC.66) fails. Then some coalition $S^\star \in \pi$ satisfies

$$p\Delta(S^\star) > \eta|\mathcal{L} \setminus S^\star|. \tag{EC.67}$$

In particular, $\Delta(S^\star) > 0$. Suppose first that

$$p\big[\Delta(H) - \underline{c}(H)\big] < \Delta(H) \qquad \forall H \in \pi \setminus \{S^\star\}. \tag{EC.68}$$

Then $d = 0$ and $B = 0$ form an equilibrium. If $H \neq S^\star$ deviates by having a nonempty set $R \subseteq H$ deforest, select the continuation in which $S^\star$ blocks every member of R. This is a blocking-stage equilibrium by (EC.67), and H obtains normalized payoff

$$p\big[\Delta(H) - c(R)\big] \leq p\big[\Delta(H) - \underline{c}(H)\big] < \Delta(H).$$

If $S^\star$ has a nonempty set $R \subseteq S^\star$ deforest, select any pure equilibrium of the resulting blocking subgame. Its normalized payoff is either at most zero, if compliance fails, or at most

$$p\big[\Delta(S^\star) - c(R)\big] < \Delta(S^\star),$$

if compliance holds. Thus, no coalition profitably deviates.

It remains to consider the case in which (EC.68) fails. Choose $H \neq S^\star$ satisfying

$$p\big[\Delta(H) - \underline{c}(H)\big] \geq \Delta(H), \tag{EC.69}$$

and choose $h \in \arg\min_{i\in H} c_i$. Consider the outcome in which only h deforests and $S^\star$ makes one attempt to block h. This is a blocking-stage equilibrium by (EC.67). The normalized on-path payoffs of H and $S^\star$ are, respectively,

$$p\big[\Delta(H) - c_h\big] \quad \text{and} \quad \bar{v} := p\Delta(S^\star) - \eta > 0.$$

Coalition H does not benefit by setting $d_H = 0$, by (EC.69); equality is resolved in favor of $d_h = 1$. If instead a nonempty set $R \subseteq H$ deforests, let $S^\star$ block every member of R. Coalition H then obtains

$$p\big[\Delta(H) - c(R)\big] \leq p\big[\Delta(H) - c_h\big].$$

Similarly, if some coalition $T \notin \{H, S^\star\}$ has a nonempty set $R \subseteq T$ deforest land, let $S^\star$ block h and every member of R. This is feasible because

$$1 + |R| \leq |\mathcal{L} \setminus S^\star|,$$

and it reduces T's normalized payoff by $pc(R) > 0$.

Finally, suppose that a nonempty set $R \subseteq S^\star$ deforests, and define $\gamma(R) := p\big[\Delta(S^\star) - c(R)\big]$. If no blocking is an equilibrium after this deviation, select it; $S^\star$ then receives zero, which is below $\bar{v}$. Otherwise, there exists a coalition $K \notin \{H, S^\star\}$ satisfying

$$p\Delta(K) > \eta(1 + |R|).$$

If $\gamma(R) \geq \eta$, select the blocking equilibrium in which $S^\star$ blocks h and K blocks every member of R. Coalition $S^\star$ then receives

$$\gamma(R) - \eta = \bar{v} - pc(R) < \bar{v}.$$

If $\gamma(R) < \eta$, let K block h and every member of R. Coalition $S^\star$ then receives $\gamma(R) < \bar{v}$. To see the latter inequality, otherwise

$$pc(R) \leq \eta \quad \text{and hence} \quad p\Delta(S^\star) = \gamma(R) + pc(R) < 2\eta.$$

But H and K are two distinct coalitions outside $S^\star$, so $|\mathcal{L} \setminus S^\star| \geq 2$, contradicting (EC.67).

Thus, no coalition profitably deviates from the equilibrium wherein only local h deforests land. For every remaining off-path deforestation decisions, select any pure equilibrium of the corresponding blocking subgame, whose existence was established above. The resulting strategy profile is a pure-strategy subgame-perfect equilibrium. The three cases are exhaustive, proving the result. □

The next result shows that *only* three types of equilibria are possible in the non-cooperative game: equilibria where all locals deforest land ($d^\star = 1$), equilibria where no locals deforest land ($d^\star = 0$), or equilibria where some locals do not deforest land and attempt to block all locals who deforest land.

LEMMA EC.18. *For any $(d^\star, B^\star) \in \mathcal{Q}_p^{\mathrm{NC}}(\pi, \bar{\mathbb{U}})$, either (i) $d^\star = 1$ or (ii) $d_i^\star = 0$ for some $i \in \mathcal{L}$ and $\max_{\ell \in \mathcal{L}} B_{\ell j}^\star = 1$, for all $j \in \mathcal{L}$ such that $d_j^\star = 1$.*

Proof. Assume by contradiction that there exists an equilibrium with $d_\ell^\star = 0$ and $d_j^\star(1 - \max_{\ell \in \mathcal{L}} B_{\ell j}^\star) = 1$. But then, $\bar{\mathbb{U}}_g(d^\star, B^\star, \theta) = \text{no}$ for any θ and any $g \in \mathcal{L}$. We claim that the coalition S that contains ℓ can profitably deviate by setting $d_\ell = 1$. Specifically, one can show that:

(i) If some local who deforests land is not blocked, then no blocking should occur:

$$d_\ell \cdot \big(1 - \max_{g \in \mathcal{L}} B_{g\ell}^\star(d)\big) = 1 \text{ for some } \ell \in \mathcal{L} \quad \Rightarrow \quad B^\star(d) = 0.$$

(ii) If the coalition S that contains ℓ sets $d_\ell = 1$, resulting in $d' = [d_\ell = 1, d_{-\ell}^\star]$, then the same blocking decisions remain optimal:

$$B^\star(d') = B^\star(d^\star) = 0.$$

Indeed, the originally unblocked local who deforests remains unblocked under d', so compliance still fails for every θ, and any blocking attempt would only incur a cost. Coalition S therefore receives the same expected net income with $d_\ell^\star = 0$ and with $d'_\ell = 1$, contradicting the deforestation tie-breaking rule.

Proving these steps follows the same ideas as the proof of claims (iii) and (iv)-(a) in Lemma 1, respectively (the only change needed is to account for expectations in the net income expressions, but the ideas readily carry over). We omit the details for brevity. □

The next result mirrors Lemma 2 and shows that with full cooperation or if blocking attempts are sufficiently likely to succeed, the set of equilibria $\mathcal{Q}_p^{\mathrm{NC}}(\pi,\bar{\mathbb{U}})$ has the same structure as in the base case model, i.e., every equilibrium has $d^\star \in \{0^{\mathcal{L}}, 1^{\mathcal{L}}\}$ and no blocking occurs ($B^\star = 0$).

Lemma EC.19 **(Extension of Lemma 2)**. *The set of equilibria $\mathcal{Q}_p^{\mathrm{NC}}(\pi,\bar{\mathbb{U}})$ satisfies:*

i) If $\pi = \{\mathcal{L}\}$:

$$\mathcal{Q}_p^{\mathrm{NC}}(\pi,\bar{\mathbb{U}}) = \begin{cases} \{(d^\star = 0, B^\star = 0)\} & \text{if } I(\mathcal{L}) > D(\mathcal{L}) \\ \{(d^\star = 1, B^\star = 0)\} & \text{if } I(\mathcal{L}) \le D(\mathcal{L}). \end{cases}$$

ii) If $\pi \neq \{\mathcal{L}\}$ and $p > p_2$,

$$\mathcal{Q}_p^{\mathrm{NC}}(\pi,\bar{\mathbb{U}}) = \begin{cases} \{(d^\star = 0, B^\star = 0)\} & \text{if } \eta < \eta_1^{\mathrm{p}}(\pi) \\ \{(d^\star = 1, B^\star = 0)\} & \text{if } \eta > \eta_2^{\mathrm{p}}(\pi) \\ \{(d^\star = 0, B^\star = 0),\, (d^\star = 1, B^\star = 0)\} & \text{otherwise,} \end{cases}$$

where the thresholds $\eta_1^{\mathrm{p}}(\pi), \eta_2^{\mathrm{p}}(\pi)$ satisfying $\eta_1^{\mathrm{p}}(\pi) \le \eta_2^{\mathrm{p}}(\pi)$ are given by:

$$\eta_1^p(\pi) := \sup\big\{\eta : \exists S \in \pi \text{ with } p(I(S) - D(S)) > \eta\,(|\mathcal{L} \setminus S|)\big\} \tag{EC.70a}$$

$$\eta_2^p(\pi) := \sup\Big\{\eta : \sum_{S \in \pi : I(S) > D(S)} \Big\lfloor \frac{p(I(S) - D(S))}{\eta} \Big\rfloor \ge |H|, \forall H \in \pi \text{ with } I(H) \le D(H)\Big\}. \tag{EC.70b}$$

Proof. We prove the two cases separately.

Case (i): $\pi = \{\mathcal{L}\}$**.** Because a coalition cannot block its own members and π contains the single coalition $\mathcal{L}$, no blocking attempts are feasible, so $B \equiv 0$ and the grand coalition chooses $d \in \{0,1\}^{\mathcal{L}}$ to maximize $V_{\mathcal{L}}(d,0)$. If $d \neq 0$, some deforesting local is not blocked, so locals do not comply with $\bar{\mathbb{U}}$ for any θ, and (EC.64) yields $V_{\mathcal{L}}(d,0) = \sum_{\ell \in \mathcal{L}} v_\ell^{\mathrm{sq}} + D(\mathcal{L})$ for every $d \neq 0$, whereas $V_{\mathcal{L}}(0,0) = \sum_{\ell \in \mathcal{L}} v_\ell^{\mathrm{sq}} + I(\mathcal{L})$. If $I(\mathcal{L}) > D(\mathcal{L})$, the unique maximizer is $d = 0$, so the unique equilibrium is $(d^\star = 0, B^\star \equiv 0)$. If $I(\mathcal{L}) \le D(\mathcal{L})$, every $d \neq 0$ attains the maximum (as does $d = 0$ in case of equality), and the deforestation tie-breaking rule selects $d = 1$, so the unique equilibrium is $(d^\star = 1, B^\star \equiv 0)$.

Case (ii): $\pi \neq \{\mathcal{L}\}$ **and** $p > p_2$**.** We first prove that in any subgame perfect equilibrium, either all locals deforest land or no locals deforest land, i.e.,

$$d^\star \in \{0,1\}, \forall\, (d^\star, B^\star) \in \mathcal{Q}_p^{\mathrm{NC}}(\pi,\bar{\mathbb{U}}). \tag{EC.71}$$

The proof ideas largely mirror those in the proof of Lemma 1, but we repeat the important steps here to show the impact of the condition $p > p_2$.

Consider $\big(d^\star, B^\star(d^\star)\big) \in \mathcal{Q}_p^{\rm NC}(\pi, \bar{\mathbb{U}})$ and assume by contradiction that $d^\star \neq 1$ and $\exists S \in \pi$ with $d^\star_S \neq 0$. Let $R := \{i \in S : d^\star_i = 1\} \neq \emptyset$ denote the members of S who deforest land. By Lemma EC.18, all deforesting locals are tentatively blocked, so locals would comply with $\bar{\mathbb{U}}$ if blocking is successful ($\theta = 1$). We claim that coalition S can strictly benefit by setting $d_S = 0$. This would result in deforestation decisions $d' = [d_S = 0, d^\star_{-S}]$. We claim that blocking attempts $B'(d')$:

$$B'_{g\ell}(d') = \begin{cases} 0 & \text{if } \ell \in S \\ B^\star_{g\ell}(d^\star) & \text{otherwise} \end{cases}$$

would be sub-game perfect for d', so that d' leads to a profitable deviation for S. We first show how this leads to a contradiction. Note that B' are identical to $B^\star$ except that no locals from S are blocked. With $[d', B'(d')]$, locals would continue to comply with $\bar{\mathbb{U}}$ if $\theta = 1$. Therefore, if $\theta = 1$, coalition S would be strictly better off than under $\big(d^\star, B^\star_{g\ell}(d^\star)\big)$ because it would save the deforestation cost $c(R)$. When $\theta = 0$, we distinguish two cases:

- if some other coalition $H \neq S$ also deforests land, $d_H = d'_H \neq 0$, then locals would not comply with $\bar{\mathbb{U}}$ under (d', B') and $\theta = 0$, and coalition S would derive the same net income as under $\big(d^\star, B^\star(d^\star)\big)$; this implies that the expected net income of S would be strictly larger with d' because $p > 0$;
- if $d'_H = 0, \forall H \in \pi$, then the locals would comply with $\bar{\mathbb{U}}$ under d' even if $\theta = 0$. But in this case, the difference in expected net income of coalition S under d' and under $d^\star$ is:

$$v^{\rm sq}(S) + I(S) - \big[v^{\rm sq}(S) + p(I(S) - c(R)) + (1-p)D(S)\big] = p\, c(R) - (1-p)\big(D(S) - I(S)\big).$$

  But note that the expression above is strictly positive: if $I(S) \geq D(S)$, it is at least $p\, c(R) > 0$; and if $I(S) < D(S)$, then $p > p_2 \geq \frac{D(S) - I(S)}{\min_{i \in S} c_i + D(S) - I(S)}$ implies $p \min_{i \in S} c_i > (1-p)\big(D(S) - I(S)\big)$, and $c(R) \geq \min_{i \in S} c_i$ because $R \neq \emptyset$. Hence S benefits by setting $d'_S = 0$.

To complete the argument, it suffices then to prove that B' are subgame-perfect blocking decisions for d'. The proof here is identical to that in Lemma 1 (that proof relies on results (EC.1) and (EC.2), which hold here with identical proofs.)

The proof that $B^\star \equiv 0$ is also identical to that in Lemma 1; that proof relies on (EC.3), which holds here as well, with an identical proof.

We are now ready to prove the structural results on $\mathcal{Q}^{\rm NC}(\pi, \bar{\mathbb{U}})$ depending on the value of the blocking cost η. We first prove the following result about the equilibrium with $d^\star = 1$:

$$\big(d^\star = 1, B^\star(d^\star) = 0\big) \in \mathcal{Q}_p^{\rm NC}(\pi, \bar{\mathbb{U}}) \quad \Leftrightarrow \quad \eta \geq \eta_1^{\rm p}(\pi). \tag{EC.72}$$

This is an equilibrium if and only if no coalition $S \in \pi$ would benefit by deviating to set $d_S = 0$. Under such a deviation, the deforestation decisions would become $d' = [d_S = 0, d^\star_{-S} = 1]$. For such a deviation to be profitable, we claim that the optimal blocking decisions are exactly $B^\star(d') = [B'_S, B^\star_{-S} = 0]$ where B'_S is such that coalition S blocks every other local in $\mathcal{L} \setminus S$ exactly once. To see this, note first that every other coalition $H \neq S$ consists of locals who deforest land under d' and cannot block its own members, so blocking by H cannot restore compliance when the locals in H are otherwise unblocked; blocking attempts by these coalitions would then only incur costs, so their rational response is to not engage in any blocking. If S then attempts blocking only $k < |\mathcal{L} \setminus S|$ other locals, at least one local is not blocked and coalition S's expected net income is $v^{\text{sq}}(S) + D(S) - \eta \cdot k$, which is strictly less than the $v^{\text{sq}}(S) + D(S)$ it earns under $\big(d^\star = 1, B^\star(d^\star) = 0\big)$. Thus, the deviation is profitable for S if and only if it earns strictly higher expected net income when setting $d_S = 0$ and attempting to block every other local once rather than setting $d_S = 1$ and not attempting any blocking. The former option results in a higher expected net income than the latter if and only if:

$$v^{\text{sq}}(S) + pI(S) + (1-p)D(S) - \eta|\mathcal{L} \setminus S| > v^{\text{sq}}(S) + D(S) \;\Leftrightarrow\; p(I(S) - D(S)) - \eta|\mathcal{L} \setminus S| > 0. \quad \text{(EC.73)}$$

But note that the condition $\eta < \eta_1^{\text{p}}(\pi)$ is exactly equivalent to the existence of a coalition $S \in \pi$ for which inequality (EC.73) holds, which proves our claim.

The next result concerns the equilibrium with $d^\star = 0$. We claim that:

$$\big(d^\star = 0, B^\star(d^\star) = 0\big) \in \mathcal{Q}_p^{\text{NC}}(\pi, \bar{\mathbb{U}}) \quad \Leftrightarrow \quad \eta \leq \eta_2^{\text{p}}(\pi). \quad \text{(EC.74)}$$

This is an equilibrium if and only if no coalition $H \in \pi$ would benefit by deviating to set $d_H = 1$. This would result in deforestation decisions $d' = [d_H = 1, d^\star_{-H} = 0]$. Note that such a deviation is profitable for H if and only if H weakly prefers deforestation $D(H) \geq I(H)$ and under the subgame-perfect blocking decisions $B^\star(d')$, at least one local in H is not blocked. That H must weakly prefer deforestation follows from our tie-breaking rule (if H were indifferent, it would set $d_H = 1$). The latter condition follows because $p > p_2$, so if blocking attempts were made on all locals in H, coalition H would earn strictly less expected net income than with $d_h = 0$.

In this context, consider the options faced by a coalition $S \in \pi, S \neq H$ when choosing its blocking attempts. When attempting to block some number k_S of the locals in H, coalition S can face two circumstances. If some local from H would not be tentatively blocked (regardless of the blocking done by coalition S), then locals would not earn the incentive either regardless of θ, so coalition S would derive expected net income of $v^{\text{sq}}(S) + D(S) - \eta \cdot k_S$, which is strictly decreasing in k_S; so the rational choice would be to not attempt blocking any locals from H, i.e., set $k_S = 0$. If by blocking k_S locals from H, coalition S can ensure that blocking attempts are made on all locals in H, then locals would comply with $\bar{\mathbb{U}}$ if $\theta = 1$, and coalition S would earn expected net income $v^{\text{sq}}(S) + pI(S) + (1-p)D(S) - \eta \cdot k_S$. When comparing this with the alternative of not blocking any local from H and earning its expected net income with deforestation, $v^{\text{sq}}(S) + D(S)$, coalition S

would therefore rationally be willing to block up to $\lfloor \frac{p(I(S)-D(S))}{\eta} \rfloor$ locals from H, and only if it prefers the incentive, $I(S) > D(S)$.

Lastly, consider the condition $\eta \le \eta_2^{\mathrm{p}}(\pi)$, which is equivalent to the statement that for any family $H \in \pi$ that weakly prefers deforestation, $I(H) \le D(H)$,

$$\sum_{S\in\pi: I(S)>D(S)} \left\lfloor \frac{p(I(S)-D(S))}{\eta} \right\rfloor \ge |H|. \tag{EC.75}$$

This implies that when $\eta \le \eta_2^{\mathrm{p}}(\pi)$, the coalitions in S that prefer the incentive can make blocking attempts on all locals in H. As our previous analysis has shown, these would be rational responses for these coalitions, which will imply that coalition H in this case would not benefit by deviating, proving that $d^\star = 0$ is an equilibrium. Conversely, when $\eta > \eta_2^{\mathrm{p}}(\pi)$, there exists some coalition H for which (EC.75) does not hold, and that coalition will then have a viable deviation by setting $d_H = 1$ because the rational response of all other coalitions will be to not attempt any blocking. Therefore, $d^\star = 0$ cannot be an equilibrium, which completes the proof of (EC.74).

Noting that $\eta_1^{\mathrm{p}}(\pi) \le \eta_2^{\mathrm{p}}(\pi)$, the two claims above readily give the desired result. □

EC.6.3.3. Analysis of Cooperative Game of Coalition Formation. We now characterize the pessimistic recursive core of the cooperative game under $\bar{\mathbb{U}}$ in this extension. We first show that, when blocking is sufficiently likely to succeed ($p > p_2$), the cooperative game with probabilistic blocking reduces exactly to the base-model cooperative game with a rescaled blocking cost η.

LEMMA EC.20. *Suppose $p > p_2$. Then, for every partition $\pi \in \Pi_{\mathcal{L}}^{\prec\bar{\pi}}$ and every blocking cost $\eta > 0$:*

i) $\mathcal{Q}_p^{\mathrm{NC}}(\pi, \bar{\mathbb{U}})$ under blocking cost η equals the base-model set of subgame-perfect equilibria $\mathcal{Q}(\pi, \bar{\mathbb{U}})$ under blocking cost η/p;

ii) every equilibrium $(d^\star, B^\star = 0)$ in this set yields each coalition the same net income in both models: $w_p\big(S;\pi,(d^\star,0)\big) = w(S;d^\star)$ for every $S \in \pi$.

Consequently, the (pessimistic recursive) core of the cooperative game with probabilistic blocking under blocking cost η is the same as the base-model (pessimistic recursive) core under blocking cost η/p.

Proof. We first claim that $\eta_1^{\mathrm{p}}(\pi) = p\,\eta_1(\pi)$ and $\eta_2^{\mathrm{p}}(\pi) = p\,\eta_2(\pi)$. The first identity follows by substituting $\eta = p\eta'$ in (EC.70a) and comparing with the definition of $\eta_1(\pi)$ in Lemma 2. The second identity follows because $\lfloor p(I(S)-D(S))/\eta \rfloor = \lfloor (I(S)-D(S))/(\eta/p) \rfloor$, so the condition defining $\eta_2^{\mathrm{p}}(\pi)$ in (EC.70b) holds at blocking cost η if and only if the condition defining $\eta_2(\pi)$ in Lemma 2 holds at blocking cost η/p.

To prove part i), consider first $\pi = \{\mathcal{L}\}$: Lemma EC.19-i) and Lemma 2 prescribe the same unique equilibrium ($d^\star = 0$ if $I(\mathcal{L}) > D(\mathcal{L})$ and $d^\star = 1$ otherwise, with $B^\star = 0$), independent of η and p. Consider next $\pi \ne \{\mathcal{L}\}$: by the threshold identities, $\eta < \eta_1^{\mathrm{p}}(\pi)$ if and only if $\eta/p < \eta_1(\pi)$, and $\eta > \eta_2^{\mathrm{p}}(\pi)$

if and only if $\eta/p > \eta_2(\pi)$, so the conditions in Lemma EC.19-ii) at blocking cost η selects the same set of equilibrium indicators as the conditions in Lemma 2 at blocking cost η/p. Moreover, by Lemma 1, the base-model equilibria are exactly the pairs $(d^\star, 0)$ with $d^\star \in T(\pi, \bar{\mathbb{U}})$, so $\mathcal{Q}(\pi, \bar{\mathbb{U}})$ consists exactly of the pairs $(d^\star, 0)$ with $d^\star \in T(\pi, \bar{\mathbb{U}})$, which coincide with the equilibria prescribed by Lemma EC.19.

To prove part ii), consider an equilibrium with $d^\star = 0$ and $B^\star = 0$: no local deforests land, so $\bar{\mathbb{U}}_\ell(d^\star, B^\star, \theta) = \text{yes}$ for every $\ell \in \mathcal{L}$ and every $\theta \in \{0,1\}$, and by (EC.64) every coalition S earns $v^{\text{sq}}(S) + I(S) = w(S;0)$. In an equilibrium with $d^\star = 1$ and $B^\star = 0$, $\bar{\mathbb{U}}_\ell(d^\star, B^\star, \theta) = \text{no}$ for every ℓ and every θ, so by (EC.64) every coalition S earns $v^{\text{sq}}(S) + D(S) = w(S;1)$. In particular, these net incomes depend on neither η nor p.

Parts i) and ii) imply that the correspondence W_p under blocking cost η coincides with the base-model correspondence W under blocking cost η/p for every feasible partition. The pessimistic recursive core is determined by the family partition $\bar{\pi}$ (through the set of feasible partitions $\Pi_{\mathcal{L}}^{\prec\bar{\pi}}$) and by the correspondence (W_p, respectively W) alone, so the two games admit identical residual games and domination relations, and their (pessimistic recursive) cores are the same. □

The following result shows that under limited cooperation and if blocking is very unlikely to succeed, the core cannot have outcomes with $d^\star{=}0$. Specifically, we show that for $p < p_1$ (where p_1 is defined in the statement of Proposition EC.1), all core outcomes have *some* deforestation, i.e., $d^\star_\ell = 1$ for some $\ell \in \mathcal{L}$.

Proposition EC.2. *If $\bar{\pi} \neq \{\mathcal{L}\}$ and $p < p_1$, the core under a conditional incentive with the no-use condition $(I, \bar{\mathbb{U}})$ does not contain outcomes with $d^\star = 0$.*

Proof. For convenience, recall the definition of the threshold p_1:

$$p_1 = \max_{F,S}\left\{\frac{D(F) - I(F)}{c(S) + D(S) - I(S)} : F \in \bar{\pi}, S \subseteq F, I(F) < D(F), I(S) < D(S), I(F \setminus S) \geq D(F \setminus S)\right\}.$$

If $I(F) \geq D(F)$ for every $F \in \bar{\pi}$, the constraint set above is empty, $p_1 = -\infty$, and the claim is vacuous.

Consider then the opposite case and let $F \in \bar{\pi}$ and $S \subseteq F$ attain the maximum defining p_1, so that $I(S) < D(S)$, $I(F) < D(F)$, and $I(F \setminus S) \geq D(F \setminus S)$.

Let $\big(\pi, (d^\star, B^\star), a\big)$ be any outcome with $d^\star = 0$. We show that coalition S can profitably deviate by forming $\pi_S = \{S\}$, regardless of how the locals in $\mathcal{L} \setminus S$ re-partition and of which equilibrium results, so that no such outcome can belong to the core.

Step 1 (lower bound on the deviation value). For every partition $\pi_{\mathcal{L}\setminus S} \in \Pi_{\mathcal{L}\setminus S}^{\prec\bar{\pi}}$ and every equilibrium $e \in \mathcal{Q}_p^{\text{NC}}(\pi_S \cup \pi_{\mathcal{L}\setminus S}, \bar{\mathbb{U}})$,

$$w_p\big(S; \pi_S \cup \pi_{\mathcal{L}\setminus S}, e\big) \geq v^{\text{sq}}(S) + p\, I(S) + (1-p)\, D(S) - p\, c(S). \tag{EC.76}$$

Indeed, after setting $d_S = 1$, coalition S can choose $B_S = 0$ in the ensuing blocking subgame. If the blocking attempts of the other coalitions cover every deforesting local, then S obtains

$$v^{\text{sq}}(S) + p\big(I(S) - c(S)\big) + (1-p) D(S).$$

Otherwise, compliance fails and S obtains $v^{\text{sq}}(S)+D(S)$, which is strictly larger than the displayed lottery payoff because $D(S)>I(S)$ and $c(S)>0$. Since S chooses a best response in every continuation, (EC.76) follows.

Step 2 (upper bound on the allocation of S). We claim that

$$\sum_{\ell\in S} a_\ell \le v^{\text{sq}}(S)+I(S)+\big(I(F\setminus S)-D(F\setminus S)\big). \tag{EC.77}$$

First, since $d^\star=0$, no local deforests land and compliance with $\bar{\mathbb{U}}$ holds for every θ irrespective of the blocking attempts; blocking attempts are then a pure cost, so $B^\star=0$ and every coalition $H\in\pi$ earns $w_p\big(H;\pi,(d^\star,B^\star)\big)=v^{\text{sq}}(H)+I(H)$. So summing over coalitions contained in F gives $\sum_{\ell\in F} a_\ell = v^{\text{sq}}(F)+I(F)$. Second, if $F\setminus S\neq\emptyset$, then $\sum_{\ell\in F\setminus S} a_\ell \ge v^{\text{sq}}(F\setminus S)+D(F\setminus S)$: otherwise, the coalition $F\setminus S$ would profitably deviate, because by setting $d_{F\setminus S}=0$ and avoiding any blocking, it earns, in every residual equilibrium and for every realization of θ, either $v^{\text{sq}}(F\setminus S)+I(F\setminus S)$ (when compliance with $\bar{\mathbb{U}}$ holds) or, through deforestation, $v^{\text{sq}}(F\setminus S)+D(F\setminus S)$ (when it does not), and both are at least $v^{\text{sq}}(F\setminus S)+D(F\setminus S)$ since $I(F\setminus S)\ge D(F\setminus S)$. Subtracting the second bound from the first yields (EC.77). (If $F\setminus S=\emptyset$, (EC.77) holds directly with $\sum_{\ell\in S} a_\ell = v^{\text{sq}}(F)+I(F)=v^{\text{sq}}(S)+I(S)$.)

Step 3 (comparison). The inequality

$$v^{\text{sq}}(S)+p\,I(S)+(1-p)\,D(S)-p\,c(S)>v^{\text{sq}}(S)+I(S)+\big(I(F\setminus S)-D(F\setminus S)\big)$$

rearranges to $D(F)-I(F)>p\big(c(S)+D(S)-I(S)\big)$, which holds because $p<p_1$ and (F,S) attains the maximum defining p_1. Combining this with (EC.76) and (EC.77) gives

$$\sum_{\ell\in S} a_\ell < w_p\big(S;\pi_S\cup\pi_{\mathcal{L}\setminus S},e\big)$$

for every $\pi_{\mathcal{L}\setminus S}\in\Pi^{\prec\bar\pi}_{\mathcal{L}\setminus S}$ and every $e\in\mathcal{Q}^{\text{NC}}_p(\pi_S\cup\pi_{\mathcal{L}\setminus S},\bar{\mathbb{U}})$. Hence the deviation by S strictly increases its net income against every residual reaction, so no outcome with $d^\star=0$ belongs to the core. □

In Proposition EC.3, we show that under limited cooperation, when the probability of successful blocking is low but not negligible ($p_1<p<p_2$), the core may not include outcomes with $d^\star=0$ for every blocking cost $\eta>0$. The mechanism rests on recourse: when a local who prefers deforestation is tentatively blocked, his entire family earns its deforestation net income with probability $1-p$, so the family treats his blocked deforestation as a gamble on enforcement failure whose only price is the cost c_ℓ wasted when blocking succeeds. When p is small enough, the family prefers sustaining this gamble to complying, and no blocking cost eliminates the resulting outcomes from the core.

Proposition EC.3. *There exist instances with $\bar\pi\neq\{\mathcal{L}\}$ and $p_1<p<p_2$ for which, for every $\eta>0$, the core under a conditional incentive with the no-use condition $(I,\bar{\mathbb{U}})$ contains an outcome with $d_i^\star=1$ for some $i\in\mathcal{L}$. In particular, this holds for every instance with $\mathcal{L}=\{\ell,m,g\}$ and $\bar\pi=\{F,\{g\}\}$, $F=\{\ell,m\}$, whose values satisfy* *(a)* $I_m > D_m$ *and* $I_g > D_g$*, while* $D_\ell - I_\ell > I_m - D_m$ *(so that* $D(F) > I(F)$*);*

(b) $c_\ell \le c_m$ *and* $I_m - D_m \le 2c_g$*;*

(c) $I_m - D_m \le I_g - D_g$*;*

and for every p *with*

$$p_1 < p < \frac{D(F) - I(F)}{c_\ell + D(F) - I(F)}. \tag{EC.78}$$

Proof. We first verify that the interval in (EC.78) is nonempty and contained in (p_1, p_2). By condition (a), the only family preferring deforestation is F, and the only coalitions S with $I(S) < D(S)$ are $\{\ell\}$ and F. Hence

$$p_1 = \max\left\{\frac{D(F) - I(F)}{c_\ell + D_\ell - I_\ell},\ \frac{D(F) - I(F)}{c_\ell + c_m + D(F) - I(F)}\right\},$$
$$p_2 = \max\left\{\frac{D_\ell - I_\ell}{c_\ell + D_\ell - I_\ell},\ \frac{D(F) - I(F)}{c_\ell + D(F) - I(F)}\right\} = \frac{D_\ell - I_\ell}{c_\ell + D_\ell - I_\ell},$$

where the cheapest member of both $\{\ell\}$ and F is ℓ (condition (b)), and the last equality holds because $x \mapsto x/(c_\ell + x)$ is increasing and $D(F) - I(F) < D_\ell - I_\ell$ by condition (a). Both expressions defining p_1 are strictly smaller than the upper bound in (EC.78): the first because $D_\ell - I_\ell > D(F) - I(F)$ by condition (a), and the second because $c_m > 0$. The upper bound is the second expression defining p_2 and is in turn strictly smaller than p_2, because $x \mapsto x/(c_\ell + x)$ is increasing and $D(F) - I(F) < D_\ell - I_\ell$. In particular, $p < p_2$ implies that $\{\ell\}$ strictly prefers deforesting while tentatively blocked over complying when no other local deforests land:

$$v_\ell^{\mathrm{sq}} + p\big(I_\ell - c_\ell\big) + (1-p)D_\ell > v_\ell^{\mathrm{sq}} + I_\ell, \tag{EC.79}$$

and (EC.78) states that the same comparison holds at the family level with the cost of ℓ alone:

$$(1-p)\big(D(F) - I(F)\big) > p\, c_\ell. \tag{EC.80}$$

We exhibit, for every $\eta > 0$, an outcome with deforestation in the core: one outcome for $\eta < \frac{p(I_g - D_g)}{2}$ and another for $\eta \ge \frac{p(I_g - D_g)}{2}$.

Case 1: $\eta < \frac{p(I_g - D_g)}{2}$. Consider the outcome with $\pi = \{\{\ell\}, \{m\}, \{g\}\}$, $d_\ell = 1$, $d_m = d_g = 0$, $B_{g\ell} = 1$ and all other blocking attempts equal to zero, with the (unique feasible) allocation

$$a_\ell = v_\ell^{\mathrm{sq}} + p\big(I_\ell - c_\ell\big) + (1-p)D_\ell, \quad a_m = v_m^{\mathrm{sq}} + p\, I_m + (1-p)D_m, \quad a_g = v_g^{\mathrm{sq}} + p\, I_g + (1-p)D_g - \eta.$$

This is an equilibrium. The blocking decisions are subgame-perfect: g benefits from blocking ℓ since $p(I_g - D_g) \ge 2\eta > \eta$, and any other blocking attempt would not affect compliance because blocking is costly. Coalition $\{\ell\}$ does not deviate to $d_\ell = 0$ by (EC.79), and adding blocking attempts would not affect compliance because blocking is costly. Coalition $\{m\}$ does not deviate to deforesting: the subgame-perfect blocking decisions either block both deforesters (which benefits g since $2\eta \le$

$p(I_g - D_g))$, in which case m loses $p\,c_m$ (note that this is due to the recourse of deforesting if blocking of ℓ fails), or block neither, in which case m earns $v_m^{\rm sq} + D_m < v_m^{\rm sq} + pI_m + (1-p)D_m = a_m$ (where the strict inequality comes from the assumption that $D_m < I_m$). Coalition $\{g\}$ does not deviate to deforesting: if the subgame-perfect blocking decisions leave g unblocked, g earns $v_g^{\rm sq} + D_g < a_g$ since g prefers the incentive; if instead m blocks both deforesters, which requires $2\eta \le p(I_m - D_m)$, then g earns $a_g + \eta - p\,c_g \le a_g$, because condition (b) gives $\eta \le \frac{p(I_m - D_m)}{2} \le p\,c_g$.

No coalition profitably deviates in the cooperative game. Because a deviation must make every coalition in the deviating partition strictly better off under every element of the corresponding assumption set, it suffices to exhibit one admissible continuation under which at least one deviating coalition receives no more than its current allocation.

Consider first a singleton deviation by $H \in \{\{\ell\}, \{m\}, \{g\}\}$. Retain the equilibrium constructed above and the allocations of the residual locals. If $H = \{\ell\}$ or $H = \{m\}$, the two residual locals belong to different families and cannot merge. The retained residual outcome is in the residual core: any residual singleton deviation faces an assumption set containing the retained equilibrium, under which that singleton receives exactly its current allocation. If $H = \{g\}$, the same argument treats singleton deviations by $\{\ell\}$ and $\{m\}$. A residual deviation by F also cannot dominate. If its members remain separate, the equilibrium constructed above gives each of them exactly its current allocation. If they form coalition F, the partition $\{F, \{g\}\}$ admits an equilibrium in which only ℓ deforests and g blocks him. By (EC.80), $c_\ell \le c_m$, and the Case 1 inequality, this is an equilibrium, and it gives F expected net income

$$v^{\rm sq}(F) + p\,I(F) + (1-p)D(F) - p\,c_\ell = a_\ell + a_m.$$

Thus, the retained outcome belongs to the relevant residual core in every singleton deviation, and the deviating singleton receives exactly its current allocation in one admissible continuation.

It remains to consider a deviation by F. If its members remain separate, the residual set is $\{g\}$, and its singleton residual core contains the equilibrium constructed above, under which ℓ and m receive a_ℓ and a_m. If its members form coalition F, the singleton residual core of $\{g\}$ contains the equilibrium just described in which only ℓ deforests, and g blocks him, under which F receives $a_\ell + a_m$. Hence, neither partition of F makes all of its constituent coalitions strictly better off under every admissible continuation. No feasible coalition therefore dominates the proposed outcome, so it is in the core.

Case 2: $\eta \ge \frac{p(I_g - D_g)}{2}$. Consider the outcome with $\pi = \bar{\pi}$, $d = 1$, $B = 0$, and allocation $a_i = v_i^{\rm sq} + D_i$ for $i \in \{\ell, m, g\}$.

This is an equilibrium. The blocking decisions are subgame-perfect: blocking both members of F does not benefit g since $2\eta \ge p(I_g - D_g)$, and blocking only one of them would not affect compliance while blocking is costly; not deforesting leaves g with the same income $v_g^{\rm sq} + D_g$ through recourse, and the deforestation tie-break selects $d_g = 1$. Coalition F earns $v^{\rm sq}(F) + D(F)$; complying yields

$v^{\rm sq}(F)+I(F)$, which is smaller by condition (a), and deforesting only one member yields at most $v^{\rm sq}(F)+D(F)$ (exactly this if unblocked; less the expected loss $p\big(D(F)-I(F)+c_\ell\big)$ if blocked).

No coalition profitably deviates. As in Case 1, it suffices to show that each deviation leads to some deviating coalition deriving a net income that does not exceed its current allocation.

For a deviation by $\{\ell\}$ or $\{m\}$, retain the all-deforestation equilibrium in the singleton partition. The two residual locals belong to different families, and the associated residual outcome is in the residual core: any residual singleton deviation faces an assumption set containing this same equilibrium, under which it receives its deforestation allocation. Thus, the original deviator receives exactly a_ℓ or a_m in an admissible continuation.

For a deviation by $\{g\}$, retain the all-deforestation outcome with residual coalition F. This residual outcome is in the residual core. Singleton deviations by $\{\ell\}$ or $\{m\}$ face the all-deforestation equilibrium in the singleton partition, while a deviation by F faces either that same equilibrium if its members separate or the all-deforestation equilibrium in $\{F,\{g\}\}$ if they remain together. In each case, at least one coalition receives exactly its deforestation allocation. Hence, the all-deforestation indicator belongs to the assumption set faced by $\{g\}$ and gives it exactly a_g.

Finally, if F deviates, its members may remain together or separate. In the former case, the singleton residual core of $\{g\}$ contains the all-deforestation equilibrium in $\{F,\{g\}\}$, which gives F exactly $v^{\rm sq}(F)+D(F)$. In the latter case, it contains the all-deforestation equilibrium in the singleton partition, which gives ℓ and m exactly a_ℓ and a_m. Therefore, no feasible deviation is strictly profitable under every admissible continuation, and the proposed outcome is in the core.

In both cases, the core has an outcome with $d_i^\star=1$ for some $i\in\mathcal{L}$, and the cases cover every $\eta>0$. $\square$

Finally, we prove proposition EC.1, which establishes our main result for this setting.

Proof of Proposition EC.1. The ordering $p_1\le p_2$ was justified before the statement: each pair (F,S) admissible for p_1 satisfies $D(F)-I(F)\le D(S)-I(S)$ and $c(S)\ge \min_{i\in S} c_i$.

Case (i). The result is immediate from Lemma EC.20.

Case (ii). Let $\bar\pi=\{\mathcal{L}\}$ and $I(\mathcal{L})>D(\mathcal{L})$, and consider any outcome in which some local deforests land; let $S\neq\emptyset$ denote the set of deforesting locals and a the outcome's allocation. If some local in S is not tentatively blocked, then $\bar{\mathrm{U}}_\ell(d,B,\theta)=\text{no}$ for every $\ell\in\mathcal{L}$ and every θ, so the total expected net income satisfies $a(\mathcal{L})\le v^{\rm sq}(\mathcal{L})+D(\mathcal{L})<v^{\rm sq}(\mathcal{L})+I(\mathcal{L})$. If instead every local in S is tentatively blocked, with $k\ge|S|\ge1$ blocking attempts in total, then by (EC.64),

$$a(\mathcal{L})=p\big(v^{\rm sq}(\mathcal{L})+I(\mathcal{L})-c(S)\big)+(1-p)\big(v^{\rm sq}(\mathcal{L})+D(\mathcal{L})\big)-\eta k<v^{\rm sq}(\mathcal{L})+I(\mathcal{L}),$$

because $c(S)>0$, $\eta k>0$, and $D(\mathcal{L})<I(\mathcal{L})$. In both cases, the outcome is dominated by the grand coalition $\{\mathcal{L}\}$, which is feasible because $\bar\pi=\{\mathcal{L}\}$: by Lemma EC.19-i), the unique equilibrium

under $\{\mathcal{L}\}$ is $(d^\star = 0, B^\star = 0)$ for every p and every η, yielding the grand coalition a net income of $v^{\rm sq}(\mathcal{L}) + I(\mathcal{L}) > a(\mathcal{L})$. Hence, every core outcome has $d^\star = 0$, for every $p \in (0,1]$.

Case (iii). Let $\bar\pi \neq \{\mathcal{L}\}$ and $I(F) > D(F)$ for some $F \in \bar\pi$. When $p < p_1$, Proposition EC.2 shows that no core outcome has $d^\star = 0$, so $(I, \bar{\mathbb{U}})$ does not prevent deforestation. When $p_1 < p < p_2$, Proposition EC.3 exhibits instances in which the core contains outcomes with $d^\star \neq 0$ for every $\eta > 0$.

Case (iv). Suppose $p \le p_2$ and $I(F) \le D(F)$ for every $F \in \bar\pi$. We first note that $\pi = \bar\pi, d^\star = 1, B^\star = 0$ constitutes a subgame-perfect equilibrium. Indeed, following any deviation by a family F that leaves at least one of its members deforesting, no coalition can unilaterally induce compliance because each family contains one of its own deforesting members and cannot block that member. If F instead sets $d_F = 0$, then F is the only coalition that could unilaterally induce compliance by blocking all remaining deforesters; when $F \neq \mathcal{L}$, its gain from doing so is

$$p\big(I(F) - D(F)\big) - \eta|\mathcal{L} \setminus F| \le 0.$$

When $F = \mathcal{L}$, choosing $d_F = 0$ changes its net income by $I(F) - D(F) \le 0$. Thus no family profitably deviates, and the tie-breaking rule selects $d_F = 1$ in case of indifference.

Now suppose the core is nonempty, and let $x := \big(\pi, (d', B'), a'\big)$ be any core outcome. For each $F \in \bar\pi$, let

$$R_F := \{i \in F : d'_i = 1\}.$$

Because every feasible partition refines $\bar\pi$, the total allocation to family F satisfies

$$\begin{aligned} a'(F) &\le \begin{cases} v^{\rm sq}(F) + I(F), & \text{if } d' = 0, \\ v^{\rm sq}(F) + p\big(I(F) - c(R_F)\big) + (1-p)D(F), & \text{if every deforester is tentatively blocked,} \\ v^{\rm sq}(F) + D(F), & \text{otherwise,} \end{cases} \\ &\le v^{\rm sq}(F) + D(F), \end{aligned}$$

where the nonnegative blocking costs have been omitted from the upper bounds. Define

$$s_F := v^{\rm sq}(F) + D(F) - a'(F) \ge 0$$

and, for each $\ell \in F$, set $a_\ell := a'_\ell + \frac{s_F}{|F|}$. Then, $a_\ell \ge a'_\ell$ for every ℓ, and

$$a(F) = v^{\rm sq}(F) + D(F) \qquad \forall F \in \bar\pi.$$

Hence, $y := \big(\bar\pi, (d^\star = 1, B^\star = 0), a\big)$ is a feasible outcome.

If y were dominated by some group H forming a partition π_H, then $a(S) \ge a'(S)$ for every $S \in \pi_H$. Because the admissible residual continuations associated with the deviation (H, π_H) do not depend on the starting outcome, the same deviation would also dominate x. This contradicts x being in the core. Therefore, y is in the core, proving that any nonempty core has an outcome with $d^\star = 1$. □

EC.6.4. Family-Dependent Blocking Cost

Consider an extension of our base model in §2 with the sole modification that the cost of blocking depends on the blocking local's family: for any family $F \in \bar{\pi}$, the cost incurred by any local $\ell \in F$ when blocking another local is η_F. We will subsequently show that the central results from our base model readily generalize, with minor modifications in the writing of the key condition $\eta \le \eta_2$ that governs whether an outcome with $d^\star = 0$ is in the core.

This modification only impacts the results under $\bar{\mathbb{U}}$ and in the case with limited cooperation. (With full cooperation, the model would reduce to the base model with blocking cost $\eta = \eta_{\mathcal{L}}$.) Consider therefore the case with $\bar{\pi} \neq \{\mathcal{L}\}$. For $\ell \in F$, the net income (1) becomes:

$$V_\ell(d,B) := \begin{cases} v_\ell^{\mathrm{sq}} + I_\ell - c_\ell \cdot d_\ell - \eta_F \sum_{i \in \mathcal{L}} B_{\ell i} & \text{if } \mathbb{C}_\ell(d,B) = \text{yes} \\ v_\ell^{\mathrm{sq}} + D_\ell - \eta_F \sum_{i \in \mathcal{L}} B_{\ell i} & \text{if } \mathbb{C}_\ell(d,B) = \text{no}, \end{cases} \tag{EC.81}$$

and each coalition $S \in \pi$ continues to maximize its aggregate net income $V_S(d,B) = \sum_{\ell \in S} V_\ell(d,B)$. The non-cooperative game of §3.2 and the cooperative game of §3.3 are otherwise unchanged.

The key observation is that all of the machinery of §3 extends because blocking decisions are made coalition by coalition. In the proofs of Lemma 1 and Lemma 2, every comparison between the benefit and the cost of blocking pertains to the blocking decisions of a *single* coalition $S \subseteq F$, and therefore carries over verbatim with η replaced by the cost η_F of the family F. The blocking costs of different families interact only where blocking capacities are *aggregated* across coalitions, as in the threshold $\eta_2(\pi)$. Concretely, Lemma 1 holds unchanged, and Lemma 2 holds for the area no-use condition with the two characterizations: for any partition $\pi \in \Pi_{\mathcal{L}}^{\preceq \bar{\pi}}$,

$$1 \in T(\pi) \iff I(S) - D(S) \le \eta_F \, |\mathcal{L} \setminus S| \quad \text{for every } F \in \bar{\pi} \text{ and } S \in \pi \text{ with } S \subseteq F, \tag{EC.82a}$$

$$0 \in T(\pi) \iff \sum_{F \in \bar{\pi}} \sum_{\substack{S \in \pi : S \subseteq F, \\ I(S) > D(S)}} \left\lfloor \frac{I(S) - D(S)}{\eta_F} \right\rfloor \ge \max_{\substack{H \in \pi: \\ I(H) \le D(H)}} |H|. \tag{EC.82b}$$

Because π is finer than $\bar{\pi}$, every coalition $S \in \pi$ is contained in a unique family $F \in \bar{\pi}$ and therefore appears in exactly one inner sum. If no coalition in π weakly prefers deforestation, the condition in (EC.82b) holds automatically.

Let us then define, for each family $F \in \bar{\pi}$, the per-family threshold:

$$\eta_1(F) := \max_{S \subseteq F} \frac{I(S) - D(S)}{|\mathcal{L} \setminus S|}.$$

Because $\bar{\pi} \neq \{\mathcal{L}\}$, every $F \in \bar{\pi}$ is a proper subset of $\mathcal{L}$ so the denominator is strictly positive for every $S \subseteq F$. Moreover, some coalition $S \subseteq F$ can profitably block all other locals if and only if $\eta_F < \eta_1(F)$. The same per-family threshold underlies both scalar thresholds:

$$\overline{\eta}_1 = \max_{F \in \bar{\pi}} \eta_1(F) \qquad \text{and} \qquad \underline{\eta}_1 = \max_{F \in \bar{\pi} : I(F) > D(F)} \eta_1(F).$$

Note that these are direct extensions of the thresholds $\underline{\eta}_1, \overline{\eta}_1$ from the base model analysis in §3.3. In contrast, the threshold η_2 does not decompose family-by-family, because the blocking threat that maintains an outcome with $d^\star=0$ is *joint*: every family contributes as many blocks as its own surplus affords *at its own cost*. Its role is now played by the following constraint on the entire vector $(\eta_F)_{F\in\bar{\pi}}$, which requires that every group H within a family that weakly prefers deforestation can be blocked by the remaining locals, each family acting at its own cost:

$$\sum_{\substack{F\in\bar{\pi}:\\ I(F\setminus H)>D(F\setminus H)}} \left\lfloor \frac{I(F\setminus H)-D(F\setminus H)}{\eta_F} \right\rfloor \geq |H|, \qquad \forall\, H\subseteq F',\ F'\in\bar{\pi} \text{ with } I(H)\leq D(H). \quad \text{(EC.83)}$$

When all blocking costs are equal, $\eta_F=\eta$ for every $F\in\bar{\pi}$, constraint (EC.83) holds if and only if $\eta\leq\eta_2$, by the definition of η_2.

Proposition EC.4 generalizes Theorem 1 and Theorem 2 to this setting. Part (b) shows that under $\bar{\mathbb{U}}$, constraint (EC.83) plays the same role as $\eta\leq\eta_2$ in the base model: it ensures that the core is non-empty and contains an outcome with $d^\star=0$. Part (c) shows that the core may be empty if neither (EC.83) nor $\eta_F\geq\eta_1(F), \forall F\in\bar{\pi}$ are satisfied; under equal blocking costs, this specializes to the result in Theorem 2 that the core can be empty for $\eta\in(\eta_2,\overline{\eta}_1)$. Parts (d) and (e) then provide conditions ensuring that the core only has outcomes with $d^\star=0$. Conditional on core non-emptiness, all core outcomes have $d^\star=0$ if and only if $\eta_F<\eta_1(F)$ for some $F\in\bar{\pi}$ satisfying $I(F)>D(F)$; this is a family-by-family condition involving only one family's cost. Separately, condition (EC.83), which aggregates all families' blocking capacities, is sufficient to guarantee that the core contains at least an outcome with $d^\star=0$.

Proposition EC.4. *Suppose that $\bar{\pi}\neq\{\mathcal{L}\}$. The core under $(I,\bar{\mathbb{U}})$ and its outcomes satisfy:*

(a) If (EC.83) *does not hold and $\eta_F\geq\eta_1(F)$ does not hold for all $F\in\bar{\pi}$, the core can be empty; this is possible even when $\eta_F=\eta$ for all $F\in\bar{\pi}$.*

(b) If $\{\eta_F\}_{F\in\bar{\pi}}$ satisfy (EC.83)*, the core is non-empty and contains an outcome with $d^\star=0$ wherein every local earns his net income with the incentive, $a_\ell=v_\ell^{\text{sq}}+I_\ell$.*

(c) If $\eta_F\geq\eta_1(F)$ for every $F\in\bar{\pi}$, the core contains an outcome with $d^\star=1$.

(d) A non-empty core contains only outcomes with $d^\star=0$ if and only if $\eta_F<\eta_1(F)$ for some $F\in\bar{\pi}$ with $I(F)>D(F)$.

(e) If (EC.83) *holds and $\eta_F<\eta_1(F)$ for some $F\in\bar{\pi}$ with $I(F)>D(F)$, then the core is non-empty and every core outcome has $d^\star=0$.*

Proof of Proposition EC.4. For this proof, define $\Delta_\ell:=I_\ell-D_\ell$ for all $\ell\in\mathcal{L}$. As explained before the proposition, Lemma 1 extends verbatim and Lemma 2 extends in the form (EC.82a)–(EC.82b), because both proofs compare blocking benefits and costs one coalition at a time.

Part (a). For the possibility of an empty core, uniform-cost instances suffice: Lemma EC.3 exhibits instances and values $\eta \in (\eta_2, \overline{\eta}_1)$ with an empty core. With $\eta_F = \eta$ for every $F \in \bar{\pi}$, (EC.83) fails because $\eta > \eta_2$, while $\eta < \overline{\eta}_1 = \max_{F \in \bar{\pi}} \eta_1(F)$ implies $\eta_F < \eta_1(F)$ for a maximizing family.

Part (b). We prove by induction on $|R|$ the following residual-game claim. For every nonempty residual set $R \subseteq \mathcal{L}$ and every fixed partition $\pi_{\mathcal{L}\setminus R} \in \Pi^{\prec \bar{\pi}}_{\mathcal{L}\setminus R}$ satisfying

$$\Delta(H) \leq 0, \qquad \forall\, H \in \pi_{\mathcal{L}\setminus R},$$

the residual core $C(R; \pi_{\mathcal{L}\setminus R})$ contains the outcome

$$\pi_R = \{\, \mathcal{G} \cap R \cap F : F \in \bar{\pi},\ \mathcal{G} \cap R \cap F \neq \emptyset \,\} \cup \{\, \{\ell\} : \ell \in R \setminus \mathcal{G} \,\}, \tag{EC.84a}$$

$$d^\star = 0, \tag{EC.84b}$$

$$a_\ell = v_\ell^{\text{sq}} + I_\ell, \qquad \forall\, \ell \in R. \tag{EC.84c}$$

We first verify that this is a valid residual outcome. Let

$$\sigma := \pi_R \cup \pi_{\mathcal{L}\setminus R}.$$

The coalitions in σ that strictly prefer the incentive are exactly the nonempty coalitions $\mathcal{G} \cap R \cap F$, $F \in \bar{\pi}$. Every other coalition in σ weakly prefers deforestation: this holds by hypothesis for the coalitions in $\pi_{\mathcal{L}\setminus R}$ and by the definition of $\mathcal{G}$ for the singleton coalitions $\{\ell\}$ with $\ell \in R \setminus \mathcal{G}$.

If σ contains no coalition that weakly prefers deforestation, then $0 \in T(\sigma)$ follows immediately from (EC.82b). Otherwise, let $\bar{H}$ be a largest such coalition, so that

$$|\bar{H}| = \max_{\substack{H \in \sigma: \\ \Delta(H) \leq 0}} |H|.$$

Because $\bar{H}$ is contained in one family and $\Delta(\bar{H}) \leq 0$, condition (EC.83) applied to $\bar{H}$ gives

$$\sum_{\substack{F \in \bar{\pi}: \\ \Delta(F \setminus \bar{H}) > 0}} \left\lfloor \frac{\Delta(F \setminus \bar{H})}{\eta_F} \right\rfloor \geq |\bar{H}|. \tag{EC.85}$$

For every family $F \in \bar{\pi}$, we have

$$\Delta(F \setminus \bar{H}) \leq \Delta(F \cap R \cap \mathcal{G}). \tag{EC.86}$$

Indeed, decompose F into $F \cap R \cap \mathcal{G}$, $F \cap (R \setminus \mathcal{G})$, and $F \cap (\mathcal{L} \setminus R)$. The second set has negative total Δ, while the third set is partitioned by coalitions in $\pi_{\mathcal{L}\setminus R}$, each with weakly negative total Δ. If $\bar{H} \subseteq F$, removing $\bar{H}$ removes one of these weakly negative components; if $\bar{H} \nsubseteq F$, no component is removed. This proves (EC.86).

Crucially, both sides of (EC.86) concern subsets of the same family F and therefore use the same blocking cost η_F. Consequently, for every family appearing in the sum in (EC.85),

$$\left\lfloor \frac{\Delta(F \setminus \bar{H})}{\eta_F} \right\rfloor \leq \left\lfloor \frac{\Delta(F \cap R \cap \mathcal{G})}{\eta_F} \right\rfloor .$$

Summing these inequalities and using (EC.85) yields

$$\sum_{\substack{F \in \bar{\pi}: \\ F \cap R \cap \mathcal{G} \neq \emptyset}} \left\lfloor \frac{\Delta(F \cap R \cap \mathcal{G})}{\eta_F} \right\rfloor \geq |\bar{H}| = \max_{\substack{H \in \sigma: \\ \Delta(H) \leq 0}} |H|.$$

The left-hand side is exactly the aggregate blocking capacity of the coalitions in σ that strictly prefer the incentive, with each coalition evaluated using the blocking cost of its own family. Hence (EC.82b) gives $0 \in T(\sigma)$. The allocation in (EC.84c) is therefore feasible for the partition and deforestation decision in (EC.84a)–(EC.84b).

For $|R| = 1$, this valid outcome belongs to the residual core by its definition. Suppose inductively that the claim holds for every residual game with at most k residual locals, and consider $|R| = k+1$. Consider any deviating group $S \subseteq R$ that forms a partition $\pi_S \in \Pi_S^{\prec \bar{\pi}}$.

If some coalition $S_i \in \pi_S$ satisfies $\Delta(S_i) \geq 0$, then

$$w(S_i; d^\star) \leq w(S_i; 0) = \sum_{\ell \in S_i} \left(v_\ell^{\text{sq}} + I_\ell\right) = a(S_i), \qquad \forall\, d^\star \in \{0,1\}.$$

Thus, the deviation cannot make every coalition formed by the deviators strictly better off.

Suppose instead that $\Delta(S_i) < 0$ for every $S_i \in \pi_S$. Then every coalition in the fixed partition $\pi_S \cup \pi_{\mathcal{L} \setminus R}$ weakly prefers deforestation. Moreover, $R \setminus S \neq \emptyset$. Otherwise, if $S = R$, this fixed partition would partition every family into coalitions with weakly negative total Δ, implying $\Delta(F) \leq 0$ for every $F \in \bar{\pi}$. But then condition (EC.83) would fail when applied to $H = F'$ for any $F' \in \bar{\pi}$: because the families are disjoint, $\Delta(F \setminus F') \leq 0$ for every $F \in \bar{\pi}$, so the left-hand side of (EC.83) would equal zero, whereas its right-hand side would equal $|F'| > 0$.

We may therefore apply the inductive hypothesis to the nonempty residual set $R \setminus S$ and the fixed partition $\pi_S \cup \pi_{\mathcal{L} \setminus R}$. It follows that

$$0 \in A(R \setminus S; \pi_S \cup \pi_{\mathcal{L} \setminus R}).$$

Consequently, for every $S_i \in \pi_S$,

$$\min_{d^\star \in A(R \setminus S; \pi_S \cup \pi_{\mathcal{L} \setminus R})} w(S_i; d^\star) \leq w(S_i; 0) = a(S_i).$$

Thus, the deviation also fails under pessimism. This completes the induction.

Applying the residual-game claim with $R = \mathcal{L}$ and $\pi_{\mathcal{L} \setminus R} = \emptyset$ proves that the core contains the outcome

$$\pi = \{\, \mathcal{G} \cap F : F \in \bar{\pi},\ \mathcal{G} \cap F \neq \emptyset \,\} \cup \{\, \{\ell\} : \ell \in \mathcal{L} \setminus \mathcal{G} \,\}, \qquad d^\star = 0, \qquad a_\ell = v_\ell^{\text{sq}} + I_\ell \quad \forall\, \ell \in \mathcal{L}.$$

In particular, the core is non-empty.

Finally, if $\eta_F \leq \eta_2$ for every $F \in \bar{\pi}$, then (EC.83) holds. Indeed, the uniform vector with every component equal to η_2 satisfies the required inequalities by the definition of η_2, and replacing any denominator η_2 with a weakly smaller η_F can only weakly increase the corresponding floor term.

Part (c). If $\eta_F \geq \eta_1(F)$ for every $F \in \bar{\pi}$, then for every partition $\pi \in \Pi_{\mathcal{L}}^{\preceq \bar{\pi}}$ and every $S \in \pi$ with $S \subseteq F$: either $\Delta(S) \leq 0 \leq \eta_F |\mathcal{L} \setminus S|$, or $\Delta(S)/|\mathcal{L} \setminus S| \leq \eta_1(F) \leq \eta_F$ (note $S \neq \mathcal{L}$ because $\bar{\pi} \neq \{\mathcal{L}\}$). By (EC.82a), $1 \in T(\pi)$ for every $\pi \in \Pi_{\mathcal{L}}^{\preceq \bar{\pi}}$. The induction in the proof of Lemma EC.2(b) involves blocking costs only through this threshold comparison, and hence applies verbatim and yields the stated outcome in the core.

Part (d). *(If.)* Suppose $\eta_F < \eta_1(F)$ for some $F \in \bar{\pi}$ with $\Delta(F) > 0$. Then there exists $S \subseteq F$ with

$$I(S) - D(S) > \eta_F \, |\mathcal{L} \setminus S| \geq 0,$$

and, as in the proof of Lemma EC.6(i), we can take $S \supseteq \mathcal{G} \cap F$. The argument there applies verbatim with η_F in place of η: coalition S's threat of blocking all other locals uses only its own family's cost, so by (EC.82a)–(EC.82b), $\{0\} = T(\{S\} \cup \pi_{\mathcal{L} \setminus S})$ for every $\pi_{\mathcal{L} \setminus S} \in \Pi_{\mathcal{L} \setminus S}^{\preceq \bar{\pi}}$, and every outcome with $d^\star\!=\!1$ is dominated by S deviating (the allocation accounting, which uses Lemma EC.1, involves no blocking costs, and that lemma's proof – a singleton deviation – is unchanged).

(Only if.) Suppose $\eta_F \geq \eta_1(F)$ for every $F \in \bar{\pi}$ with $\Delta(F) > 0$, and suppose the core is non-empty. If $\Delta(F) \leq 0$ for every $F \in \bar{\pi}$, the core contains an outcome with $d^\star\!=\!1$ by Lemma EC.13(i), whose proof involves no blocking costs. Otherwise, the induction in the proof of Lemma EC.6(ii) applies: every partition arising there has all its coalitions with positive Δ inside families that prefer the incentive, so for each such coalition $S \subseteq F$, $\Delta(S)/|\mathcal{L} \setminus S| \leq \eta_1(F) \leq \eta_F$, and (EC.82a) yields $1 \in T(\pi)$ wherever the original proof invoked $\eta \geq \eta_1(\pi)$ for the partition π at hand; deviations by groups within families with $\Delta(F) \leq 0$ are handled by the allocation comparison (EC.35), which involves no blocking costs. Hence $0 \in A(\mathcal{L}; \emptyset)$ implies $1 \in A(\mathcal{L}; \emptyset)$: a non-empty core containing an outcome with $d^\star\!=\!0$ also contains an outcome with $d^\star\!=\!1$.

Part (e). By Part (b), the core contains an outcome with $d^\star\!=\!0$; in particular it is non-empty. By Part (d) (if direction), every outcome with $d^\star\!=\!1$ is dominated, so all core outcomes have $d^\star\!=\!0$. $\square$

EC.6.5. When Some Locals Would Not Benefit From Deforestation

Whereas our base model assumed that all locals have strictly positive net gain from deforestation, we now relax this assumption by allowing $D_\ell = 0$ for some locals $\ell \in \mathcal{L}$. We will subsequently show that any such instance can be mapped into one satisfying the base-model assumption by adding $\varepsilon > 0$ to both I_ℓ and D_ℓ for every such local. This transformation will preserve all differences $I(S) - D(S)$

for any $S \subseteq \mathcal{L}$ and add only a decision-independent constant to each affected local's net income. Consequently, all our base-model results will be applicable for analyzing this changed model.

To formalize the setting, let $\mathcal{L}^0 := \{\ell \in \mathcal{L} : D_\ell = 0\}$ denote the set of locals who do not increase their net income by deforesting land. Every other element of the model is unchanged. In particular: (i) deforesting land remains costly, $c_\ell > 0$, even for a local ℓ with $D_\ell = 0$;[8] (ii) both area conditions remain defined with respect to *all* locals in $\mathcal{L}$: under $\bar{\mathbb{D}}$, no local receives the incentive if $d_i = 1$ for some $i \in \mathcal{L}$, and under $\bar{\mathbb{U}}$, no local receives the incentive if $d_i\big(1 - \max_{g\in\mathcal{L}} B_{gi}\big) = 1$ for some $i \in \mathcal{L}$; (iii) net incomes are given by (1)[9]; and (iv) every local retains a strict preference, $I_\ell \neq D_\ell$.

All of our results translate to this extension. Rather than repeating each proof, we observe that every instance of the extension is *strategically equivalent* to a base-model instance in which D_ℓ and I_ℓ are shifted by the same constant for all locals in $\mathcal{L}^0$.

Lemma EC.21. *Fix $\varepsilon > 0$ and consider the modified instance with values $\tilde{I}_\ell := I_\ell + \varepsilon$ and $\tilde{D}_\ell := D_\ell + \varepsilon$ for $\ell \in \mathcal{L}^0$ and values $\tilde{I}_\ell := I_\ell$ and $\tilde{D}_\ell := D_\ell$ for $\ell \notin \mathcal{L}^0$, and all other primitives $(v^{\mathrm{sq}}, c, \eta, \bar{\pi})$ unchanged. Then:*

(i) The modified instance satisfies the base-model assumption $\tilde{D}_\ell > 0$ for all $\ell \in \mathcal{L}$ and preserves all value differences: $\tilde{I}(S) - \tilde{D}(S) = I(S) - D(S)$ for every $S \subseteq \mathcal{L}$. In particular, the set $\mathcal{G}$, every coalition's preference between the incentive and deforestation, and the thresholds $\eta_1(\pi), \eta_2(\pi)$ of Lemma 2 and $\overline{\eta}, \eta_2, \overline{\eta}_1, \underline{\eta}_1$ of (6)–(9) coincide across the two instances.

(ii) For every partition π, condition $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$, and decisions (d, B), the net income (1) of each local ℓ in the modified instance equals $V_\ell(d, B) + \varepsilon \cdot \mathbb{1}\{\ell \in \mathcal{L}^0\}$. Consequently, each local's preference between $d_\ell = 0$ and $d_\ell = 1$ under $\mathbb{I}$ is the same in the two instances and, for $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$, the two instances have identical sets of subgame-perfect equilibria $\mathcal{Q}(\pi, \mathbb{C})$ and identical equilibrium indicators $T(\pi, \mathbb{C})$.

(iii) An outcome $(\pi, d^\star, a)$ is in the core of the original instance if and only if $(\pi, d^\star, \tilde{a})$ with $\tilde{a}_\ell := a_\ell + \varepsilon \cdot \mathbb{1}\{\ell \in \mathcal{L}^0\}$ is in the core of the modified instance. Corresponding outcomes have the same $d^\star$, and one compensates if and only if the other does.

Proof of Lemma EC.21. (i) For $\ell \in \mathcal{L}^0$ we have $\tilde{D}_\ell = \varepsilon > 0$, and the modification adds ε to both values of each $\ell \in \mathcal{L}^0$, so $\tilde{I}(S) - \tilde{D}(S) = I(S) - D(S)$ for every $S \subseteq \mathcal{L}$. Each object listed in (i) depends on the values only through these differences (together with cardinalities and η), so each is unchanged.

(ii) Whether the locals comply with $\mathbb{C}$ depends only on (d, B), not on the values, and each branch of (1) contains exactly one of I_ℓ (if $\mathbb{C}_\ell = \text{yes}$) or D_ℓ (if $\mathbb{C}_\ell = \text{no}$). Hence, for every (d, B), the

[8] We also conducted the analysis under the alternative assumption that a local ℓ with $D_\ell = 0$ would not deforest land in any equilibrium and would not incur the cost c_ℓ. The qualitative insights are similar and are omitted for brevity.

[9] The rationale for the recourse assumption underlying (1) extends here: a local ℓ with $D_\ell = 0$ who does not receive the incentive is indifferent between subsequently deforesting land or not, and either way earns net income $v_\ell^{\mathrm{sq}} + D_\ell = v_\ell^{\mathrm{sq}}$.

modification raises local ℓ's net income by exactly $\varepsilon \cdot \mathbb{1}\{\ell \in \mathcal{L}^0\}$. Under $\mathbb{I}$, each local's comparison between $d_\ell = 0$ and $d_\ell = 1$ therefore shifts by the same constant on both sides and is unchanged. Under $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$, the modification raises every coalition S's net income by the constant $\varepsilon\,|S \cap \mathcal{L}^0|$, independently of the decisions. Adding a decision-independent constant to a coalition's objective changes neither the best responses in (2a)–(2b) nor the indifferences resolved by the tie-breaking rule, so the two instances have identical sets of subgame-perfect equilibria and identical $T(\pi, \mathbb{C})$.

(iii) By (ii), the partition-function values satisfy $\tilde{w}(S; d^\star) = w(S; d^\star) + \varepsilon\,|S \cap \mathcal{L}^0|$ for every coalition S and equilibrium indicator $d^\star$. The map $a \mapsto \tilde{a}$ is a bijection between the feasible allocations of the two instances for any $(\pi, d^\star)$, and it preserves every domination comparison (4) because both sides of each comparison shift by the same constant $\varepsilon\,|S \cap \mathcal{L}^0|$. By induction on the size of the residual set in Definition 2, the residual cores of the two instances correspond under this map, and hence so do the cores $C(\mathcal{L}; \emptyset)$. Finally, $\tilde{a}_\ell \geq v_\ell^{\mathrm{sq}} + \tilde{D}_\ell$ if and only if $a_\ell \geq v_\ell^{\mathrm{sq}} + D_\ell$, so an outcome compensates in one instance if and only if the corresponding outcome does in the other. □

Proposition EC.5. *Suppose $D_\ell \geq 0$ for every $\ell \in \mathcal{L}$, with the remaining assumptions as in the base model (in particular, $c_\ell > 0$ and $I_\ell \neq D_\ell$ for every $\ell \in \mathcal{L}$). Then the characterization of the individual condition $\mathbb{I}$ in §3.1, Lemma 1, Lemma 2 (with the thresholds $\eta_1(\pi), \eta_2(\pi)$ unchanged), Proposition 1, and Theorems 1 and 2 (with the thresholds* (6)–(9) *unchanged) hold as stated, and the recommendations in §3.4 are unchanged.*

Proof of Proposition EC.5. Consider first the assertions that must hold for *every* instance. Apply each base-model result to the modified instance of Lemma EC.21 and translate back. Every such assertion is invariant under the translation, because it refers only to: (a) value differences $I(S) - D(S)$ (through $\mathcal{G}$, conditions such as $I(F) > D(F)$, and the thresholds), (b) the blocking cost η and cardinalities of sets of locals, (c) objects preserved by Lemma EC.21(ii)–(iii) (equilibrium decisions, membership of $d^\star$ in $T(\pi, \mathbb{C})$, non-emptiness of the core), and (d) the compensation status of core outcomes. The one exception is the bound $a_\ell \geq v_\ell^{\mathrm{sq}}$ in Proposition 1, whose right-hand side is not translated; but its proof in §EC.5.2 applies analogously, as it only requires $I_\ell \geq 0$ and $D_\ell \geq 0$. The remaining assertions state that some property is *possible* (for instance, that the core *can be* empty, or that it is *possible* that no outcome has $d^\star = 0$); each is established by exhibiting an instance, and every base-model instance is also an instance of this extension, so these assertions carry over unchanged. □

Two remarks are in order. First, under the maintained tie-breaking rule, locals with $D_\ell = 0$ remain *potential deforesters*. By Lemma EC.21(ii), for every partition π and condition $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$, the original and modified instances have identical sets of subgame-perfect equilibria. Because the modified instance satisfies $\tilde{D}_\ell > 0$ for every $\ell \in \mathcal{L}$, Lemma 1 implies that any equilibrium with $d^\star = 1$ has $d_\ell^\star = 1$

for every $\ell \in \mathcal{L}$. The same conclusion therefore holds in the original instance, including for locals in $\mathcal{L}^0$. Consequently, the cardinalities appearing in the blocking-cost thresholds – e.g., $|\mathcal{L} \setminus S|$ in (8)–(9) and $|H|$ in (7) – correctly count *all* locals, not only those with $D_\ell > 0$: under $\bar{\mathbb{U}}$, a coalition that sustains an outcome with $d^\star = 0$ through a blocking threat must be prepared to block deviations by locals with $D_\ell = 0$ as well.

Under $\bar{\mathbb{U}}$, treating such locals as potential deforesters can also have strategic significance. If a coalition deviates toward deforestation, having an additional member $\ell \in \mathcal{L}^0$ set $d_\ell = 1$ increases the number of deforesting locals whom the other coalitions must block to restore compliance. This can make blocking unprofitable and thereby sustain the deviation. The benefit is not automatic, however: if the deforestation attempt is nevertheless blocked and compliance is restored, local ℓ incurs the strictly positive cost c_ℓ.

Second, although locals with $D_\ell = 0$ add nothing to the value of deforestation, they are not passive in the analysis. Each such local belongs to $\mathcal{G}$, so his value $I_\ell > 0$ enters every relevant surplus $I(S) - D(S)$ – strengthening his coalition's preference for the incentive and its capacity for blocking threats – while his own compensation requirement, $a_\ell \geq v_\ell^{\text{sq}} + D_\ell = v_\ell^{\text{sq}}$, is met in every core outcome with $d^\star = 0$ by Proposition 1.

EC.6.6. Hybrid Conditions

We allow the buyer to partition the area into non-overlapping sub-areas and apply one condition to all locals in each sub-area. Formally, let $\mathcal{A}$ be a partition of the locals, where $H \in \mathcal{A}$ is the set of locals who can deforest in the corresponding sub-area. For every $H \in \mathcal{A}$, define the following conditions:

1. *Sub-area No-Deforestation Condition*, $\bar{\mathbb{D}}^H$. Every local $\ell \in H$ receives the incentive if and only if no local in H engages in deforestation:

$$\bar{\mathbb{D}}_\ell^H(d, B) = \text{yes} \quad \Longleftrightarrow \quad d_i = 0 \quad \text{for every } i \in H.$$

2. *Sub-area No-Use Condition*, $\bar{\mathbb{U}}^H$. Every local $\ell \in H$ receives the incentive if and only if no local in H generates income on deforested land:

$$\bar{\mathbb{U}}_\ell^H(d, B) = \text{yes} \quad \Longleftrightarrow \quad d_i\big(1 - \max_{g \in \mathcal{L}} B_{gi}\big) = 0 \quad \text{for every } i \in H.$$

A *hybrid conditional incentive* consists of a sub-area partition $\mathcal{A}$, incentive values $I_\ell \geq 0$, and one common condition for the locals in each sub-area:

$$\mathbb{C}_\ell = \mathbb{C}^H \in \{\bar{\mathbb{D}}^H, \bar{\mathbb{U}}^H\} \quad \text{for every } H \in \mathcal{A} \text{ and } \ell \in H. \tag{EC.87}$$

The uniform area conditions are obtained by taking $\mathcal{A} = \{\mathcal{L}\}$, while the individual condition is obtained by taking the singleton partition because $\bar{\mathbb{D}}_\ell^{\{\ell\}} = \mathbb{I}_\ell$.

For any coalition partition, the non-cooperative objective separates across sub-areas. A deforestation or blocking decision in H can affect only compliance with the condition in H, and every coalition's aggregate net income is the sum of its net incomes from the sub-areas in which it has members. The equilibrium decisions can therefore be obtained by solving the corresponding base-model non-cooperative game separately in every sub-area and combining the solutions. Different sub-areas may select different equilibrium deforestation decisions, so throughout this subsection one should think of $d^\star$ as the vector from $\{0,1\}^{\mathcal{L}}$. Then, $d^\star = 0^{\mathcal{L}} \equiv 0$ means that no local in any sub-area deforests, and $d^\star = 1^{\mathcal{L}} \equiv 1$ means that every local deforests.

Moreover, because each family belongs entirely to one sub-area, the cooperative game also separates into one cooperative game in each sub-area. Specifically, within each sub-area, we can define a cooperative game with a suitable partition function coming from the associated non-cooperative game for that sub-area, and characterize its core using the results from §3.3. The formalism and derivations are straightforward and omitted for brevity. We use the term *hybrid core* subsequently to refer to the cartesian product of the cores corresponding to sub-area cooperative games; the hybrid core corresponds to area-level outcomes.

We focus only on preventing deforestation. As in §3.4, the relevant requirement depends on whether the buyer can induce a preferred type of core outcome: the hybrid core must be non-empty and contain at least one outcome with $d^\star=0$ when the buyer can induce such an outcome, whereas the hybrid core must be non-empty and all its outcomes must have $d^\star=0$ otherwise.

Our first result concerns the case with full cooperation.

Proposition EC.6 **(Hybrid conditions under full cooperation)**. *Suppose $\bar{\pi} = \{\mathcal{L}\}$ and consider any hybrid conditional incentive in the class* (EC.87).

(i) If the hybrid core contains an outcome with $d^\star=0$, then $I(H) \geq D(H)$ for every $H \in \mathcal{A}$. In particular, $I(\mathcal{L}) \geq D(\mathcal{L})$.

(ii) If the hybrid core contains an outcome with $d^\star=0$ and $I(\mathcal{L}) = D(\mathcal{L})$, then the core also contains an outcome with $d^\star=1$.

(iii) If the hybrid core is non-empty and every hybrid core outcome has $d^\star=0$, then $I(\mathcal{L}) > D(\mathcal{L})$.

Proof of Proposition EC.6. Consider any outcome $(\pi, 0, a)$. Because no local deforests, every sub-area condition is met and every local receives the incentive. Feasibility therefore implies

$$a(\mathcal{L}) = v^{\text{sq}}(\mathcal{L}) + I(\mathcal{L}). \tag{EC.88}$$

For part (i), fix $H \in \mathcal{A}$. Because the grand coalition is feasible under full cooperation, all locals can deviate and form $\{\mathcal{L}\}$. The grand coalition can choose a decision profile in which every local in H

deforests, no local outside H deforests, and no blocking occurs. Hence its optimal net income from deviating is at least

$$v^{\mathrm{sq}}(\mathcal{L})+D(H)+I(\mathcal{L}\setminus H).$$

If $I(H)<D(H)$, this amount is strictly greater than the right-hand side of (EC.88). The grand coalition would then dominate $(\pi,0,a)$. Hence, any hybrid core outcome with $d^\star=0$ requires $I(H)\geq D(H)$ for every $H\in\mathcal{A}$.

For (ii), summing the inequalities from (i) over the sub-area partition and using $I(\mathcal{L})=D(\mathcal{L})$ gives

$$I(H)=D(H) \qquad \text{for every } H\in\mathcal{A}.$$

Thus, when all locals form the grand coalition, it is indifferent between compliance and deforestation in each sub-area. Under the maintained tie-breaking convention, $\{\mathcal{L}\}$ chooses deforestation for every local, so the outcome with partition $\{\mathcal{L}\}$ and $d^\star=1$ is feasible. Its total net income is

$$v^{\mathrm{sq}}(\mathcal{L})+D(\mathcal{L})=v^{\mathrm{sq}}(\mathcal{L})+I(\mathcal{L})=a(\mathcal{L}).$$

Because utility is transferable within the grand coalition, the allocation a is feasible with this outcome. If a deviation dominated $(\{\mathcal{L}\},1,a)$, the same deviation would dominate $(\pi,0,a)$: the dominance test depends on a, the deviating partition, and the resulting residual games, none of which depends on the original partition or deforestation profile. This contradicts $(\pi,0,a)$ being in the hybrid core. Hence $(\{\mathcal{L}\},1,a)$ is also in the hybrid core.

Part (iii) follows immediately. If every core outcome has $d^\star=0$, part (ii) rules out equality, while part (i) implies $I(\mathcal{L})\geq D(\mathcal{L})$. Hence the aggregate inequality must be strict. The converse for the uniform condition $\bar{\mathbb{D}}$ follows from Theorem 1-A–B. Finally, if $I(\mathcal{L})>D(\mathcal{L})$ but $I(H)<D(H)$ for some sub-area H, the same theorem gives a non-empty core consisting only of outcomes with $d^\star=0$ under uniform $\bar{\mathbb{D}}$, whereas part (i) rules out every such core outcome under the hybrid scheme. □

The first key implication of Proposition EC.6 comes by recalling the results in Theorem 1. Under a uniform area no-deforestation condition $\bar{\mathbb{D}}$, the core is non-empty and every core outcome has $d^\star=0$ if $I(\mathcal{L})>D(\mathcal{L})$. Part (iii) of Proposition EC.6 shows that if the hybrid core only contains outcomes with $d^\star=0$, so will the core under the area no-deforestation condition. Moreover, there are problem instances wherein the latter contains such an outcome but the former does not: if $I(\mathcal{L})>D(\mathcal{L})$ holds but $I(H)<D(H)$ for some $H\in\mathcal{A}$ does not hold, the uniform condition $\bar{\mathbb{D}}$ has a non-empty core consisting only of outcomes with $d^\star=0$, whereas a hybrid scheme would have *no* core outcome with $d^\star=0$.

Next, we consider the case with limited cooperation. Here, we make the additional assumption that families are not divided across sub-areas: for every $F\in\bar{\pi}$, there exists $H\in\mathcal{A}$ such that $F\subseteq H$. (Note that we did not impose this restriction in Proposition EC.6; imposing it under full cooperation would have forced $\mathcal{A}=\{\mathcal{L}\}$, mooting any comparisons.)

Proposition EC.7 **(No-deforestation conditions on sub-areas)**. *Suppose every family is contained in one sub-area and every sub-area $H \in \mathcal{A}$ is assigned the condition $\bar{\mathbb{D}}^H$. Then the following statements hold:*

(i) The hybrid core is non-empty.

(ii) At least one hybrid core outcome has $d^\star = 0$ if and only if

$$I(F) > D(F) \qquad \text{for every } F \in \bar{\pi}.$$

(iii) Every hybrid core outcome has $d^\star = 0$ if and only if

$$\mathcal{A} = \bar{\pi} \qquad \text{and} \qquad I(F) > D(F) \quad \text{for every } F \in \bar{\pi}.$$

The proof follows from a straightforward application of Theorem 1 for each relevant sub-area, and is omitted for brevity.

Proposition EC.7 gives a direct comparison with the area no-deforestation condition $\bar{\mathbb{D}}$. Under limited cooperation, every partition into sub-areas governed by $\bar{\mathbb{D}}^H$ has the same requirement for the hybrid core to contain at least one outcome with $d^\star = 0$: every family must prefer the incentive. However, with more than two families, the area no-deforestation condition would fail in that case due to coordination problems (recall that the best recommendation for an area-wide incentive would then be $\mathbb{I}$). In contrast, that hybrid condition is uniquely effective in that case: with an incentive that ensures that $I(H) > D(H)$ for $H \in \mathcal{A}$, it would remove the coordination problem and would ensure that every hybrid core outcome has $d^\star = 0$. (And it would also be strictly more robust than a perfect incentive with the individual condition $\mathbb{I}$.)

We next consider a mixed scheme that assigns separate no-deforestation conditions to selected families and a single no-use condition to all remaining families. This structure is sufficient to capture the central benefit and cost of hybridization under limited cooperation.

Proposition EC.8 **(Hybrid $\bar{\mathbb{D}} - \bar{\mathbb{U}}$ with limited cooperation)**. *Suppose $\bar{\pi} \neq \{\mathcal{L}\}$. Let $\mathcal{H} \subsetneq \bar{\pi}$ and $\mathcal{L}' := \mathcal{L} \setminus \bigcup_{F' \in \mathcal{H}} F'$. Assign the condition $\bar{\mathbb{D}}^{F'}$ to every family $F' \in \mathcal{H}$ and the condition $\bar{\mathbb{U}}^{\mathcal{L}'}$ to all locals in $\mathcal{L}'$. Consider the base-model game restricted to $\mathcal{L}'$, with family partition $\bar{\pi} \setminus \mathcal{H}$, incentive values restricted to $\mathcal{L}'$, and condition $\bar{\mathbb{U}}$.*

(i) The hybrid core is non-empty if and only if the core of the restricted no-use game is non-empty.

(ii) The hybrid core has at least one outcome with $d^\star = 0$ if and only if $I(F') > D(F')$ for every $F' \in \mathcal{H}$ and the restricted no-use core contains at least one outcome with no deforestation in $\mathcal{L}'$.

(iii) The hybrid core is non-empty and every hybrid core outcome has $d^\star = 0$ if and only if $I(F') > D(F')$ for every $F' \in \mathcal{H}$ and the restricted no-use core is non-empty and every one of its outcomes has no deforestation in $\mathcal{L}'$.

Proof of Proposition EC.8. The non-cooperative game separates into one game for each family $F' \in \mathcal{H}$ and one game for $\mathcal{L}'$. Each family $F' \in \mathcal{H}$ plays the base-model game under $\bar{\mathbb{D}}$ with full cooperation, while the locals in $\mathcal{L}'$ play exactly the restricted base-model game under $\bar{\mathbb{U}}$. The $\bar{\mathbb{D}}^{F'}$ component has a non-empty core in every residual game, by the argument in the proof of Proposition EC.7.

We claim that, in every residual game, the hybrid core is obtained by combining the component cores whenever the restricted no-use core is non-empty, and is empty whenever the restricted no-use core is empty. The proof is straightforward and is omitted. Part (i) follows.

By Theorem 1-B, the core of the F' component contains only outcomes with no deforestation if $I(F') > D(F')$, and contains no outcome with no deforestation otherwise. Combining this fact with the componentwise core characterization proves parts (ii) and (iii). □

Thus, all results in Theorem 2 apply directly to the reduced instance on $\mathcal{L}'$. In particular, suppose the reduced incentive is imperfect, at least two families remain in $\mathcal{L}'$, and at least one remaining family prefers the incentive. Evaluate η_2, $\underline{\eta}_1$, and $\overline{\eta}_1$ using only the locals and families in $\mathcal{L}'$. If every family in $\mathcal{H}$ prefers the incentive, then:

(a) if $\eta \le \eta_2$, the hybrid core is non-empty and contains an outcome with $d^\star = 0$;

(b) if $\eta \le \eta_2$ and $\eta < \underline{\eta}_1$, the hybrid core is non-empty and every core outcome has $d^\star = 0$;

(c) if $\eta \ge \underline{\eta}_1$ and the reduced core is non-empty, the hybrid core contains an outcome with $d^\star \ne 0$;

(d) the hybrid core is guaranteed to be non-empty also when $\eta \ge \overline{\eta}_1$, but in that range it need not contain an outcome with $d^\star = 0$.

The remaining cases – a perfect incentive on $\mathcal{L}'$, no remaining local who prefers the incentive, or only one remaining family – are likewise given directly by the corresponding parts of Theorem 2.

For a fixed choice of families assigned separate no-deforestation conditions, the comparison with a uniform area no-use condition is not monotone. Restricting the no-use condition to $\mathcal{L}'$ reduces the number of outsiders whom an enforcing coalition must threaten to block and removes the families in $\mathcal{H}$ from the reduced no-use game; both effects can improve performance. However, it also removes those families' value from the incentive from the resources available to support blocking in the no-use game, which can reduce η_2 and worsen performance. For example, consider two singleton families $F_1 = \{\ell_1\}$ and $F_2 = \{\ell_2\}$ with $D_{\ell_1} = D_{\ell_2} = 1$, $I_{\ell_1} = 3$, $I_{\ell_2} = 0$, and $\eta = 1$. Under the uniform condition $\bar{\mathbb{U}}$, the thresholds are $\eta_2 = \underline{\eta}_1 = 2$, so the core is non-empty and every core outcome has no deforestation. If F_1 is instead assigned $\bar{\mathbb{D}}^{F_1}$ and F_2 is assigned $\bar{\mathbb{U}}^{F_2}$, the latter component has full cooperation and strictly prefers deforestation, so the hybrid core contains no outcome with no deforestation. The following instance shows that the opposite comparison can also hold: assigning separate no-deforestation conditions can strictly improve performance when the buyer does not induce a preferred type of core outcome.

Instance 2 (A mixed hybrid can strictly improve robust performance). Consider the following four-local, three-family instance at blocking cost $\eta = 1$:

$$\mathcal{L} = \{\ell_1, \ell_2, \ell_3, \ell_4\}, \bar{\pi} = \{F_1, F_2, F_3\}, \qquad F_1 = \{\ell_1, \ell_2\}, \quad F_2 = \{\ell_3\}, \quad F_3 = \{\ell_4\}.$$

For some $0 < \epsilon < 1/2$, let

$$D_{\ell_1} = D_{\ell_2} = D_{\ell_3} = \tfrac{1}{4}, \quad D_{\ell_4} = 10,$$
$$I_{\ell_1} = I_{\ell_2} = \tfrac{1}{4} + \epsilon, \qquad I_{\ell_3} = \tfrac{5}{4} + \epsilon, \qquad I_{\ell_4} = 0.$$

Assign $\bar{\mathbb{D}}^{F_1}$ to F_1 and $\bar{\mathbb{U}}^{F_2 \cup F_3}$ to $F_2 \cup F_3$. The hybrid core is non-empty and every hybrid core outcome has $d^\star = 0$. Under the same incentive, no uniform condition in $\{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$ has a non-empty core consisting only of outcomes with $d^\star = 0$.

Proof of Instance 2. For family F_1,

$$I(F_1) - D(F_1) = 2\epsilon > 0,$$

so Theorem 1-B implies that the $\bar{\mathbb{D}}^{F_1}$ component has a non-empty core consisting only of outcomes with no deforestation.

In the reduced no-use game on $F_2 \cup F_3$, $I_{\ell_3} - D_{\ell_3} = 1 + \epsilon$ and local ℓ_4 prefers deforestation. Directly from (7) and (9), evaluated for this two-local reduced instance,

$$\eta_2 = \underline{\eta}_1 = 1 + \epsilon.$$

Because $\eta = 1 < 1 + \epsilon$, Part A-ii of Theorem 2 implies that the reduced core is non-empty and every reduced core outcome has no deforestation. Part (iii) of Proposition EC.8 therefore gives the claimed performance of the hybrid scheme.

Now consider the uniform conditions. The individual condition $\mathbb{I}$ does not prevent deforestation because $I_{\ell_4} < D_{\ell_4}$, by §3.1. The uniform area no-deforestation condition $\bar{\mathbb{D}}$ has no core outcome with $d^\star = 0$ because family F_3 prefers deforestation, by Theorem 1-C.

For the uniform area no-use condition $\bar{\mathbb{U}}$, the thresholds for the full four-local instance are

$$\eta_2 = 1 + \epsilon, \qquad \underline{\eta}_1 = \max\left\{\epsilon, \frac{1+\epsilon}{3}\right\} < 1.$$

Indeed, the binding coalition in the definition of η_2 is $F_3 = \{\ell_4\}$; local ℓ_3 alone supplies one unit of blocking capacity up to cost $1 + \epsilon$. In the definition of $\underline{\eta}_1$, family F_1 supplies at most the ratio $2\epsilon/(4-2) = \epsilon$, while local ℓ_3 supplies the ratio $(1+\epsilon)/(4-1) = (1+\epsilon)/3$. Because $\eta = 1 \leq \eta_2$, the uniform no-use core is non-empty and contains an outcome with $d^\star = 0$. But because $\eta = 1 \geq \underline{\eta}_1$, the same core also contains an outcome with $d^\star = 1$, by Part A-ii of Theorem 2. Hence the uniform no-use condition works only if the buyer can induce a preferred type of core outcome, whereas the mixed hybrid works even when the buyer cannot. $\square$

Proposition EC.9 **(Protecting sub-areas when area-wide prevention fails)**. *Under the mixed scheme in Proposition EC.8, suppose the hybrid core is non-empty. For every $F' \in \mathcal{H}$ satisfying $I(F') > D(F')$, no local in F' deforests in any hybrid core outcome, regardless of whether some hybrid core outcomes involve deforestation in $\mathcal{L}'$.*

Proof of Proposition EC.9. By Theorem 1-B, every core outcome of the $\bar{\mathbb{D}}^{F'}$ component has no deforestation when $I(F') > D(F')$. The result follows from Proposition EC.8. □

Proposition EC.9 clarifies a benefit of hybrid conditions that is not captured by the paper's binary requirement of preventing all deforestation. Even when the (restricted) sub-area no-use condition admits outcomes with deforestation, the families governed by successful separate no-deforestation conditions remain protected. If sub-areas differ in forest area, this can reduce the amount of forest cleared relative to a uniform condition whose failure causes the incentive to be lost throughout the entire area. Formally optimizing this objective would require weighting locals or sub-areas by the amount of forest at risk, so we leave that for future research.

EC.6.7. Optimistic Recursive Core

We next define the *optimistic* recursive core. Under optimism, deviating locals need to benefit in only one plausible outcome of the residual game. Our next proposition shows that every optimistic recursive core outcome is also a pessimistic recursive core outcome. Thus, conditional on the optimistic recursive core being non-empty, any performance requirement satisfied by every pessimistic recursive core outcome is also satisfied by every optimistic recursive core outcome.

Definition EC.1 (Optimistic Recursive Core). We define this recursively, based on the size of the residual set R. For a residual game where one local $R = \{\ell\}$ responds to a partition $\pi_{\mathcal{L}\setminus\{\ell\}}$ of the other locals, define

$$C_o(\{\ell\};\pi_{\mathcal{L}\setminus\{\ell\}}) = \Big\{\big(\{\{\ell\}\}, d^\star, a_\ell\big) : a_\ell = w\big(\{\ell\}; d^\star\big),\ d^\star \in T\big(\{\{\ell\}\} \cup \pi_{\mathcal{L}\setminus\{\ell\}}\big)\Big\}.$$

For some integer $k < |\mathcal{L}|$, suppose that optimistic recursive cores are defined for every residual game faced by at most k residual locals, i.e., $C_o(R;\pi_{\mathcal{L}\setminus R})$ is defined for every $R \subseteq \mathcal{L}$ with $|R| \le k$ and every $\pi_{\mathcal{L}\setminus R} \in \Pi^{\prec\bar{\pi}}_{\mathcal{L}\setminus R}$. For a residual game with $|R| = k+1$, the optimistic recursive core $C_o(R;\pi_{\mathcal{L}\setminus R})$ is the set of residual-game outcomes that are not *optimistically dominated*. An outcome $(\pi_R, d^\star, a)$ is optimistically dominated if there exists a non-empty group of locals $H \subseteq R$ from the same family ($H \subseteq F$ for some $F \in \bar{\pi}$) that forms a partition $\pi_H \in \Pi^{\prec\bar{\pi}}_H$, and there exist $\hat{\pi}_{R\setminus H} \in \Pi^{\prec\bar{\pi}}_{R\setminus H}$, an equilibrium indicator $\hat{d}$, and an allocation $\hat{a} \in \mathbb{R}^{R\setminus H}$ such that

$$w\big(S;\hat{d}\big) > a(S), \qquad \forall S \in \pi_H, \tag{EC.89}$$

and

$$\begin{cases} \big(\hat{\pi}_{R\setminus H}, \hat{d}, \hat{a}\big) \in C_o\big(R\setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R}\big) & \text{if } H \subsetneq R \text{ and } C_o\big(R\setminus H; \pi_H \cup \pi_{\mathcal{L}\setminus R}\big) \neq \emptyset, \\ \hat{d} \in T\big(\hat{\pi}_{R\setminus H} \cup \pi_H \cup \pi_{\mathcal{L}\setminus R}\big) & \text{otherwise.} \end{cases} \tag{EC.90}$$

The optimistic recursive core of the TU cooperative game among all locals is $C_o(\mathcal{L};\emptyset)$.

The optimistic and pessimistic recursive cores admit the same feasible deviations. They differ in the residual outcomes against which a deviation is evaluated. Under pessimism, every deviating coalition must be strictly better off under *every* plausible residual outcome. Under optimism, *there must exist some* plausible residual outcome under which every deviating coalition is strictly better off. The plausible outcomes are generated recursively by the corresponding residual core, with all feasible equilibrium outcomes used when that residual core is empty.

Proposition EC.10. *Under either condition $\mathbb{C} \in \{\bar{\mathbb{D}}, \bar{\mathbb{U}}\}$, for every $R \subseteq \mathcal{L}$ and every $\pi_{\mathcal{L}\setminus R} \in \Pi^{\prec\bar{\pi}}_{\mathcal{L}\setminus R}$,*

$$C_o(R;\pi_{\mathcal{L}\setminus R}) \subseteq C(R;\pi_{\mathcal{L}\setminus R}).$$

In particular, $C_o(\mathcal{L};\emptyset) \subseteq C(\mathcal{L};\emptyset)$.

Proof of Proposition EC.10. We proceed by induction on $|R|$. If $|R| = 1$, the two definitions coincide, so $C_o(R;\pi_{\mathcal{L}\setminus R}) = C(R;\pi_{\mathcal{L}\setminus R})$.

Suppose the inclusion holds for every residual game with at most k residual locals. Fix $R \subseteq \mathcal{L}$ with $|R| = k+1$ and a feasible fixed partition $\pi_{\mathcal{L}\setminus R}$. Suppose, toward a contradiction, that a residual-game outcome belongs to $C_o(R;\pi_{\mathcal{L}\setminus R})$ but not to $C(R;\pi_{\mathcal{L}\setminus R})$. The outcome is then pessimistically dominated by some non-empty $H \subseteq R$ from one family, forming some $\pi_H \in \Pi^{\prec\bar{\pi}}_H$. Hence every $S \in \pi_H$ is strictly better off for every equilibrium indicator in $A(R\setminus H;\pi_H \cup \pi_{\mathcal{L}\setminus R})$.

We show that the same H and π_H constitute an optimistic deviation. If $H = R$, then $A(\emptyset;\pi_H \cup \pi_{\mathcal{L}\setminus R}) = T(\pi_H \cup \pi_{\mathcal{L}\setminus R})$; choosing any equilibrium indicator in this non-empty set satisfies (EC.89).

Now suppose $H \subsetneq R$. If $C_o(R\setminus H;\pi_H \cup \pi_{\mathcal{L}\setminus R})$ is non-empty, the inductive hypothesis implies that it is contained in $C(R\setminus H;\pi_H \cup \pi_{\mathcal{L}\setminus R})$. Choose any outcome in this optimistic residual core. Its equilibrium indicator belongs to $A(R\setminus H;\pi_H \cup \pi_{\mathcal{L}\setminus R})$, so the strict inequalities from pessimistic dominance imply (EC.89) for that same residual outcome.

Finally, suppose $C_o(R\setminus H;\pi_H \cup \pi_{\mathcal{L}\setminus R})$ is empty. The optimistic definition then permits any feasible equilibrium outcome of the residual game. If the corresponding pessimistic residual core is non-empty, choose any outcome in that core; its equilibrium indicator belongs to the Assumption set A. If the pessimistic residual core is also empty, A itself contains all feasible residual equilibrium indicators. In either case, choose one such feasible residual outcome. Pessimistic dominance again implies (EC.89). Thus the same deviation optimistically dominates the starting outcome in every case, contradicting its membership in $C_o(R;\pi_{\mathcal{L}\setminus R})$. This completes the induction. □

The inclusion result in Proposition EC.10 has important consequences regarding conclusions about performance (of conditional incentives) under optimistic rather than pessimistic behavioral assumptions. On the one hand, the inclusion result makes the pessimistic assumption conservative: if all core outcomes under pessimism satisfy some desirable performance requirements, then all core outcomes under optimism would also satisfy those performance requirements. However, the inclusion result also means that the optimistic core may be empty even though the pessimistic core is nonempty (simple examples exist to confirm this possibility). As such, if core non-emptiness itself is a desirable requirement, then one cannot readily conclude that the pessimistic assumption is conservative because the same set of circumstances that lead to a nonempty core and desirable outcomes under pessimism may lead to an empty core under optimism.

EC.7. Illustration in Indonesia

East Kalimantan, Indonesia, has extensive forests at risk of conversion to palm oil farms. To gauge whether a conditional price premium for palm-fruit bunches could both prevent deforestation and compensate, we conducted detailed household surveys in several villages. This section describes the resulting dataset and explains, step by step, how we deploy our analytical framework–offering a template for practitioners who wish to replicate the approach elsewhere.

In §EC.7.1, we describe the survey data and other datasets needed for our study. In §EC.7.2, we develop a procedure to estimate each farmer's value with deforestation and the incentive. In §EC.7.3, we apply our model separately for each village, with the incentive being a price premium and area conditions based on each village's perimeter. For each village, with $\mathcal{L}$ denoting the set of all locals in that village, we consider two cases: a case with full cooperation ($\bar{\pi} = \{\mathcal{L}\}$) and an extreme case with *no* cooperation, where each family is a singleton ($\bar{\pi} = \{\{\ell\} : \ell \in \mathcal{L}\}$). §EC.7.4 then examines a special case with limited cooperation of practical importance, where $\bar{\pi} = \{S\} \cup \{\{\ell\} : \ell \in \mathcal{L} \setminus S\}$; here, locals in S form a tight-knit group (e.g., residents of the same village, members of the same cooperative) whereas all others can be thought of as "outsider entrants" who are unable to cooperate with the tight-knit group or among themselves. This configuration allows us to examine the robustness of the area no-use condition with respect to entry by those outside the community. §EC.7.4 also conducts a few other important robustness checks.

EC.7.1. Description of the data

Our survey included 60 villages in two regencies of East Kalimantan, Indonesia, mapped in Figure EC.2 (the figure shows all villages in East Kalimantan, and those in our survey). In total, 420 farmers were surveyed, but we retain a subset of 391 farmers from 58 villages for our study. These are all farmers in our data with total land less than 20 hectares (ha) and all villages with at least two observations.

Table EC.3 provides brief summary statistics and Figure EC.3 shows histograms for all data fields. Each farmer in the data has multiple plots. We replace missing prices, costs, and interest rate values

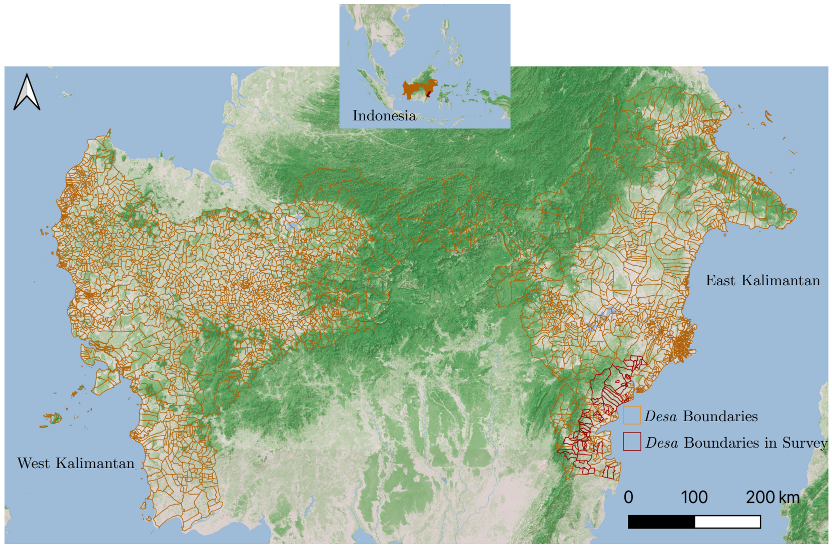

Figure EC.2 Map of East and West Kalimantan, Indonesia, showing the divisions into all rural villages (*desa*) in orange, and those in the survey in red. Darker shades of green denote more forest cover.

for each farmer with the median of the corresponding field over all the available data. Finally, because some farmers reported overly optimistic production values in our survey, we limit their maximum production quantities using the maximum attainable yields for palm trees in Indonesia (as a function of the age of the trees) according to Hoffmann et al. (2014).

Definition	Mean	Range
plot area $A_{\ell,i}$ (hectares)	2	0.3-17.5
tree age $a_{\ell,i}$ (years)	9.9	1-40
production $q_{\ell,i}$ (tFFB/year)	41.3	0.5-480
price $p_{\ell,i}$ (USD/tFFB)	90	51-129
transport cost $\tau_{\ell,i}$ (USD/tFFB)	11.8	1-29.6
harvest cost $h_{\ell,i}$ (USD/tFFB)	16	0.2-29.6
interest rate β_ℓ (% /year)	16.5	2.6-102

Table EC.3 Data by farmer-plot pairs (ℓ, i): The 391 survey participants produce palm fruit on 683 separate plots: $i \in \mathcal{P}_\ell$ indicates that plot i is among farmer ℓ's plots.

Additional Data Sources. Palm fruit production varies with tree age. Production is zero for the first two years after planting, peaks after eight years, and declines thereafter. We use yield data from Hoffmann et al. (2014) to account for the change in productivity over the whole time horizon. Based on this, Figure EC.4 shows a *normalized* value of the yield y_a as a function of the tree age a (i.e., years elapsed after replanting). This normalized measure takes values between 0 and 1 and is obtained by dividing the yield for trees of age a by the maximum attainable yield (at age 8 years).

To generate maps (as in Figure EC.2 and Figure EC.9), we combine global data on forest canopy cover from Potapov et al. (2021) with the village boundaries extracted from the sub-national administrative boundaries for Indonesia from OCHA (2020).

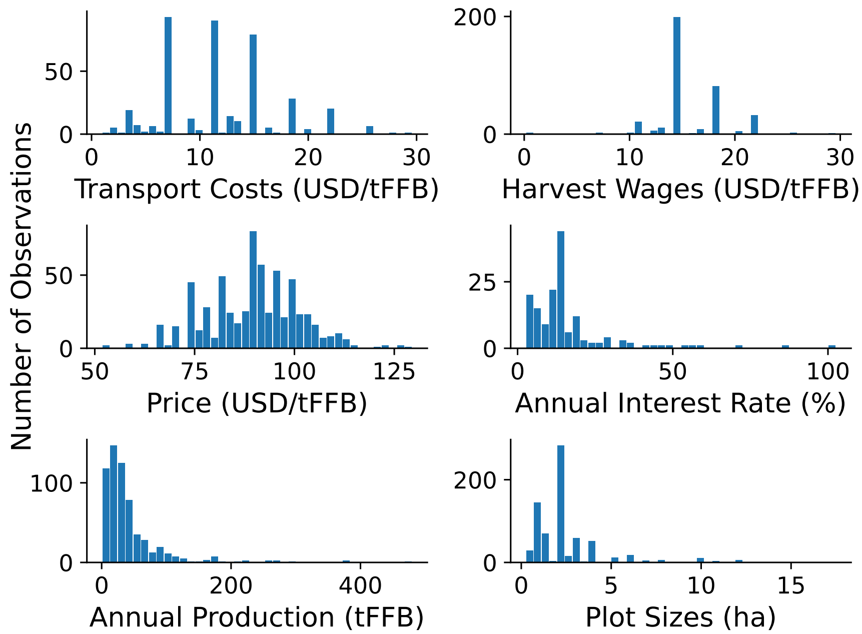

Figure EC.3 Distributions of key parameters in our data set. Each observation corresponds to a particular farmer and plot, except for the interest rates, where each observation corresponds to a specific farmer.

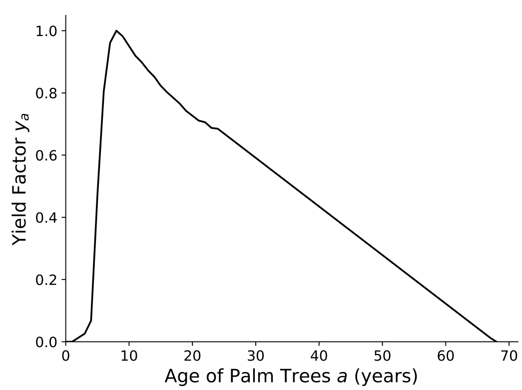

Figure EC.4 Yield Multiplier: The normalized attainable yield for palm oil in Indonesia as a function of tree age a. We calculate this by dividing the attainable yield in year a with the maximum attainable yield, which occurs at age 8 years.

EC.7.2. Model Calibration

A local ℓ is a palm farmer. Recall that his value from engaging in deforestation, D_ℓ, and his value from the incentive, I_ℓ, are gains relative to his status quo income v_ℓ^{sq}: $(v_\ell^{\text{sq}} + I_\ell)$ is his income with no deforestation and the incentive, and $v_\ell^{\text{sq}} + D_\ell$ is his net income with deforestation. We develop a structural model to calibrate these model parameters for each ℓ so as to be representative of one of the 391 palm farmers in our survey. (These are all farmers in the 58 villages; the number of farmers $|\mathcal{L}|$ per village – estimated from Indonesian census data as detailed in §EC.7.4.2 – ranges from 35 to 295.) We apply robust Data Envelopment Analysis (DEA) to *overestimate* each farmer's idiosyncratic value with deforestation D_ℓ, so as to be conservative in predicting the performance of an incentive and condition.

Table EC.3 describes and assigns notation for the data we use from our survey. We use plot-level data because half of the surveyed farmers have more than one plot, and tree ages, production yields, and (in some cases) selling prices and production costs differ among a farmer's plots. Production is measured in metric tons of fresh fruit bunches (tFFB). Farmers generally reported their significant production costs as those for harvest labor and transport of fruit from the plot to the mill for sale. Half of the surveyed farmers did not report an interest rate to borrow money, so we substitute the median reported interest rate for β_ℓ; we use an analogous procedure in the few cases that the price, transport cost, or harvest cost is missing for a plot.

Lastly, we assume that any farmer can convert forest to a new plot by incurring cost $c^{\text{def}} = 375$ USD per hectare, comprised of the costs to clear forest by fire (5 USD/ha, per Falcon et al. 2022) and to plant saplings (2.96 USD/sapling and 125 saplings per hectare, per our survey).

Status Quo Income from Existing Plots. We estimate the net cash flow for farmer ℓ from existing plot $i \in \mathcal{P}_\ell$ in a future year when the trees reach age $a \geq a_{\ell,i}$ and with fruit price p as

$$I^e_\ell(i,a,p) = (p - h_{\ell,i} - \tau_{\ell,i}) \cdot q_{\ell,i} \cdot y_a / y_{a_{\ell,i}}, \text{ for any } a \geq a_{\ell,i}. \tag{EC.91}$$

Here, $y_a/y_{a_{\ell,i}}$ accounts for the predictable variation in fruit production with the age of the trees. We thus estimate the farmer's status-quo income without deforestation and without the incentive as

$$v^{\text{sq}}_\ell = \textstyle\sum_{t=1}^T (1+\beta_\ell)^{-t} \sum_{i \in \mathcal{P}_\ell} I^e_\ell(i, a_{\ell,i}+t, p_{\ell,i}). \tag{EC.92}$$

We assume that a farmer discounts cash flows according to his interest rate to borrow money and uses a finite planning horizon T. All results in this section are for $T = 20$ years, and §EC.7.4.1 shows that (due to discounting) the results exhibit remarkably little variation with any choice of planning horizon larger than T=15. We assume a farmer's expected future prices and costs are the same as those he reported in the survey. This is not an unreasonable representation of information available to a smallholder farmer.

Income from Existing Plots with the Incentive. The incentive is a *price premium* $p^\star$ per tFFB (tonne of FFB), so we estimate farmer ℓ's income without deforestation and with the incentive as

$$(v^{\text{sq}}_\ell + I_\ell) = \textstyle\sum_{t=1}^T (1+\beta_\ell)^{-t} \sum_{i \in \mathcal{P}_\ell} I^e_\ell(i, a_{\ell,i}+t, p_{\ell,i}+p^\star), \tag{EC.93}$$

and his value from the incentive as

$$I_\ell(p^\star) = (v^{\text{sq}}_\ell + I_\ell) - v^{\text{sq}}_\ell = p^\star \cdot \textstyle\sum_{t=1}^T (1+\beta_\ell)^{-t} \sum_{i \in \mathcal{P}_\ell} q_{\ell,i} \cdot y_{(a_{\ell,i}+t)} / y_{a_{\ell,i}}. \tag{EC.94}$$

The maximum RSPO price premium contemporaneous with our survey is 30 USD/tFFB.

Income with Deforestation. We conservatively *overestimate* the area of forest x_ℓ that farmer ℓ would convert to a palm farm, and his resulting net income $v^{\text{sq}}_\ell + D_\ell$.

The first step is to estimate an efficient production frontier $u(A)$: maximum annual production quantity of palm fruit for a farmer with *total* land area A and trees at peak productive age 8 years. Scaling up the production quantity $q_{\ell,i}$ reported by farmer ℓ (for trees aged $a_{\ell,i}$) by a factor $y_8/y_{a_{\ell,i}} = 1/y_{a_{\ell,i}} \geq 1$, we estimate the *total peak production quantity* that farmer ℓ could have produced on all his existing plots if the trees were at the peak productive age:

$$\hat{q}_\ell := \sum_{i\in\mathcal{P}_\ell} q_{\ell,i}/y_{a_{\ell,i}}. \tag{EC.95}$$

Each point in Figure EC.5 (left) plots a farmer's total peak production quantity $\hat{q}_\ell$ and total land $A_\ell = \sum_{i\in\mathcal{P}_\ell} A_{\ell,i}$. The estimated efficient production frontier $u(A)$ (blue line) is the robust concave envelope of those points calculated by m-estimator robust DEA, as detailed below.

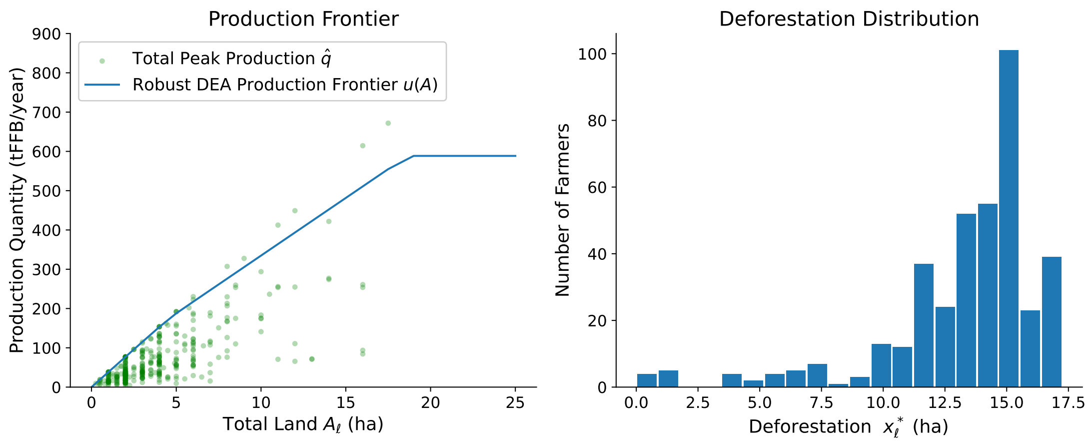

Figure EC.5 (Left) Scatter plot of each farmer's total peak production quantity $\hat{q}_\ell$ and total land $A_\ell = \sum_{i\in\mathcal{P}_\ell} A_{\ell,i}$, with $u(A)$, the robust efficient production frontier of these points estimated using m-estimator DEA. (Right) Histogram of a farmer's estimated optimal area to deforest x^*.

Second, we use that efficient production frontier to *overestimate* the net income that farmer ℓ could generate by deforesting a plot of area x_ℓ. The estimated annual production quantity for farmer ℓ from the deforested plot of area x_ℓ with trees at peak productive age is:

$$\hat{q}^{\text{def}}_\ell(x_\ell) := u\Big(\sum_{i\in\mathcal{P}_\ell} A_{\ell,i} + x_\ell\Big) - u\Big(\sum_{i\in\mathcal{P}_\ell} A_{\ell,i}\Big), \tag{EC.96}$$

and with trees of arbitrary age a it is $\hat{q}^{\text{def}}_\ell(x_\ell)\cdot y_a$. Hence, in the a^{th} year after deforesting the land and planting seedlings, the estimated net cash flow from the new plot is:

$$I^d_\ell(x_\ell, a) = (\overline{p}_\ell - \underline{h}_\ell - \underline{\tau}_\ell)\cdot \hat{q}^{\text{def}}_\ell(x_\ell)\cdot y_a, \text{ for any } a \geq 0, \tag{EC.97}$$

where $\overline{p}_\ell := \max_{i\in\mathcal{P}_\ell} p_{\ell,i}$, $\underline{h}_\ell := \min_{i\in\mathcal{P}_\ell} h_{\ell,i}$, and $\underline{\tau}_\ell := \min_{i\in\mathcal{P}_\ell} \tau_{\ell,i}$ denote the largest price obtained and lowest costs incurred by farmer ℓ on his existing plots, respectively.

Finally, we *overestimate* the net income for ℓ with deforestation and without the incentive by

$$v^{\text{sq}}_\ell + D_\ell = \max_{x_\ell\geq 0}\Big\{\sum_{t=1}^T (1+\beta_\ell)^{-t}\Big[\sum_{i\in\mathcal{P}_\ell} I^e_\ell(i, a_{\ell,i}+t, p_{\ell,i}) + I^d_\ell(x_\ell, t)\Big] - c^{\text{def}}\cdot x_\ell\Big\}, \tag{EC.98}$$

and the value from deforestation D_ℓ – the difference between (EC.98) and the status-quo income (EC.92) – as

$$D_\ell = \sum_{t=1}^{T}(1+\beta_\ell)^{-t} I_\ell^d(x_\ell^\star, t) - c^{\text{def}} \cdot x_\ell^\star, \tag{EC.99}$$

where $c^{\text{def}} = 375$ USD/hectare is the cost to clear forest and plant seedlings. With x_ℓ^* denoting the solution to (EC.98), Figure EC.5 (right) shows the distribution of x_ℓ^* among farmers. Nearly all farmers (387 out of 391) would benefit from clearing some forest.[10] None would clear more than 20 ha.

Data Envelopment Analysis. To obtain the production frontier while systematically accounting for outliers, we use m-estimator Data Envelopment Analysis from Aragon et al. (2005). We obtain the production frontier $u(A)$ by applying Algorithm 1 to the set of points $\{(A_\ell, \hat{q}_\ell) : \ell \in \cup_j \mathcal{L}_j\}$, where $\cup_j \mathcal{L}_j$ is the union of all the villages in the dataset.

Algorithm 1 Production Frontier $u(x)$

Require: $x \geq 0$, $\{(A_\ell, \hat{q}_\ell) : \ell \in \cup_j \mathcal{L}_j\}$

procedure U(x)
    for $A \in \cup_{\ell \in \cup_j \mathcal{L}_j} A_\ell$ **do**
        for $b = 1$ to B **do**
            $\{q_b^1, ..., q_b^m\} \leftarrow$ A random sample with replacement of size m from $\{\hat{q}_\ell : A_\ell \leq A\}$
            $h_b(A) \leftarrow \max\{q_b^1, ..., q_b^m\}$
        end for
        $h(A) \leftarrow \frac{1}{B}\sum_{b=1}^{B} h_b(A)$
    end for
    $\{(A, \hat{h}(A))\} \leftarrow$ convex hull of $\{(A, h(A)) : A \in \cup_{\ell \in \cup_j \mathcal{L}_j} A_\ell\}$
    if $x \leq \max_{\ell \in \cup_j \mathcal{L}_j} A_\ell$ **then**
        $u(x) \leftarrow$ linear interpolation of $\{(A, \hat{h}(A))\}$ at x
    else
        $u(x) \leftarrow u(\max_{\ell \in \cup_j \mathcal{L}_j} A_\ell)$
    end if
end procedure

The procedure for obtaining the production frontier $u(x)$, for any $x \geq 0$, detailed in Algorithm 1, works in three parts: (i) first, it obtains the expected value of the DEA frontier defined by a sub-sample of m points with total area $A_\ell \leq x$. It uses a Monte-Carlo simulation, sampling B times and taking the average. We set $B = 500$, and $m = 150$. (ii) Because the sampling procedure in (i) need not

[10] For the remaining four farmers, $D_\ell = 0$, whereas our base model maintains $D_\ell > 0$; §EC.6.5 shows that our results extend to locals with $D_\ell = 0$.

produce a concave function, this step computes the convex hull of the points obtained. (iii) Finally, a linear interpolation of the points in the convex hull leads to the value of $u(x)$. We assume the production would be constant for any x larger than the maximum total land registered in our dataset.

Blocking Cost. We consider a range of blocking cost values $\eta \in (0, 3000]$ USD. Given the difficulty of estimating the cost of preventing deforestation by fire in the Indonesian context, we focus on the cost of blocking palm production. The upper bound of 3000 USD is the cost to use bulldozers and excavators to clear 20 hectares of light forest (Falcon et al. 2022). Lower blocking cost could be achieved, for instance, by cutting palm seedlings with a chainsaw (Villadiego 2017).

EC.7.3. Performance of a Price Premium Conditional on $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$

For each village in the dataset, we apply the results in §3 with the price premium as the incentive and with each condition $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$. Importantly, we characterize the entire population of palm farmers in each village by replicating the surveyed farmers from that village up to the village's census-estimated size (as detailed in §EC.7.4.2); provided that the sample is representative, this should not bias our results. (Proposition EC.11 in §EC.7.5 formalizes the underlying invariance: duplicating a problem instance leaves all results unchanged.)

Figure EC.6 illustrates the *mismatch* inherent in a price premium incentive to prevent deforestation: the largest value from the incentive goes to farmers with the least value from deforestation, whereas farmers with the least land and largest value from deforestation gain the least value from the incentive. Due to this mismatch and the low value of the RSPO price premium $p^\star = 30$ USD/tFFB, 94% of the farmers prefer deforestation (have $I_\ell < D_\ell$).

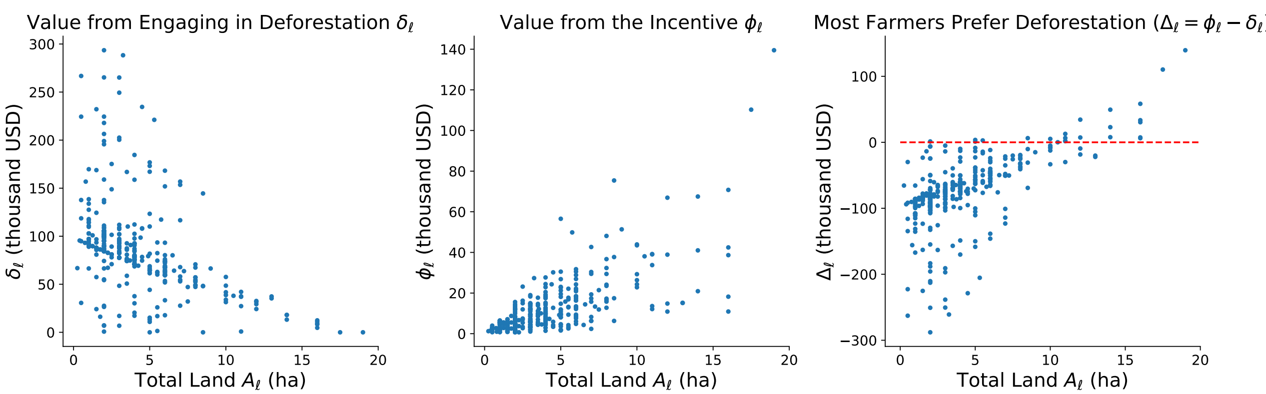

Figure EC.6 Scatter plots of a farmer's value D_ℓ from engaging in deforestation, value I_ℓ from a price premium $p^\star = 30$ USD/tFFB, and preference for deforestation $\Delta_\ell = (I_\ell - D_\ell)$ as a function of the farmer's total land A_ℓ.

We apply the area No-Deforestation Condition $\bar{\mathbb{D}}$ and area No-Use Condition $\bar{\mathbb{U}}$ at a village level. In each of the 58 villages, all palm farmers in the village get the price premium on all their production if and only if the specified condition holds for that village. For each of those villages, §EC.7.4.2 details how we estimate the number of palm-farming households from Indonesian census data (Table EC.4), and we set $|\mathcal{L}|$ for the village to that number. We assume that the surveyed farmers from each village are representative of the other farmers in their village and replicate their model parameters to characterize the set of farmers $\ell \in \mathcal{L}$ in the village.

With the RSPO price premium $p^\star = 30$ USD/tFFB, only the area no-use condition $\bar{\mathbb{U}}$ can possibly prevent deforestation. In each of the 58 villages, farmers collectively prefer deforestation ($I(\mathcal{L}) < D(\mathcal{L})$), so $\bar{\mathbb{U}}$ can prevent deforestation in a village *only* if the farmers don't have full cooperation, as shown by Theorem 2-(A-i). In the case where families consist of singletons $\bar{\pi} = \{\{\ell\} : \ell \in \mathcal{L}\}$, every family trivially has homogeneous preferences, so the core is non-empty at every blocking cost (see the discussion following Theorem 2), and by Theorem 2-(A-ii) every core outcome has $d^\star\!=\!0$ – that is, $\bar{\mathbb{U}}$ prevents deforestation – if and only if some farmer g satisfies $I_g > D_g + \eta \cdot (|\mathcal{L}| - 1)$, i.e., $\eta < \underline{\eta}_1$ in (9).[11] Under this criterion, $\bar{\mathbb{U}}$ prevents deforestation in 23 villages if the blocking cost $\eta \le 10$ USD, in only one village at $\eta = 500$ USD, and in none for $\eta \ge 1{,}060$ USD.

What is the minimum (uniform) price premium that would prevent deforestation and compensate, respectively, in *all* 58 villages? From (EC.94), the value of the incentive $I_\ell(p^\star)$ to each farmer ℓ increases linearly with the price premium $p^\star$. Depending on the condition $\mathbb{C} \in \{\mathbb{I}, \bar{\mathbb{D}}, \bar{\mathbb{U}}\}$ and the setting, we determine the minimum value of $p^\star$ that satisfies the corresponding requirement reported in Figure 3.2, for all villages: for the individual condition $\mathbb{I}$, a perfect incentive, $I_\ell > D_\ell$ for every $\ell \in \mathcal{L}$ (§3.1); for either area condition with full cooperation, $I(\mathcal{L}) > D(\mathcal{L})$ (Theorem 1-(B) and Theorem 2-(A-i)); and for $\bar{\mathbb{U}}$ with no cooperation, the blocking requirement $I_g > D_g + \eta \cdot (|\mathcal{L}| - 1)$ for some farmer g, as derived above.[12] Figure EC.7 shows the minimum price premium for each condition and case considered (except $\bar{\mathbb{D}}$ in the setting with no cooperation, where $\bar{\mathbb{D}}$ cannot prevent deforestation, by Theorem 1-(C)). The minimum price premium to prevent deforestation is much lower for area No-Use Condition $\bar{\mathbb{U}}$ (and $\bar{\mathbb{D}}$ with full cooperation) than for the individual condition $\mathbb{I}$. Under full cooperation, the premium is the same for the two area conditions, but the caveat from §3.4 applies to $\bar{\mathbb{U}}$: at that premium the core under $\bar{\mathbb{U}}$ can be empty at intermediate blocking costs $\eta \in (\eta_2, \overline{\eta}_1)$, by Theorem 2-(A), whereas under $\bar{\mathbb{D}}$ the core is always non-empty and every core outcome has $d^\star\!=\!0$ and compensates (Theorem 1); for $\eta > \overline{\eta}$ the two area conditions perform identically. The full-cooperation curve is therefore best interpreted as the premium for $\bar{\mathbb{D}}$, the condition that §3.4 recommends in that scenario. For $\bar{\mathbb{U}}$ it is lower with full cooperation than in the case with no cooperation if and only if the blocking cost η exceeds 630 USD. As in §3.4, full cooperation reduces the cost to prevent deforestation if and only if the blocking cost is sufficiently large. For $\bar{\mathbb{U}}$ and no cooperation, the minimum price premium

[11] In the computations, the number of locals that farmer g must be able to block counts only the locals who would engage in deforestation ($D_\ell > 0$): a local with $D_\ell = 0$ never deforests and need not be blocked (cf. §EC.6.5). With 387 of 391 surveyed farmers having $D_\ell > 0$, this refinement is immaterial.

[12] This last requirement corresponds to the scenario in which the buyer does not induce a preferred type of core outcome: it makes *every* core outcome have $d^\star\!=\!0$. A buyer who induces a preferred type of core outcome needs only the weaker requirement $\eta_2(I) \ge \eta$ from Figure 3.2 – with singleton families, $I_g \ge D_g + \eta$ for some farmer g – which would lower the minimum price premiums for $\bar{\mathbb{U}}$ with no cooperation below those reported here.

to prevent deforestation increases with η, from 471 USD/tFFB at η=1 USD to 2496 USD/tFFB at η=3000 USD. For $\bar{\mathbb{U}}$ with full cooperation, the minimum price premium is invariant with η. To compensate, the minimum price premium is the same for $\bar{\mathbb{U}}$ and $\mathbb{I}$. Indeed, under $\bar{\mathbb{U}}$ with an imperfect incentive ($\mathcal{G} \neq \mathcal{L}$), no core outcome with $d^\star=0$ compensates: with no cooperation, some singleton family strictly prefers deforestation, which rules out compensating outcomes (Theorem 2-(A-ii)); with full cooperation, compensating outcomes require $\eta \geq \overline{\eta}_1$ (Theorem 2-(A-i)), and at the relevant premiums $\overline{\eta}_1$ far exceeds the largest blocking cost we consider (as the blocking surpluses in §EC.7.4.2 imply). With a perfect incentive, every core outcome with $d^\star=0$ compensates, by Theorem 2-(B). Under full cooperation, the minimum premium to compensate is much lower for $\bar{\mathbb{D}}$: by Theorem 1, compensation is automatic at the premium that prevents deforestation.

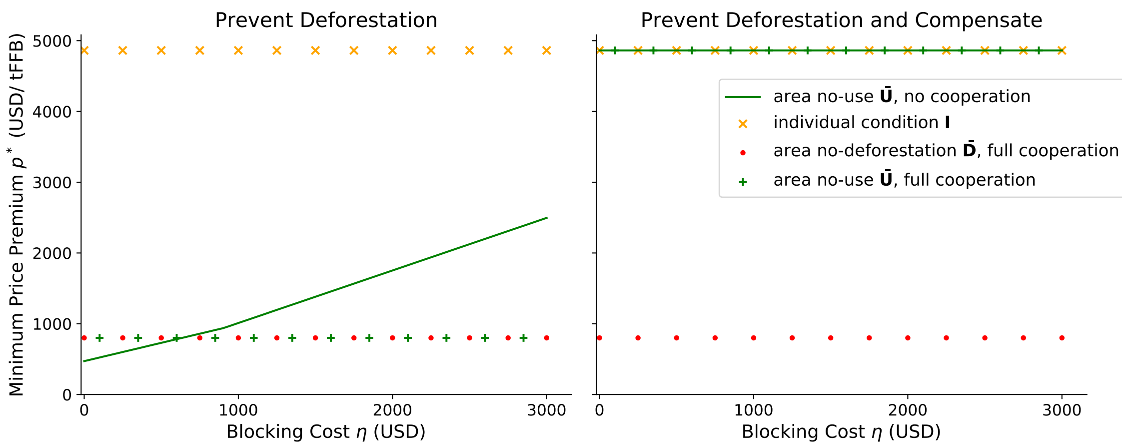

Figure EC.7 Minimum uniform price premium to prevent deforestation (left) and compensate (right) in all villages.

Figures EC.8 and EC.9 depict the *village-specific* minimum price premiums to prevent deforestation and compensate, respectively, which differ among the villages. If farmers have full cooperation, a price premium below 250 USD prevents deforestation and compensates all locals (under $\bar{\mathbb{D}}$) in most villages. That is less than a third of the 800 USD/tFFB *uniform* price premium needed to prevent deforestation in all villages, which suggests that village-specific price premiums can meaningfully reduce the cost of preventing deforestation (and achieving compensation).

Taken together, our findings in this section indicate that – even under deliberately conservative opportunity-cost assumptions – area conditions, especially when coupled with tailored area-specific incentives, remain a powerful instrument for curbing deforestation.

Two caveats about our results also deserve highlighting. First, we discourage practitioners from applying our estimated price premiums verbatim, because those figures are probably inflated by the assumptions used to value deforestation benefits in (EC.98). A real-world application should start with richer, site-specific data to construct a localized efficiency frontier and may also adopt production-function estimation techniques that are less conservative. Second, we acknowledge that implementing *village-specific* price premiums in practice may face some practical challenges, such as side-selling and equity concerns (particularly when villages neighbor each other). However, REDD+ projects have recently been successfully implemented in Indonesia at a sub-national jurisdictional level, including village-specific projects (see Irawan et al. 2019, Wahyudi et al. 2024), which shows that such conditional incentives may remain viable.

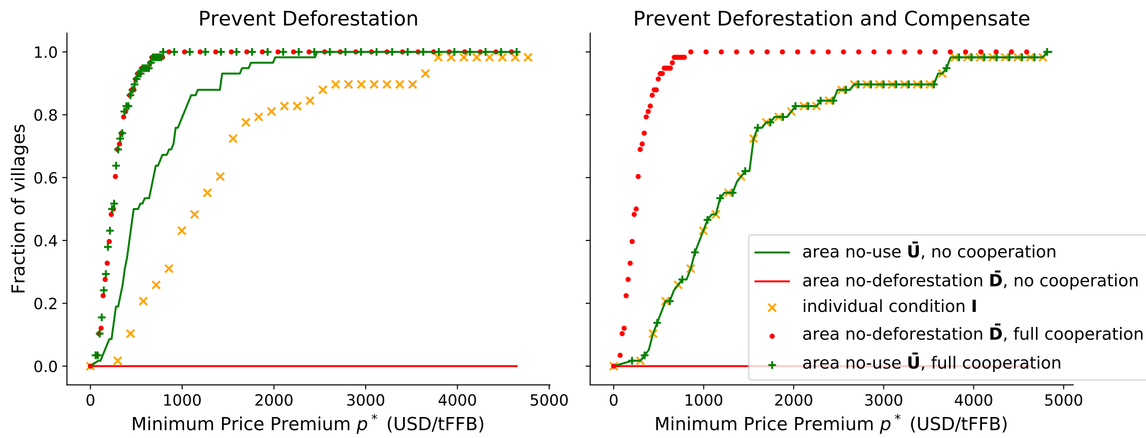

Figure EC.8 Fraction of villages in which a condition prevents deforestation (left) and compensates (right) as a function of the price premium. For $\bar{\mathbb{U}}$, this is at $\eta = 3,000$ USD; the fraction of villages would be higher at lower blocking cost.

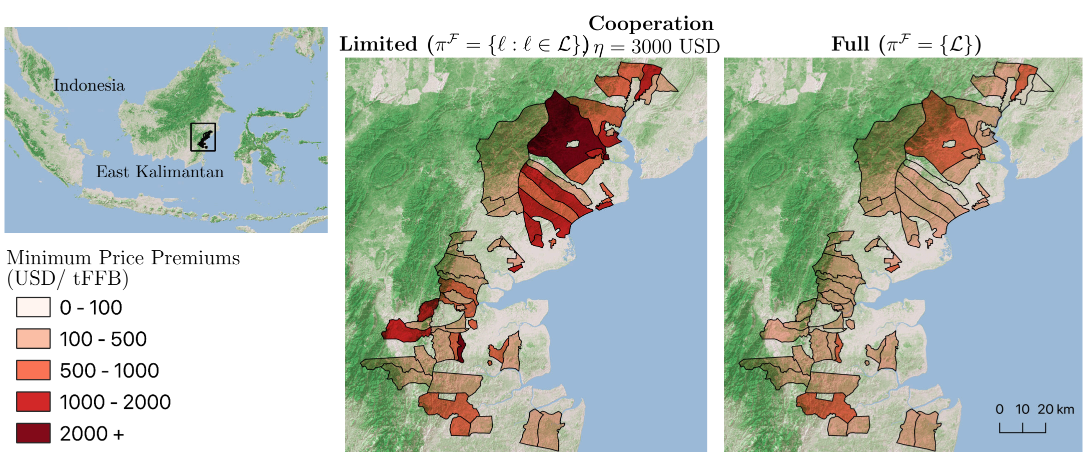

Figure EC.9 Minimum price premium to prevent deforestation and, under $\bar{\mathbb{D}}$ with full cooperation, compensate.

EC.7.4. Robustness Checks

We conduct a few important robustness checks for these empirical results.

EC.7.4.1. Robustness with respect to the planning horizon T. Our base analysis considered a planning horizon of $T = 20$ years. We now relax this assumption and consider other values of T. Due to the heavy discounting, this does not carry much impact on the results presented in §EC.7. We re-computed all our results with $T \in [15, 60]$ and Figure EC.10 shows that the deforestation distribution remains virtually identical for any T in that range. Figures EC.11 and EC.12 show that the uniform and village-specific minimum prices would also remain virtually unchanged. The

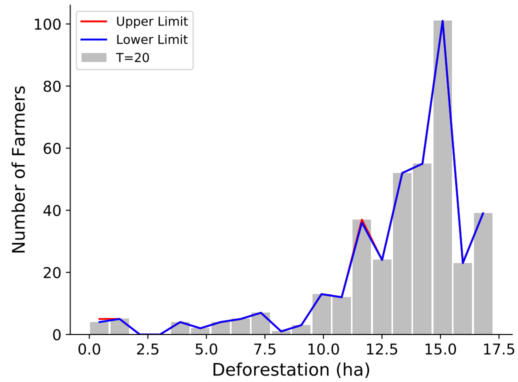

Figure EC.10 Histogram of the estimated optimal area to deforest x^* at $T=20$ years, with upper limit (lower limit) showing the maximum (minimum) number of farmers in each bin for $T\in[15,60]$ years.

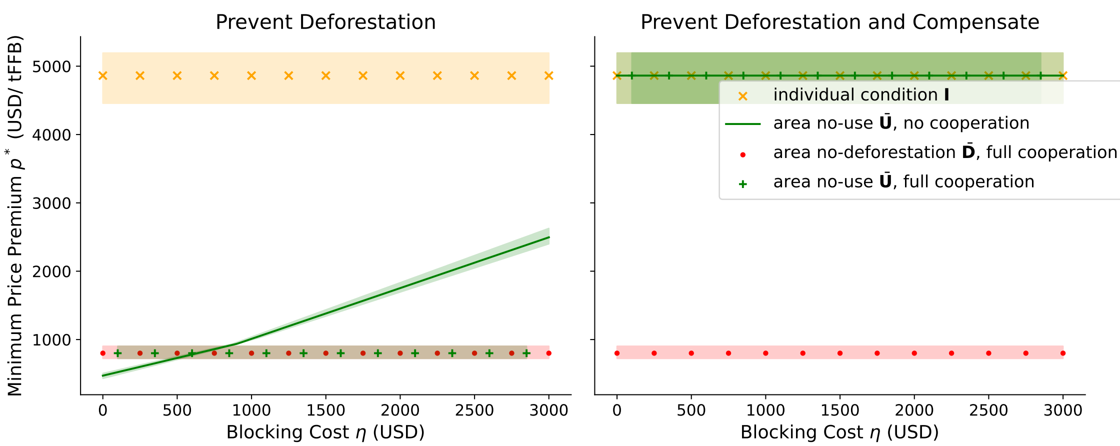

Figure EC.11 Minimum price premium p^* that would prevent deforestation (left) and compensate (right) in all villages, for $T=20$ years, together with shaded areas showing the variation for $T\in[15,60]$ years.

most meaningful change is in the uniform minimum price that would prevent deforestation with compensation in all 58 villages under the individual incentive; this changes from 4,456 USD/tFFB at $T=15$ years to 5,193 USD/tFFB at $T=60$ years.

EC.7.4.2. Deterring Entrants: Robustness of area no-use condition $\bar{\mathrm{U}}$. We now consider a special case with limited cooperation of practical significance. Specifically, we take the partition of families as $\bar{\pi}=\{S\}\cup\{\{\ell\}:\ell\in\mathcal{L}\setminus S\}$ and we only consider incentives with $I_\ell=0$ for all $\ell\in\mathcal{L}\setminus S$ (each such local strictly prefers deforestation, since $D_\ell>0=I_\ell$). This emulates a practical setting where a tight-knit group S (e.g., suppliers to a commodity buyer, residents of the same village, members of the same cooperative) are the only ones who receive the incentive, whereas locals in $\mathcal{L}\setminus S$ are "outsiders" who cannot receive any incentive and cannot cooperate with the tight-knit group. We refer to locals in $\mathcal{L}\setminus S$ as *entrants*.[13]

[13] That entrants cannot cooperate among themselves is irrelevant in what follows: taking an arbitrary partition of entrants into families would not change any results.

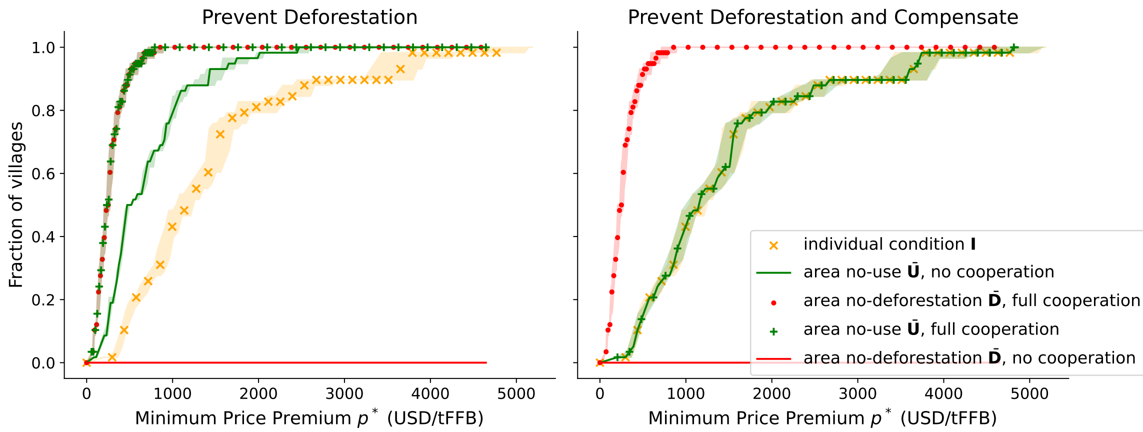

Figure EC.12 Fraction of villages in which each condition prevents deforestation (left) and compensates (right) as a function of the price premium $p^\star$, considering $\eta = 3,000$ USD and $T = 20$ years. The shaded areas show the variation of these curves for $T \in [15, 60]$ years.

In many settings, it is challenging to determine the number of entrants in a community. Rather than attempting to estimate this number, we examine how robust an incentive is to the presence of entrants when the price premium $p^\star$ is calculated *as if there are no entrants*, i.e., under the assumption that $\mathcal{L} = S$. We first calculate the maximum number M of entrants for which every core outcome, if any, has $d^\star\!=\!0$. This bound does not by itself guarantee that the core is non-empty. We then consider the higher premiums that achieve compensation absent entrants and derive a stronger bound $\bar{M}$ under which the core is non-empty, every core outcome has $d^\star\!=\!0$, and every local in the community S is compensated.

One can readily verify that Theorem 2-(A-ii) implies the following. Suppose a price premium $p^\star$ conditional on $\bar{\mathrm{U}}$ prevents deforestation without any entrants, i.e., satisfies (EC.102) below. With up to

$$M := \left\lceil \textstyle\sum_{\ell\in H}\big(I_\ell(p^\star) - D_\ell\big)/\eta \right\rceil - 1 - |S| + |H|, \qquad H = \{\ell \in S : I_\ell(p^\star) > D_\ell - \eta\}, \tag{EC.100}$$

entrants, every core outcome, if any, has $d^\star\!=\!0$. Indeed, with at most M entrants, coalition H retains a credible threat to block all other locals, so $\eta < \underline{\eta}_1$ in (9). Consequently, every outcome with $d^\star\!=\!1$ is dominated.

The qualifier "if any" is essential: the bound M alone does not guarantee that the core is non-empty. By Theorem 2-(A), core non-emptiness is additionally guaranteed when $\eta \leq \eta_2$.

A stronger conclusion holds when the price premium compensates all locals absent entrants, so that $I_\ell(p^\star) > D_\ell$ for every $\ell \in S$. With up to

$$\bar{M} := \left\lceil \textstyle\sum_{\ell\in S}\big(I_\ell(p^\star) - D_\ell\big)/\eta \right\rceil - 1 \geq M, \tag{EC.101}$$

entrants, the core is non-empty, every core outcome has $d^\star\!=\!0$, and every core outcome compensates every local in S. Because each $\ell \in S$ prefers the incentive, an outcome that allocated him less than

$v_\ell^{\text{sq}} + D_\ell$ would be dominated by ℓ forming a singleton coalition, as in the discussion of Theorem 2-(B). The entrants themselves receive no incentive and strictly prefer deforestation, so no outcome compensates *them*. Once entrants are present, an incentive restricted to S therefore cannot compensate all of $\mathcal{L}$; the compensation guarantee applies to the community S.

Both entrant bounds M and $\bar{M}$ grow without bound as the blocking cost η decreases toward zero, decrease as η increases, and increase with the price premium $p^\star$.

Estimating the number of farmers in the village. Recall from §EC.7.3 that our analysis characterizes each village by its census-estimated number of palm-farming households $|\mathcal{L}|$; as long as our sample is properly stratified and representative, this does not bias the results, in view of Proposition EC.11. These estimates matter especially here because the number of entrants that can be deterred grows with the village's population.

As in §EC.7.3, we assume that the full population of palm farmers in the village can be obtained by suitably duplicating the farmers in our dataset from that village. (That is, if the village has N palm farmers and we have n farmers in our dataset, we simply duplicate each farmer in the dataset $\lceil N/n \rceil$ times.) The census-based estimates of N are constructed as follows. Because each farmer really represents a farming household, we first estimate the number of households in each village: we divide the total population of each district by the number of villages in the district (to estimate an average village population) and then divide that value by 5.12, which is the average household size in East Kalimantan according to BPS, Indonesia (2010). Finally, we multiply the estimated number of households by 38%, which is the mean fraction of households that farm palm, according to BPS, Indonesia (2013). Table EC.4 shows our estimates.

District	District Population (people)	Number of Villages	Village Population (people)	Village Households	Palm Households $\lvert\mathcal{L}\rvert$
Tanah Grogot	63,311	16	3,957	773	295
Waru	15,643	4	3,911	764	292
Penajam	66,983	23	2,912	569	217
Batu Sopang	22,540	9	2,504	489	187
Babulu	29,434	12	2,453	479	183
Sepaku	30,863	15	2,058	402	154
Pasir Belengkong	23,543	15	1,570	307	117
Kuaro	23,934	13	1,841	360	137
Long Ikis	36,701	26	1,412	276	105
Tanjung Harapan	7,720	7	1,103	215	82
Batu Engau	11,662	13	897	175	67
Muara Samu	4,221	9	469	92	35

Table EC.4 District and village information, including the estimated number of palm farming households, which we take as the total number of locals $|\mathcal{L}|$ in the village for our model.

Results. The entrant bounds are remarkably *large*, even though the price premiums are calculated assuming no entrants. In Table EC.5, the first two columns report M, using the minimum uniform or village-specific price premium conditional on $\bar{\mathbb{U}}$ that is calculated to prevent deforestation absent entrants. These values identify the maximum number of entrants for which every core outcome, if any, has $d^\star = 0$; they do not by themselves guarantee that the core remains non-empty. The last two columns report $\bar{M}$, using the corresponding price premiums calculated to achieve compensation absent entrants. For up to $\bar{M}$ entrants, the core is non-empty, every core outcome has $d^\star = 0$, and every local in S is compensated.

For each village, we calculate M and $\bar{M}$ at blocking costs $\eta = 1000$ and $\eta = 3000$, and the table reports the range across villages. A premium calculated to achieve compensation is generally higher than one calculated only to prevent deforestation absent entrants, and a uniform premium for all villages is generally higher than the corresponding village-specific premium; both differences increase the resulting entrant bound. With the uniform prevention premium, M is large except in the one village whose village-specific minimum prevention premium equals the uniform premium. With village-specific compensation premiums, $\bar{M}$ is large except in the 13 villages where the compensation premium equals the corresponding minimum prevention premium. Both bounds are very large across the remaining settings and villages. Even the village with the fewest farmers – only 35 members of S – has $M = 194$ at $\eta = 3000$ USD under its village-specific prevention premium. Thus, with as many as 194 entrants, no outcome with deforestation can belong to the core, although this M value alone does not guarantee that the core is non-empty.

How does cooperation in S make the performance of a price premium conditional on $\bar{\mathbb{U}}$ so highly robust to potential entrants? A price premium conditional on $\bar{\mathbb{U}}$ must be large enough that

$$\textstyle\sum_{\ell \in S} (I_\ell(p^\star) - D_\ell) > 0 \tag{EC.102}$$

to prevent deforestation absent entrants (Theorem 2-(A-i)). In each of the 58 villages, at the village-specific minimum price premium to prevent deforestation (determined by (EC.102)), the subset of farmers $H = \{\ell \in S : I_\ell(p^\star) > D_\ell - \eta\}$ have large $\sum_{\ell \in H} (I_\ell(p^\star) - D_\ell)$ and hence a credible threat to block all the other farmers plus a large number of potential entrants, even at the maximum blocking cost $\eta = 3000$ USD. This entry deterrence rests on the heterogeneity of the farmers. Additionally, for any price premium that satisfies (EC.102), cooperation and utility transfer enable farmers to deter more entrants. Without cooperation in S, blocking of entrants would have to be done by just one farmer with the greatest $I_\ell(p^\star) - D_\ell$, whereas through cooperation and utility transfer, farmers can pool their resources to block, which deters more entrants. This echoes the observation in §3.4 that the ability to cooperate reduces the cost to prevent deforestation when the blocking cost is not too large.

Taken together, the results show convincingly that a village-specific price premium conditional on $\bar{\mathbb{U}}$ can be highly robust to the presence of outsiders. At the prevention premiums, up to M entrants can be present without any outcome with deforestation belonging to the core, conditional on the core

	Every core outcome, if any, has $d^\star\!=\!0$ (M)		Core non-empty; every core outcome has $d^\star\!=\!0$ and compensates S ($\bar{M}$)	
	$\eta = 1{,}000$	$\eta = 3{,}000$	$\eta = 1{,}000$	$\eta = 3{,}000$
Uniform $p^\star$	2,744 - 264,362	905 - 88,120	27,775 - 1,707,419	9,258 - 569,139
Village-specific $p^\star$	626 - 15,261	194 - 4,918	2,470 - 357,614	823 - 119,204

Table EC.5 Range across villages of the entrant bounds M and $\bar{M}$, with price premiums calculated assuming no entrants. The first two columns report M, the maximum number of entrants for which every core outcome, if any, has no deforestation. The last two columns report $\bar{M}$, the maximum number for which the core is non-empty, every core outcome has no deforestation, and every local in the community S is compensated.

being non-empty. At the compensation premiums, the stronger bound $\bar{M}$ additionally guarantees core non-emptiness, no deforestation in every core outcome, and compensation of every local in the community S.

EC.7.4.3. Robustness with respect to village size. Our analysis in §EC.7 considers all villages in our dataset, with the census-estimated number of palm-farming households $|\mathcal{L}|$ (Table EC.4) varying from 35 to 295. Although there are many agricultural cooperatives in Indonesia with memberships within these ranges–in particular, the first cooperative to be RSPO certified in 2021 consists of 209 farmers (RSPO 2021)–it is reasonable to expect that having full cooperation would be simpler in smaller villages. So this section repeats our analysis for those villages that have fewer locals (from fewer than 200 to fewer than 50).

Table EC.6 shows the size of the reduced dataset, considering only villages with a smaller number of oil palm farmers. Our full dataset includes 58 villages with 391 observations. As we restrict the maximum number of oil palm farmers in the villages, we reduce the total number of observations. We observe only 3 villages with fewer than 50 oil palm farmers, and for these three villages, we have only 18 observations. This reduced number of observations reduces the ability to generalize results drawn from these smaller datasets.

max $\lvert\mathcal{L}\rvert$	**200**	**150**	**100**	**50**
number of villages	47	29	9	3
number of observations	313	194	56	18

Table EC.6 Number of villages in our dataset with fewer than 200, 150, 100, and 50 palm farmers and the total number of observations across all of these villages.

Table EC.7 and Figures EC.13-EC.15 summarize the results, namely the minimum uniform price premiums p^* required to prevent deforestation (and compensate in the cases of the individual condition $\mathbb{I}$ and the area no-deforestation condition $\bar{\mathbb{D}}$ with full cooperation) in villages with fewer than 200, 150, 100, and 50 oil palm farmers.

We can see that when filtering based on fewer than 200 or fewer than 150 locals, respectively, the minimum price premiums are identical, and almost the same as in the full dataset. The only difference is that for the area No-Use condition $\bar{\mathbb{U}}$, when locals have limited cooperation, the price

max $\vert\mathcal{L}\vert$	**200**	**150**	**100**	**50**
min p^*, $\mathbb{I}$ (USD/tFFB)	4863	4863	4863	2494
min p^*, $\bar{\mathbb{D}}$, full cooperation (USD/tFFB)	800	800	561	434
min p^*, $\bar{\mathbb{U}}$, $\eta = 1$USD, limited cooperation (USD/tFFB)	471	471	334	185
min p^*, $\bar{\mathbb{U}}$, $\eta = 3000$USD limited cooperation (USD/tFFB)	2022	2022	935	427

Table EC.7 Minimum homogeneous prices that prevent deforestation under each condition and ability to cooperate (where $\bar{\pi} = \{\{\ell\} : \ell \in \mathcal{L}\}$ when locals have limited cooperation), considering only villages with fewer than 200, 150, 100, and 50 oil palm farmers. Under the individual condition $\mathbb{I}$ and the area no-deforestation condition $\bar{\mathbb{D}}$, these prices prevent deforestation with compensation as well.

premium increases faster for higher values of η in the full dataset, reaching 2496 USD at $\eta = 3000$, as opposed to the 2022 USD for the reduced dataset.

Considering only villages with fewer than 100 locals, the minimum price premium to prevent deforestation (and compensate) under $\mathbb{I}$ is the same as in the full dataset ($4,863$USD). The minimum prices to prevent deforestation under $\bar{\mathbb{U}}$ and to compensate under $\bar{\mathbb{D}}$ and full cooperation are lower. Although the values are expected to be lower (we are considering a subset of the villages), they are still of the same order of magnitude as those obtained from the full dataset. Moreover, we consistently observe the same relationship between the prices for $\mathbb{I}$ and both area conditions; to prevent deforestation, both area conditions are significantly lower than the individual, and for low blocking costs (below 1130USD) $\bar{\mathbb{U}}$ with limited cooperation (and $\bar{\pi} = \{\{\ell\} : \ell \in \mathcal{L}\}$) results in lower price premiums, while to compensate, $\bar{\mathbb{D}}$ with full cooperation results in the lowest price premium.

Considering only villages with fewer than 50 locals reduces the dataset to only 3 villages and 18 observations. Nevertheless, the main observations from the whole dataset are preserved, although the estimated minimum prices are lower. In particular, we see that for $\eta < 3050$USD, $\bar{\mathbb{U}}$ results in lower price premiums to prevent deforestation when locals have limited cooperation (and $\bar{\pi} = \{\{\ell\} : \ell \in \mathcal{L}\}$). It is, nevertheless, hard to draw conclusions from this reduced dataset, as it includes a very small number of villages and observations.

We conclude then that our main observations in §EC.7 are robust to considering only villages with a small number of locals.

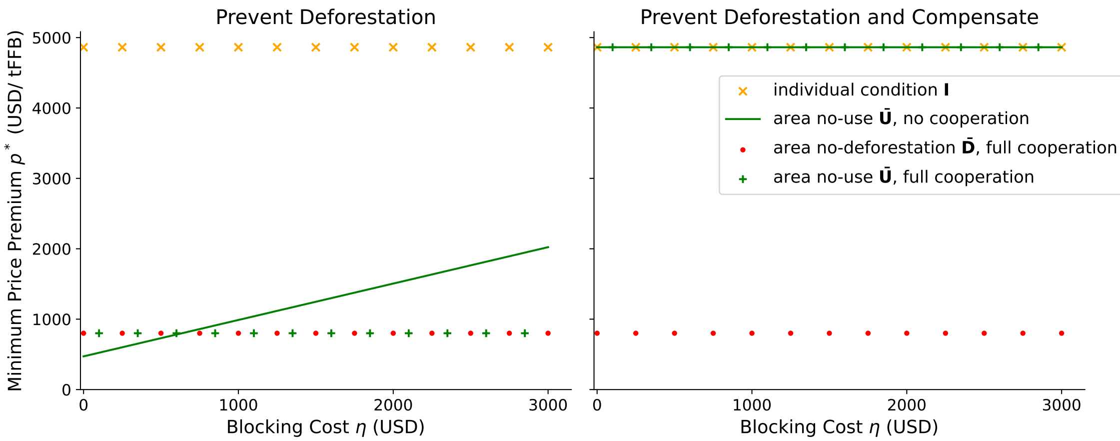

Figure EC.13 Minimum uniform price premium to prevent deforestation (left) and compensate (right) in all villages with fewer than 150 oil palm farmers (the same prices apply when considering all villages with fewer than 200 oil palm farmers).

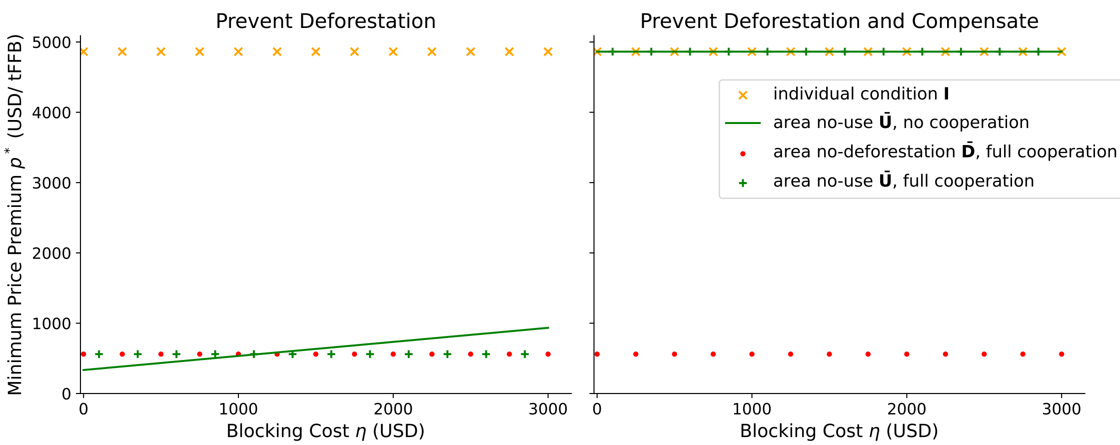

Figure EC.14 Minimum uniform price premium to prevent deforestation (left) and compensate (right) in all villages with fewer than 100 oil palm farmers.

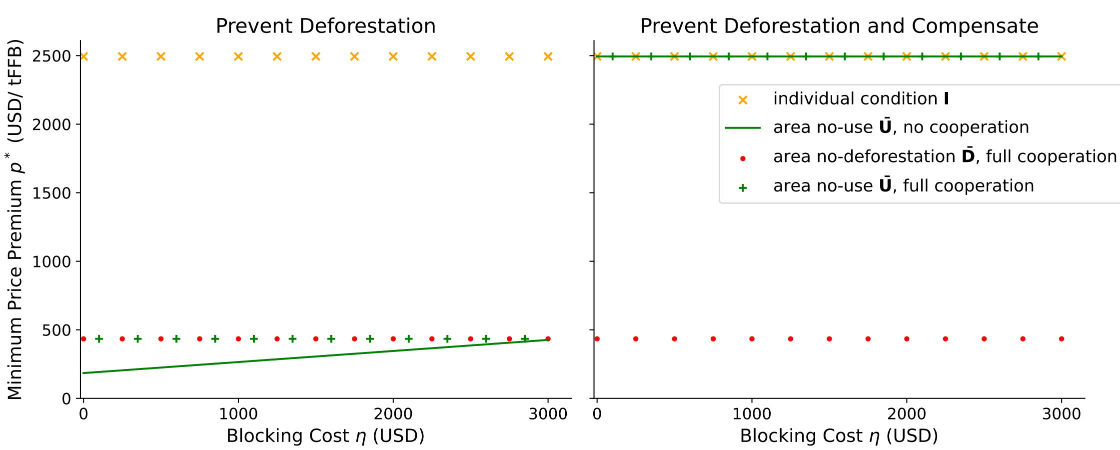

Figure EC.15 Minimum uniform price premium to prevent deforestation (left) and compensate (right) in all villages with fewer than 50 oil palm farmers.

EC.7.5. Technical Supplement

The results in §EC.7.3 implicitly assumed that the surveyed locals in each village represent *all* palm-farmers in that village. We show here that our results are consistent, provided the surveyed sample is representative. In particular, we show that given any problem instance, a condition prevents deforestation (and compensates all locals) in this instance if and only if it does so in the instance arising from duplicating the original instance k times (for any positive integer k). Throughout this subsection, "prevents deforestation" means that the core is non-empty and every core outcome has $d^\star = 0$, and "compensates" means that, in addition, every core outcome compensates; under $\bar{\mathbb{U}}$ we establish the compensation claim for the perfect-incentive criterion used in §EC.7.3, i.e., $\mathcal{G} = \mathcal{L}$.

Proposition EC.11. *Consider a problem instance characterized by $\mathcal{L} = \{1, 2, \ldots, |\mathcal{L}|\}$, $\bar{\pi}$, and given $\{I_\ell\}_{\ell\in\mathcal{L}}$, $\{D_\ell\}_{\ell\in\mathcal{L}}$. For a positive integer k, consider a new problem instance obtained by duplicating the original instance k times, that is, where the set of locals is $\tilde{\mathcal{L}} := \{1, 2, \ldots, k\cdot|\mathcal{L}|\}$, the partition into families is $\tilde{\bar{\pi}} = \left\{\bigcup_{i\in\{1,\ldots,k\}} F_i : F \in \bar{\pi}\right\}$, where $F_i = \{j + (i-1)\cdot|\mathcal{L}| : j \in F\}$ for each $i \in \{1, \ldots, k\}$, and the values from the incentives and from deforestation are $\tilde{I}_h = I_{1+((h-1) \bmod |\mathcal{L}|)}$, $\tilde{D}_h = D_{1+((h-1) \bmod |\mathcal{L}|)}$ for all $h \in \tilde{\mathcal{L}}$.*

(i) The individual condition $\mathbb{I}$ prevents deforestation *(*and compensates *all locals, respectively) for the problem instance with $\mathcal{L}$, $\bar{\pi}$ and $\{I_\ell\}_{\ell\in\mathcal{L}}$, $\{D_\ell\}_{\ell\in\mathcal{L}}$ if and only if it prevents deforestation (*and compensates *all locals, respectively) for the problem instance with $\tilde{\mathcal{L}}$, $\tilde{\bar{\pi}}$, $\{\tilde{I}_i\}_{i\in\tilde{\mathcal{L}}}$, and $\{\tilde{D}_i\}_{i\in\tilde{\mathcal{L}}}$.*

(ii) The area No-Deforestation condition $\bar{\mathbb{D}}$ prevents deforestation *(and* compensates *all locals, respectively) for the problem instance with $\mathcal{L}$, $\bar{\pi}$, $\{I_\ell\}_{\ell\in\mathcal{L}}$, and $\{D_\ell\}_{\ell\in\mathcal{L}}$ if and only if it prevents deforestation (*and compensates *all locals, respectively) for the problem instance with $\tilde{\mathcal{L}}$, $\tilde{\bar{\pi}}$, $\{\tilde{I}_i\}_{i\in\tilde{\mathcal{L}}}$, and $\{\tilde{D}_i\}_{i\in\tilde{\mathcal{L}}}$.*

(iii) Suppose every family has homogeneous preferences, $F \subseteq \mathcal{G}$ or $F \subseteq \mathcal{L}\setminus\mathcal{G}$ for every $F \in \bar{\pi}$ – as in the two cases in which this section applies the result: singleton families, and a perfect incentive ($\mathcal{G} = \mathcal{L}$). The area No-Use condition $\bar{\mathbb{U}}$ prevents deforestation *(and* compensates *all locals, respectively) for the problem instance with $\mathcal{L}$, $\bar{\pi}$, $\{I_\ell\}_{\ell\in\mathcal{L}}$, $\{D_\ell\}_{\ell\in\mathcal{L}}$, and a blocking cost of η if and only if it prevents deforestation (*and compensates all locals *all locals, respectively) for the problem instance with $\tilde{\mathcal{L}}$, $\tilde{\bar{\pi}}$ and $\{\tilde{I}_i\}_{i\in\tilde{\mathcal{L}}}$, $\{\tilde{D}_i\}_{i\in\tilde{\mathcal{L}}}$, and a blocking cost of η.*

Proof: (i) As discussed in §3.1, the individual condition $\mathbb{I}$ prevents deforestation (and also *compensates* all locals) if and only if $I_\ell > D_\ell, \forall \ell \in \mathcal{L}$, which holds if and only if $I_{\ell^i} > D_{\ell^i}, \forall \ell^i \in \tilde{\mathcal{L}}$.

(ii) By Theorem 1-(B,C), every core outcome under $\bar{\mathbb{D}}$ has $d^\star = 0$ if and only if $\bar{\pi} = \{\mathcal{L}\}$ and $I(\mathcal{L}) > D(\mathcal{L})$; because the core under $\bar{\mathbb{D}}$ is always non-empty and all its outcomes with $d^\star = 0$ compensate, this is also the characterization of preventing deforestation with compensation. This occurs if and only if $\tilde{\bar{\pi}} = \{\tilde{\mathcal{L}}\}$ and $\tilde{I}(\tilde{\mathcal{L}}) > \tilde{D}(\tilde{\mathcal{L}})$, which gives the desired result.

(iii) Because every family has homogeneous preferences, the core is non-empty at every blocking cost (see the discussion following Theorem 2), and duplication preserves homogeneity (each $\tilde{F}$ consists of k copies of the members of F), so the core of the duplicated instance is non-empty as well. Given a non-empty core, Theorem 2 shows that every core outcome under $\bar{\mathbb{U}}$ has $d^\star\!=\!0$ – with full cooperation if and only if $I(\mathcal{L}) > D(\mathcal{L})$ (parts A-i, B and C), and with limited cooperation if and only if $\eta < \underline{\eta}_1$ defined in (9) (parts A-ii,iii and B, C) – that is, $\bar{\mathbb{U}}$ prevents deforestation in the problem instance with $\mathcal{L}$, $\bar{\pi}$, $\{(I_\ell, D_\ell)\}_{\ell\in\mathcal{L}}$ if and only if

$$(I(S) - D(S)) > \eta\cdot(|\mathcal{L}| - |S|) \text{ for some } S \subseteq F \text{ and } F \in \bar{\pi} \text{ such that } I(F) > D(F). \tag{EC.103}$$

(With $\bar{\pi} = \{\mathcal{L}\}$, (EC.103) reduces to $I(\mathcal{L}) > D(\mathcal{L})$, taken at $S = F = \mathcal{L}$; with $\bar{\pi} \neq \{\mathcal{L}\}$, it states $\eta < \underline{\eta}_1$.) Similarly, $\bar{\mathbb{U}}$ prevents deforestation for the instance with $\tilde{\mathcal{L}}, \tilde{\bar{\pi}}$, $\{\tilde{I}_i\}_{i\in\tilde{\mathcal{L}}}$, and $\{\tilde{D}_i\}_{i\in\tilde{\mathcal{L}}}$ if and only if

$$\tilde{I}(\tilde{S}) - \tilde{D}(\tilde{S}) > \eta\cdot(|\tilde{\mathcal{L}}| - |\tilde{S}|) \text{ for some } \tilde{S} \subseteq \tilde{F}, \text{ and } \tilde{F} \in \tilde{\bar{\pi}}, \text{ such that } \tilde{I}(\tilde{F}) > \tilde{D}(\tilde{F}). \tag{EC.104}$$

We will show that (EC.103) and (EC.104) are equivalent. That (EC.103) implies (EC.104) is immediate, simply by taking the set $\tilde{S} = \{\ell + n\cdot|\mathcal{L}| : \ell \in S, n \in \{0,\ldots,k-1\}\} \subseteq \tilde{\mathcal{L}}$. To show that (EC.104) implies (EC.103), we first argue that if (EC.104) holds for some $\tilde{F} \in \tilde{\bar{\pi}}$ and $\tilde{S} \subseteq \tilde{F}$, it must hold for $\tilde{F}$ and $\tilde{H} = \{\ell + n\cdot|\mathcal{L}| : \ell \in H, n \in \{0,\ldots,k-1\}\}$ where $H = \{\ell \in F : (I_\ell - D_\ell) > -\eta\}$ and $F = \{i \in \tilde{F} : i \leq |\mathcal{L}|\}$, that is, F is the family in $\bar{\pi}$ that scales to $\tilde{F}$, and H is the subset in F that scales to $\tilde{H}$. From this, it is immediate that H must satisfy (EC.103).

Assume $\tilde{S} \not\subseteq \tilde{H}$, consider then $f \in \tilde{S}\setminus\tilde{H}$. We have that $(\tilde{I}(\tilde{S}\setminus\{f\}) - \tilde{D}(\tilde{S}\setminus\{f\})) > \eta\cdot(|\tilde{\mathcal{L}}| - |\tilde{S}|) - (\tilde{I}_f - \tilde{D}_f) \geq \eta\cdot(|\tilde{\mathcal{L}}| - |\tilde{S}\setminus\{f\}|)$, where the last inequality comes from $f \notin \tilde{H}$, which implies that $\tilde{D}_f \geq \tilde{I}_f + \eta$. But then, $\tilde{S}\setminus\{f\}$ satisfies (EC.104) as well. We can thus assume that $\tilde{S} \subseteq \tilde{H}$.

If $\tilde{S} \subset \tilde{H}$, consider $f \in \tilde{H}\setminus\tilde{S}$. Because $f \in \tilde{H}$ and $\tilde{S}$ satisfies (EC.104), then $(\tilde{I}(\tilde{S}) - \tilde{D}(\tilde{S})) + (\tilde{I}_f - \tilde{D}_f) > \eta\cdot(|\tilde{\mathcal{L}}| - |\tilde{S}|) + (\tilde{I}_f - \tilde{D}_f) > \eta\cdot(|\tilde{\mathcal{L}}| - |\tilde{S}\cup\{f\}|)$. Therefore, any $f \in \tilde{H}\setminus\tilde{S}$ can be included and (EC.104) would still hold, showing that $\tilde{H}$ must satisfy (EC.104).

Finally, consider compensation under the perfect-incentive criterion $\mathcal{G} = \mathcal{L}$. By Theorem 2-(B), when $I_\ell > D_\ell$ for every $\ell \in \mathcal{L}$, the core is non-empty, every core outcome with $d^\star\!=\!0$ compensates, and every core outcome has $d^\star\!=\!0$ if and only if $\bar{\pi} = \{\mathcal{L}\}$ or $\eta < \underline{\eta}_1$ – equivalently, if and only if (EC.103) holds. Hence $\bar{\mathbb{U}}$ prevents deforestation with compensation for the instance with $\mathcal{L}$, $\bar{\pi}$ and $\{(I_\ell, D_\ell)\}_{\ell\in\mathcal{L}}$ if and only if (EC.103) holds and $I_\ell > D_\ell$ for every $\ell \in \mathcal{L}$, which is equivalent to (EC.104) and $\tilde{I}_i > \tilde{D}_i$ for every $i \in \tilde{\mathcal{L}}$, implying the desired result. □

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